Chapter 2

The Electric Field

2.1 The Important Stuff

2.1.1 The Electric Field

When we solved the longer Coulomb Law problems in the previous chapter we added up the (vector) forces from charges $q_1, q_2, \ldots$ acting on a certain charge Q. Now, each one of these individual forces (and hence the *sum* of those forces) is proportional to the charge Q. If in each of those problems we divided the net force by the charge Q we would get a *force per unit charge* at the location of Q. This quantity (which is a vector, since force is a vector) would depend on the values and locations of the charges $q_1, q_2 \ldots$. This idea is represented in Fig. 2.1.

So, a given configuration of charges $q_1, q_2 \ldots$ gives rise to an **electric field E** such that the force on a charge Q is given by

$$\mathbf{F} = Q\mathbf{E} \ . \tag{2.1}$$

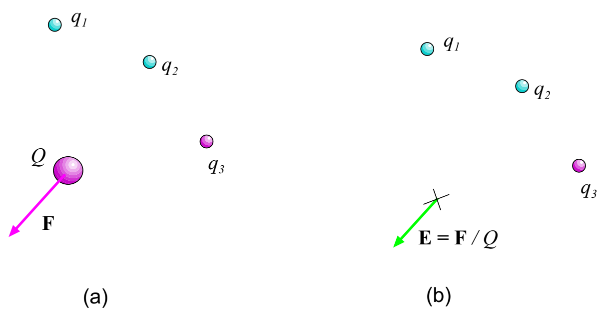

Figure 2.1: (a) Charge Q experiences a force $\mathbf{F}$ from the charges $q_1, q_2 \ldots$. (b) The quantity $\mathbf{E} = \mathbf{F}/Q$ depends only on the charges $q_1, q_2 \ldots$.

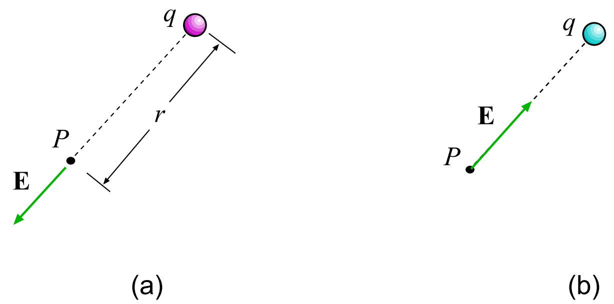

Figure 2.2: (a) Point P is a distance r away from charge q. If q is positive, the electric field points away from q. (b) If q is negative, the electric field points toward q. In both cases the magnitude of $\mathbf{E}$ is given by $E = k|q|/r^2$.

When we use this equation we mean that after we put Q in place all the little charges $q_1, q_2 \ldots$ are in the *same* places they were when we deduced the value of **E** from their values and positions! This will be true in practice if the "test charge" Q is small. Thus we give a practical *definition* of the **E** field as

$$\mathbf{E} = \frac{\mathbf{F}}{Q} \qquad \text{where } Q \text{ is a } \textit{small} \text{ charge} \tag{2.2}$$

From Eq. 2.2 we see that the electric field is a *vector* and has units of N/C.

We note that finding the electric field is more useful than finding the force on a *specific* charge since once we have the E field we simply multiply by the charge Q to get the force, as given by Eq. 2.1.

2.1.2 Finding the Electric Field

It follows from Coulomb's law that at a point which is a distance r from a point charge q, the magnitude of the electric field is

$$E_{\mathrm{pt-ch}} = k\frac{|q|}{r^2} \tag{2.3}$$

and the direction of the field is *away from* q if q is positive and *toward* q if q is negative. This is shown in Fig. 2.2.

When we need to find the electric field due a collection of point charges we find the electric field due to *each* charge and then find the (vector) sum.

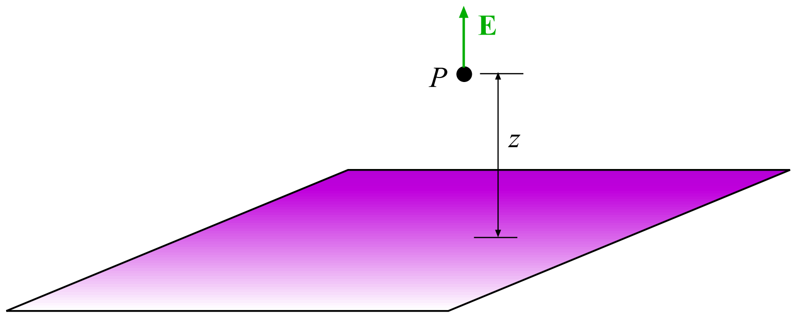

Figure 2.3: Point P is at some distance z above an infinte plane of charge with charge density σ. If σ is positive the E field points away from the sheet and has magnitude $\sigma/(2\epsilon_0)$.

