## Chapter 4

## Electric Current and Resistance

### 4.1 The Important Stuff

### 4.1.1 Electric Current

The results of the last three chapters (particularly those involving conductors) apply to the special case that electric charges are not in motion, the electrostatic case. For that case, all the points of a single conductor were at the same potential and the electric field was zero within the material of the conductor.

In certain situations we can maintain the motion of charges through a conductor, as when we connect a battery across the ends of a wire. In that case electric charge (negative charge, as it turns out) moves through the wire and there will be potential differences between the points of the conductor.

Even though it is the electrons in the material which move it turns out that it makes no difference if we think of positive charges moving in the opposite direction, and that is how we will think of current.

For a long conductor the fact that electric charge doesn't build up anywhere implies that the amount of charge per time passing any point is the same. For clarity, we can imagine a plane cutting through a conductor as shown in Fig. 4.1. We imagine counting the charge per time which crosses this plane, and the amount of charge per time is the current $I$,

$$
\begin{equation*}
I=\frac{\Delta q}{\Delta t} \tag{4.1}
\end{equation*}
$$

Electric current (as we will use it) is a scalar and from Eq. 4.1 must have units of C/s. We define this combination of units to be the ampere,

$$
\begin{equation*}
1 \text { ampere }=1 \mathrm{~A}=1 \frac{\mathrm{C}}{\mathrm{~s}} \tag{4.2}
\end{equation*}
$$



Figure 4.1: Electric current: Charge per time passing through a cross-section of a wire.

Electric current can be related to the number density of electrons in a conductor and the speed with which they move by:

$$
\begin{equation*}
I=n q v_{d} A \tag{4.3}
\end{equation*}
$$

where $n$ is the number density of charge carriers (electrons, usually), $q$ is the value of their charge, $v_{d}$ is the drift velocity, the speed with which the carriers actually move in the wire (on average) and $A$ is the cross-sectional area of the wire.

### 4.1.2 Ohm's Law

For many substances it is found that the current flowing through a wire made of the material is proportional to the potential difference across its ends: $I \propto V$. We write this relation in the following way:

$$
\begin{equation*}
\frac{V}{I}=R \quad \text { or } \quad V=I R \tag{4.4}
\end{equation*}
$$

where $R$ is constant which depends on the properties of the wire (its material and its dimensions). $R$ is called the resistance of the wire and relation 4.4 is known as Ohm's law. It is really an empirical relation, i.e. one which does not come directly from the laws of physics but which is obeyed pretty well in the real world and is very useful.

From the relation $R=V / I$ we see that the units of resistance must be $\frac{\mathrm{V}}{\mathrm{A}}$. This combination of units is called an ohm:

$$
\begin{equation*}
1 \mathrm{ohm}=1 \Omega=1 \frac{\mathrm{~V}}{\mathrm{~A}} \tag{4.5}
\end{equation*}
$$

### 4.1.3 Resistance and Resistivity

The resistance of a piece of material depends on the type and shape of the material. If the piece has length $L$ and cross-sectional area $A$, the resistance is

$$
\begin{equation*}
R=\rho \frac{L}{A} \tag{4.6}
\end{equation*}
$$

where $\rho$ is a constant (for a given material at a given temperature) known as the resistivity of the material. Some selected values for $\rho$ are:
$\rho_{\text {Copper }}=1.72 \times 10^{-8} \Omega \cdot \mathrm{~m} \quad \rho_{\text {Aluminum }}=2.82 \times 10^{-8} \Omega \cdot \mathrm{~m} \quad \rho_{\text {Carbon }}=3.5 \times 10^{-5} \Omega \cdot \mathrm{~m}$

The resistivity of a material usually increases with temperature. It generally follows an empirical formula given by:

$$
\begin{equation*}
\rho=\rho_{0}\left[1+\alpha\left(T-T_{0}\right)\right] \tag{4.7}
\end{equation*}
$$

where $\rho$ and $\rho_{0}$ are the resistivities of the material at temperatures $T$ and $T_{0}$, respectively. The constant $\alpha$ is the temperature coefficient of resistivity.

### 4.1.4 Electric Power

As charge moves through the wires of an electric circuit, they lose electric potential energy. (When charge $\Delta q$ moves through a potential difference $V$, it loses $\Delta q V$ of potential energy.) The rate of energy loss is the power $P$ delivered to the circuit elements,

$$
P=\frac{\Delta q V}{\Delta t}=\frac{\Delta q}{\Delta t} V=I V
$$

that is,

$$
\begin{equation*}
P=I V \tag{4.8}
\end{equation*}
$$

Electric power is measured in joules per second, or watts: $1 \frac{\mathrm{~J}}{\mathrm{~s}}=1 \mathrm{~W}$. (We have already met this unit when we considered mechanical work done per unit time in first-semester physics.)