2.1.3 Continuous Distributions; Sheets of Charge

Many charged objects we encounter are not sets of points charges; rather they are continuous distributions of charge. If a two-dimensional region of space contains a charge we can talk about its *charge per unit area*, or **surface charge density**.

Surface charge density is usually given the symbol σ; it has units of $\mathrm{C/m^2}$.

The simplest case of a surface charge is that of an *infinite* planar sheet of charge with uniform charge density σ. We want to know the value of the electric field $\mathbf{E}$ at a point P which is a distance z from the plane; see Fig. 2.3.

It turns out that the answer does not depend on z. If σ is positive, the electric field at P points away from the sheet and has magnitude

$$E_{\mathrm{inf-sh}} = \frac{\sigma}{2\epsilon_0} \quad . \tag{2.4}$$

Here it is easiest to express the result using the constant ϵ_0 introduced in Eq. 1.3

If the sheet has a negative charge density then the field points toward the sheet and the magnitude of the field is $E = |\sigma|/(2\epsilon_0)$.

Next we take the case of the two very large, flat, parallel sheets of charge, as shown in Fig. 2.4. A total charge of $+Q$ has been placed on one sheet and a charge of $-Q$ on the other. We assumed the charge is spread around uniformly so that the charge density of the positively-charge sheet is $\sigma = \frac{Q}{A}$.

This situation can arise when equal and opposite charges are placed on metal plates which are held apart at some distance. (Such a device is called a **parallel–plate capacitor**.) Our approximation is suitable for the case where the plates are separated by a distance which is small compared with the linear size of the plates.

From Eq. 2.4 it follows that the magnitude of the E field between the plates is twice that of the single sheet,

$$E_{\mathrm{inf-sh}} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \quad . \tag{2.5}$$

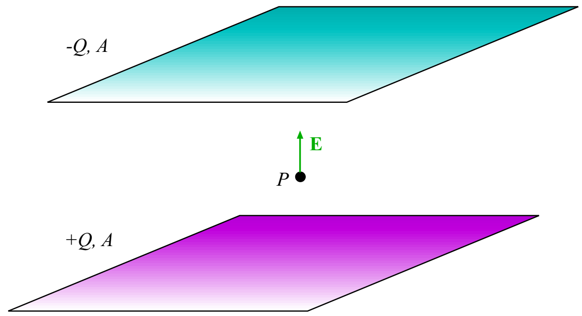

Figure 2.4: Point P is between two very large sheets of charge. On one sheet the total charge is $+Q$ and on the other it is $-Q$. Both sheets have area A. With $\sigma = Q/A$, the electric field between the plates has magnitude σ/ϵ_0.

and the field points from the positive plate to the negative plate.

This equation gives the E field *anywhere* between the plates and it is good as long we can approximate the plates as "very large". Near the edges of the plates it is not a very good approximation.

2.1.4 Electric Field Lines

While the direction of the electric field near a point charge or between two large plates has a simple answer, most charge distributions produce electric fields dependence on position can be hard to visualize.

To help in seeing the direction of the electric at all points we imagine finding the direction of the electric field at all points in space, represented by a little arrow at any point. Then if we join nearby arrows together to form a curve we get an **electric field line**. This is shown in Fig. 2.5 for a (positive) point charge; the field lines start on the charge and go outward. (For a negative charge the field lines would go inward to the charge.)

An interesting and important configuration of charges is the **electric dipole** which consists of two opposite charges $\pm q$ separated by a distance which is usually taken to be "small" in some sense. Near the charge $+q$ the electric field points mainly away from the charge and near the charge $-q$ the field points mainly toward the charge. At other points in space we have to form the sum of the field from the two charges and add. The result is shown in Fig. 2.6.

The mathematics of the electric force gives the following properties of field lines:

- Field lines begin and end *only* on charges; they start on positive charges and end on negative charges.
- Field lines cannot cross one another.

Field lines give us the *direction* of the electric field at any point, but since we have joined the arrows togther to form them, a single field line can't tell the *magnitude* of the E field.

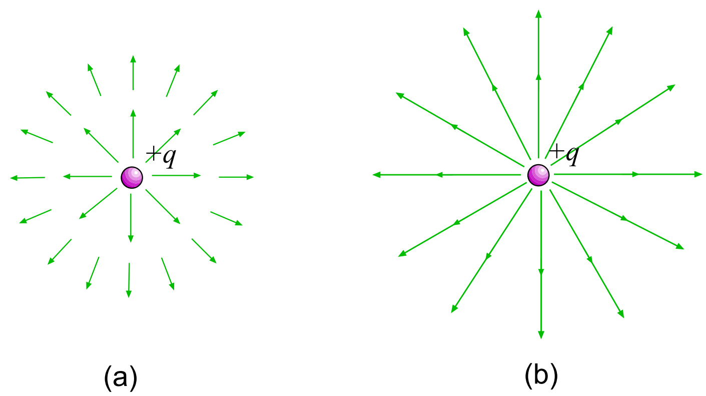

Figure 2.5: (a) A representation of the electric field around a point charge using individual vectors. (b) Representation of the electric field around a point charge using field lines.