The energy goes into heating the resistor.
Using Ohm's law, $(V=I R$, or $I=V / R)$ we can show that the power deliver to a circuit element of resistance $R$ can also be written as

$$
\begin{equation*}
P=I^{2} R \quad \text { or } \quad P=\frac{V^{2}}{R} \tag{4.9}
\end{equation*}
$$



Figure 4.2: (a) Battery connected to two resistors and a capacitor. (b) Schematic diagram for this circuit.


Figure 4.3: Circuit with two resistors in series.

### 4.1.5 Series and Parallel Circuits

We will now consider circuits which are more complicated than a single battery connected to a single resistor. To make progress we will need to use schematic diagrams as shown in Fig. 4.2. In these diagrams a battery is represented by two parallel lines; the longer line represents the positive end of the battery (the one at the higher potential). A resistor is represented by a zigzag line and a capacitor is represented by two parallel lines.

In any of these circuits, the precise shapes of the wires which connect the elements does matter; we only need to care about the circuit elements and how they are connected to the other elements.

The first kind of circuit we consider is where a battery is connected to two or more resistors which are joined end-to-end. Such a circuit is shown in Fig. 4.3. In this circuit the same current $I$ flows through $R_{1}$ and $R_{2}$ (it has nowhere else to go). From Ohm's law the drops in potential across the two resistors are $I R_{1}$ and $I R_{2}$. The sum of these potential drops must equal $V$, the gain in potential across the leads of the battery. So then:

$$
I R_{1}+I R_{2}=V \quad \Longrightarrow \quad V=I\left(R_{1}+R_{2}\right)=I R_{\text {equiv }}
$$

where the equivalent resistance of the pair is the sum, $R_{1}+R_{2}$.


Figure 4.4: Circuit with two resistors in parallel.

This result generalizes to three or more resistors in series, so we have found that when we have a number of resistors in series, then for the purposes of finding the common current through them we can replace them with the equivalent resistance given by

$$
\begin{equation*}
R_{\mathrm{ser}}=R_{1}+R_{2}+R_{3}+\cdots \tag{4.10}
\end{equation*}
$$

A different arrangement of battery and resistors is shown in Fig. 4.4. Here the end of two resistors are at a comon potential so that the potential drop across the resistors is the same (here, it is $V$, the battery volatge) but the current through each resistor is not the same. If the current through $R_{1}$ is $I_{1}$ and the current through $R_{2}$ is $I_{2}$ then Ohm's law gives

$$
V=I_{1} R_{1}=I_{2} R_{2}
$$

so that

$$
I_{1}=\frac{V}{R_{1}} \quad \text { and } \quad I_{2}=\frac{V}{R_{2}}
$$

Now if the total current which comes out of the battery is $I$, then this current splits into the two branches, so that $I=I_{1}+I_{2}$. Combining these results gives

$$
I=I_{1}+I_{2}=\frac{V}{R_{1}}+\frac{V}{R_{2}}=V\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right)
$$

We can write this as

$$
V=I\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right)^{-1}
$$

Now, this looks like Ohm's law where the equivalent resistance of the parallel resistors is

$$
R_{\text {equiv }}=\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right)^{-1} \quad \text { or } \quad \frac{1}{R_{\text {equiv }}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}
$$

So we have an addition rule for resistors in parallel: The reciprocal of the equivalent resistance is the sum of the reciprocals of the individual resistances. So here it's the reciprocals which
add together. This rule hold for any number of resistors in parallel so we give the rule for the parallel case as:

$$
\begin{equation*}
\frac{1}{R_{\mathrm{par}}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}+\cdots \tag{4.11}
\end{equation*}
$$

### 4.1.6 Kirchhoff's Rules

When analyzing fairly simple circuits the rules for series and parallel resistors along with Ohm's law can give all the currents and potential differences. For more complicated casesfor example networks where there are two or more batteries and resistors with multiple connections - we need more general rules to solve for the currents.

This help is provided by Kirchhoff's Rules. These rules are:
Junction Rule: Consider a place where several wires meet- a junction. The sum of the currents going into this junction equals the sum of currents coming out of this junction.
Loop Rule Consider any loop in the circuit. The sum of potential drops equals the sum of potential rises, or more simply with signs properly given to all potential differences, the sum of potential differences is zero.

To apply Kirchhoff's Rules to a circuit, assign a current (with magnitude and direction) to each branch in the circuit. Then after choosing a particular loop and a direction in which to go around that loop, use the potential differences given by:

- If you go from the - terminal to the + terminal of a battery of voltage $V$, the potential difference is $+V$.
- If you go from the + terminal to the - terminal of a battery of voltage $V$, the potential difference is $-V$.
- If you go across a resistor in the direction of the current $I$, the potential change is $-I R$ (that is, this is a voltage drop).
- If you go across a resistor in the direction opposite that of the current $I$, the potential change is $+I R$ (that is, this is a potential gain).

Adding up the potential differences then gives zero for any loop.

### 4.2 Worked Examples

### 4.2.1 Electric Current

1. A certain conductor has $7.50 \times 10^{28}$ free electrons per cubic meter, a crosssectional area of $4.00 \times 10^{-6} \mathrm{~m}^{2}$, and carries a current of 2.50 A . Find the drift speed of the electrons in the conductor. [SF7 17-2]

Use Eq. 4.3 and solve for $v_{d}$ :

$$
I=n q v_{d} A \quad \Longrightarrow \quad v_{d}=\frac{I}{n q A}
$$

Plug in numbers:

$$
v_{d}=\frac{2.50 \mathrm{C} / \mathrm{s}}{\left(7.50 \times 10^{28} \mathrm{~m}^{-3}\right)\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(4.00 \times 10^{-6} \mathrm{~m}^{2}\right)}=5.2 \times 10^{-5} \frac{\mathrm{~m}}{\mathrm{~s}}
$$

(While this speed may seem implausibly slow, in fact the average drifting motion of the electrons in a wire is slow.)
2. In a particular television picture tube, the measured beam current is $60.0 \mu \mathrm{~A}$. How many electrons strike the screen every second? [SF7 17-4]

The given current (in amperes, or coulombs per second) gives the charge per unit time. We can use the charge of an electron $\left(1.60 \times 10^{-19} \mathrm{C}\right.$, in absolute value) to convert this to electrons per unit time:

$$
60.0 \mu \mathrm{~A}=\left(60.0 \times 10^{-6} \frac{\mathrm{C}}{\mathrm{~s}}\right)\left(\frac{1 \text { electron }}{\left(1.60 \times 10^{-19} \mathrm{C}\right)}\right)=3.75 \times 10^{14} \frac{\text { electrons }}{\mathrm{s}}
$$

So $3.75 \times 10^{14}$ electrons hit the screen every second.

### 4.2.2 Ohm's Law

3. The filament of a light bulb has a resistance of $580 \Omega$. A voltage of 120 V is connected across the filament. How much current is in the filament? [CJ6 20-3]

Ohm's law relates $V, I$ and $R$; from it, we have $I=V / R$. Plugging in the numbers,

$$
I=\frac{V}{R}=\frac{(120 \mathrm{~V})}{(580 \Omega)}=0.21 \mathrm{~A}
$$

### 4.2.3 Resistance and Resistivity

4. A cylindrical copper cable carries a current of 1200 A. There is a potential difference of $1.6 \times 10^{-2} \mathrm{~V}$ between two points on the cable that are 0.24 m apart. What is the radius of the cable? [CJ7 20-11]


Figure 4.5: Illustration of Example 4

The problem is diagrammed in Fig. 4.5. We have the current in the cable and the potential difference for two different points, so from Ohm's law the resistance of the part of the cable between those two points is

$$
R=\frac{V}{I}=\frac{\left(1.6 \times 10^{-2} \mathrm{~V}\right)}{(1200 \mathrm{~A})}=1.33 \times 10^{-5} \Omega
$$

Then from Eq. 4.6, knowing $R, L$ and the resistivity of the material (i.e. copper) we can get the cross-sectional area:

$$
R=\rho \frac{L}{A} \quad \Longrightarrow \quad A=\frac{\rho L}{R}
$$

Plug in the numbers:

$$
A=\frac{\left(1.72 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)(0.24 \mathrm{~m})}{\left(1.33 \times 10^{-5} \Omega\right)}=3.10 \times 10^{-4} \mathrm{~m}^{2}
$$

The cable has a circular cross-section so that $A=\pi r^{2}$. Solve for $r$ :

$$
r^{2}=\frac{A}{\pi}=9.87 \times 10^{-5} \mathrm{~m}^{2} \quad \Longrightarrow \quad r=9.93 \times 10^{-3} \mathrm{~m}=9.93 \mathrm{~mm}
$$