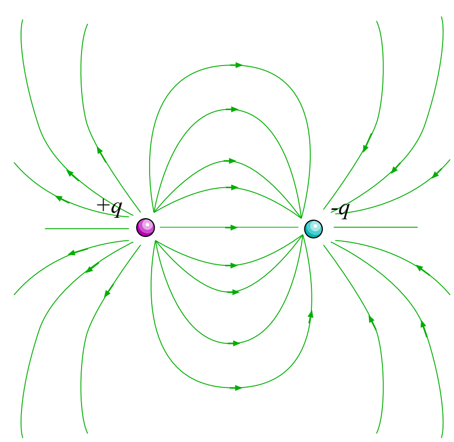

Figure 2.6: Field lines of an electric dipole.

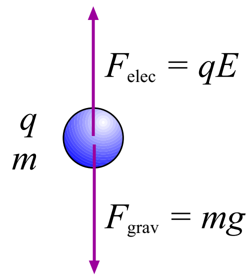

Figure 2.7: Forces on the charged mass in Example 1. The electric force is upward (in the same direction as the E field). The force of gravity is downward.

But the mathematics of the electric force tell us that the number of field lines that we draw originating on a charge should be *proportional* the the size of the charge. If we follow that rule, then the magnitude of the electric field can be judged from the density of field lines at any point. If the lines are closely space, the electric field is strong at that place.

2.1.5 Conductors

In conductors any excess charge is free to move through the material.

2.2 Worked Examples

2.2.1 The Electric Field

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1. An object with a net charge of $24\,\mu\text{C}$ is placed in a uniform electric field of $610\,\frac{\text{N}}{\text{C}}$, directed vertically. What is the mass of the object if it "floats" in the electric field? [SF7 15-17]

The forces acting on this object (of mass m and charge q are shown in Fig. 2.7. The force of gravity has magnitude mg and points downward. The electric force, from Eq. 2.1 has magnitude qE and points upward. (Here the charge q is positive so that the force points in the same direction as the E field.)

The object "floats" so the net force on it must be zero. Hence:

$$qE = mg \qquad \Longrightarrow \qquad m = \frac{qE}{g}$$

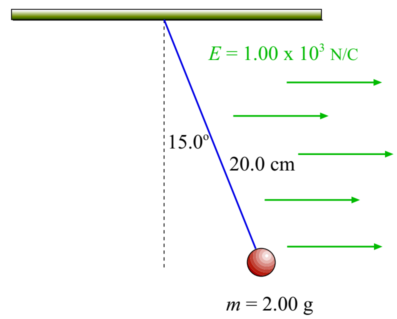

Figure 2.8: Plastic ball suspended in uniform E field, in Example 3.

Plug in the numbers:

$$m = \frac{(24 \times 10^{-6}\,\mathrm{C})(610\,\frac{\mathrm{N}}{\mathrm{C}})}{(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})} = 1.5 \times 10^{-3}\,\mathrm{kg}$$

So the mass of the object is 1.5 grams.

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2. An electric field of $260000\,\frac{\mathrm{N}}{\mathrm{C}}$ points due west at a certain spot. What are the magnitude and direction of the force that acts on a charge of $-7.0\,\mu\mathrm{C}$ at this spot? [CJ6 18-25]

From $\mathbf{F} = q\mathbf{E}$, the *magnitude* of the force is

$$F = |q|E = (7.0\,\mu\mathrm{C})(260000\,\tfrac{\mathrm{N}}{\mathrm{C}}) = (7.0 \times 10^{-6}\,\mathrm{C})(2.60 \times 10^{5}\,\tfrac{\mathrm{N}}{\mathrm{C}}) = 1.8\,\mathrm{N}$$

Since the charge q is *negative* here, the direction of the force is opposite that of the field $\mathbf{E}$, so the force points to the *East*.