5. Calculate the diameter of a $2.0-\mathrm{cm}$ length of tungsten filament in a small lightbulb if its resistance is $0.050 \Omega$. Use $\rho_{\text {Tung }}=5.6 \times 10^{-8} \Omega \cdot \mathrm{~m}$. [SF7 17-13]

We have the resistance of the sample, its length and its resistivity. Use Eq. 4.6 to get the cross-sectional area:

$$
R=\rho \frac{L}{A} \quad \Longrightarrow \quad \Longrightarrow \quad A=\frac{\rho L}{R}
$$

Use the given values:

$$
A=\frac{\left.5.6 \times 10^{-8} \Omega \cdot \mathrm{~m}\right)\left(2.0 \times 10^{-2} \mathrm{~m}\right)}{(0.050 \Omega)}=2.2 \times 10^{-8} \mathrm{~m}^{2}
$$

Then since $A=\pi r^{2}$ the radius is

$$
r^{2}=\frac{A}{\pi}=\frac{\left(2.2 \times 10^{-8} \mathrm{~m}^{2}\right)}{\pi}=7.1 \times 10^{-9} \mathrm{~m}^{2} \quad \Longrightarrow \quad r=8.4 \times 10^{-5} \mathrm{~m}
$$

and the diameter of the wire is

$$
d=2 r=1.7 \times 10^{-4} \mathrm{~m}=0.17 \mathrm{~mm}
$$

### 4.2.4 Electric Power

6. The heating element in an iron has a resistance of $24 \Omega$. The iron is plugged into a $120-\mathrm{V}$ outlet. What is the power delivered to the iron? [CJ7 20-21]
[We need to bend the rules a bit here; actually a wall outlet delivers an alternating voltage, not the constant voltage that we use through this chapter. It turns out that if we treat the given voltage value as constant we do get the right answer.]

Here we are given the potential drop across the resistor (i.e. the iron) and its resistance so that we can one of the equations from 4.9 to get

$$
P=\frac{V^{2}}{R}=\frac{(120 \mathrm{~V})^{2}}{(24 \Omega)}=600 \mathrm{~W}
$$

### 4.2.5 Resistors in Series and in Parallel

7. A $36.0-\Omega$ resistor and a $18.0-\Omega$ resistor are connected in series across a $15.0-\mathrm{V}$ battery. What is the voltage across (a) the $36.0-\Omega$ resistor and (b) the $18.0-\Omega$ resistor? [CJ7 20-41]

The circuit is shown in Fig. 4.6. As the resistors are in series, the equivalent resisance is the sum of the two:

$$
\left.R_{\text {equiv }}=R_{1}+R_{2}=36.0 \Omega+18.0 \Omega=54.0 \Omega\right\}
$$

and with this value, Ohm's law gives the current $I$ :

$$
V=I R \quad \Longrightarrow \quad I=\frac{V}{R}=\frac{15.0 \mathrm{~V}}{54.0 \Omega}=0.278 \mathrm{~A}
$$



Figure 4.6: Circuit for Example 7.

This is the current which goes through each resistor.
Using Ohm's law we can find the voltage (potential difference) across each resistor:

$$
\begin{aligned}
& V_{36.0}=I R_{1}=(0.278 \mathrm{~A})(36.0 \Omega)=10.0 \mathrm{~V} \\
& V_{18.0}=I R_{2}=(0.278 \mathrm{~A})(18.0 \Omega)=5.00 \mathrm{~V}
\end{aligned}
$$

The sum of the two potential differences is 15.0 V , as it must be since that is the same as potential difference across the terminals of the battery.
8. What resistance must be placed in parallel with a $155-\Omega$ resistor to make the equivalent resistance $115 \Omega$ ? [CJ7 20-48]

Eq. 4.11 gives the equivalent resistance for two resistors in parallel. If one of them is $155 \Omega$ and the equivalent resistance is $115 \Omega$, then

$$
\frac{1}{R_{\mathrm{par}}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}+\cdots \quad \Longrightarrow \quad \frac{1}{115 \Omega}=\frac{1}{155 \Omega}+\frac{1}{R}
$$

Solve for $R$ :

$$
\frac{1}{R}=\frac{1}{115 \Omega}-\frac{1}{155 \Omega}=2.24 \times 10^{-3} \Omega^{-1} \quad \Longrightarrow \quad R=446 \Omega
$$

## 9. Find the equivalent resistance between points $a$ and $b$ in Fig. 4.7. [SF7 18-45]

First we note that the $5.1 \Omega$ and $3.5 \Omega$ resistors are in series The picture shows a bend where they join, but that's irrelevant!) They are equivalent to a single resistor of value

$$
R_{\text {equiv }}=R_{1}+R_{2}=5.1 \Omega+3.5 \Omega=8.6 \Omega
$$

so we can draw a (new) equvalent circuit as shown in Fig. 4.8(a). The new $8.6 \Omega$ resistor is


Figure 4.7: Resistor combination for Example 9.


Figure 4.8: Steps in solving Example 9.
in parallel with the $1.8 \Omega$ resistor, so their resistances combine as given in Eq. 4.11,

$$
\frac{1}{R_{\text {equiv }}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}=\frac{1}{1.8 \Omega}+\frac{1}{8.6 \Omega}=0.672 \Omega^{-1} \quad \Longrightarrow \quad R_{\text {equiv }}=1.5 \Omega
$$

Then replace the parallel resistors in Fig. 4.8(a) with a single $1.5 \Omega$ resistor and we have the circuit shown in Fig. 4.8(b). Here there are three resistors in series so the equivalent resistance is the sum of their values:

$$
R_{\text {equiv }}=2.4 \Omega+1.5 \Omega+3.6 \Omega=7.5 \Omega
$$

So the equivalent resistance between points $a$ and $b$ is $7.5 \Omega$.
10. (a) Find the equivalent resistance between points $a$ and $b$ in Fig. 4.9. (b) Calculate the current in each resistor if a potential difference of 34.0 V is applied between points $a$ and $b$ [SF7 18-5]
(a) First, find the equivalent resistance of the pair of parallel resistors in the center. (We have to start with this; we have no other simple series or parallel combination to start with.)


Figure 4.9: Resistor combination for Example 10.


Figure 4.10: First steps in solving Example 10.

Using Eq.'4.11 we get:

$$
\frac{1}{R_{\text {rmequiv }}}=\frac{1}{7.00 \Omega}+\frac{1}{10.0 \Omega}=0.243 \Omega^{-1} \quad \Longrightarrow \quad R_{\mathrm{equiv}}=4.12 \Omega
$$

so we can replace this pair by a single $4.12 \Omega$ resistor; we now have the equivalent circuit shown in Fig. 4.10.

Now the combination is a simple series circuit and the equivalent resistance is just the sum of the individual resistances:

$$
R_{\text {equiv }}=4.00 \Omega+4.12 \Omega+9.00 \Omega=17.1 \Omega
$$

(b) With the answer to (a) we can get the total current $I$ flowing into the network at $a$ and out at $b$ :

$$
I=\frac{V}{R_{\mathrm{equiv}}}=\frac{(34.0 \mathrm{~V})}{(17.1 \Omega)}=1.99 \mathrm{~A}
$$

This must be the same as the current in the $4.00 \Omega$ and $9.00 \Omega$ resistors.
Using Ohm's law we can find the potential drops across the $4.00 \Omega$ and $9.00 \Omega$ resistors. They are

$$
V_{4.00}=I R=(1.99 \mathrm{~A})(4.00 \Omega)=7.96 \mathrm{~V} \quad \text { and } \quad V_{9.00}=I R=(1.99 \mathrm{~A})(9.00 \Omega)=17.9 \mathrm{~V}
$$

respectively. But the total drop in potential from $a$ to $b$ is 34.0 V so the drop across the resistor pair must be

$$
34.0 \mathrm{~V}-7.96 \mathrm{~V}-17.9 \mathrm{~V}=8.1 \mathrm{~V}
$$

and this is the drop in potential of each resistor in the pair. Using Ohm's law we get the current in each of the resistors:

$$
\begin{aligned}
& I_{7.00}=\frac{V}{R}=\frac{(8.1 \mathrm{~V})}{(7.00 \Omega)}=1.2 \mathrm{~A} \\
& I_{10.0}=\frac{V}{R}=\frac{(8.1 \mathrm{~V})}{(10.0 \Omega)}=0.81 \mathrm{~A}
\end{aligned}
$$

Then the currents are

$$
I_{4.00}=1.99 \mathrm{~A} \quad I_{7.00}=1.2 \mathrm{~A} \quad I_{10.0}=0.81 \mathrm{~A} \quad I_{9.00}=1.99 \mathrm{~A}
$$