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3. A small 2.00-g plastic ball is suspended by a 20.0-cm -long string in a uniform electric field, as shown in Fig. 2.8. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical as indicated, what is the net charge on the ball? [SF7 15-50]

First, make a free–body diagram of the forces acting on the ball. They are: The string tension T directed along the string; the force of gravity, mg, downward; and the electric force which must be parallel to the electric and so here it must point to the *right*. These forces are shown in Fig. 2.9. The magnitude of the electric force is qE, where q is the charge on the plastic ball; this charge must be positive since the force points in the same direction

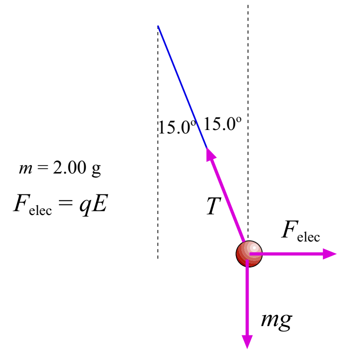

Figure 2.9: Forces acting on the plastic ball in Example 3.

as **E**. The ball is in equilibrium so the (vector) sum of the forces is zero. The condition that the vertical force components sum to zero allows us to find T:

$$T\cos 15.0^\circ - mg = 0 \qquad \Longrightarrow \qquad T = \frac{mg}{\cos 15.0^\circ} = \frac{(2.00\times10^{-3}\,\mathrm{kg})(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})}{(\cos 15.0^\circ)} = 2.03\times10^{-2}\,\mathrm{N}$$

The condition that the horizontal forces sum to zero gives us:

$$-T\sin 15^\circ + F_{\mathrm{elec}} = -T\sin 15^\circ + qE = 0 \qquad \Longrightarrow \qquad q = \frac{T\sin 15^\circ}{E}$$

Plug in the numbers and get:

$$q = \frac{(2.03\times10^{-2}\,\mathrm{N})\sin 15^\circ}{(1.00\times10^{-3}\,\frac{\mathrm{N}}{\mathrm{C}})} = 5.25\times10^{-6}\,\mathrm{C} = 5.25\,\mu\mathrm{C}$$

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4. Each of the protons in a particle beam has a kinetic energy of 3.25×10^{-15} J. What are the magnitude and direction of the electric field that will stop these protons in a distance of $1.25\,\mathrm{m}$? [SF7 15-22]

First, use the proton mass and definition of kinetic to find the initial speed of these protons. With $m_\mathrm{p} = 1.67\times10^{-27}\,\mathrm{kg}$, we find:

$$\mathrm{KE} = \tfrac{1}{2}m_\mathrm{p}v^2 = 3.25\times10^{-15}\,\mathrm{J} \qquad \Longrightarrow \qquad v^2 = \frac{2(3.25\times10^{-15}\,\mathrm{J})}{(1.67\times10^{-27}\,\mathrm{kg})} = 3.89\times10^{12}\,\frac{\mathrm{m}^2}{\mathrm{s}^2}$$

Then:

$$v = 1.97\times10^{6}\,\frac{\mathrm{m}}{\mathrm{s}}$$

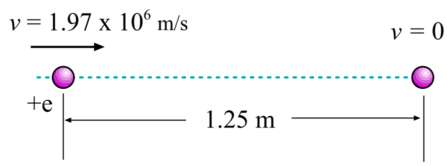

Figure 2.10: Proton slows to a halt in Example 4.

The motion of the proton is as shown in Fig. 2.10. Using our equations of kinematics, we can find the acceleration of the proton:

$$v^2 = v_0^2 + 2ax \qquad \Longrightarrow \qquad a = \frac{v^2 - v_0^2}{2x} = \frac{0 - (1.97 \times 10^6 \, \frac{\mathrm{m}}{\mathrm{s}})^2}{2(1.25\,\mathrm{m})} = -1.56 \times 10^{12} \, \frac{\mathrm{m}}{\mathrm{s}^2}$$

which should be negative since the proton's velocity decreases.

The force on the proton comes from the electric field, as given by Eq. 2.1:

$$F_x = ma_x = qE_x = +eE_x$$

where we've used the fact that a proton's charge is $+e$. Then:

$$E_x = \frac{ma_x}{e} = \frac{(1.67 \times 10^{-27}\,\mathrm{kg})(-1.56 \times 10^{12} \, \frac{\mathrm{m}}{\mathrm{s}^2})}{(1.60 \times 10^{-19}\,\mathrm{C})} = -1.62 \times 10^4 \, \frac{\mathrm{N}}{\mathrm{C}}$$

The electric field has magnitude $1.62 \times 10^4 \, \frac{\mathrm{N}}{\mathrm{C}}$ and points in the $-x$ direction, that is, opposite the initial motion of the proton.

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5. A proton accelerates from rest in a uniform electric field of $640 \, \frac{\mathrm{N}}{\mathrm{C}}$. At some time, its speed is $1.20 \times 10^6 \, \frac{\mathrm{m}}{\mathrm{s}}$. (a) Find the magnitude of the acceleration of the proton. (b) How long does it take the proton to reach this speed? (c) How far has it moved in that interval? (d) What is its kinetic energy at the later time? [SF7 15-23]

(a) The facts given in the problem are diagrammed in Fig. 2.11. If the E field points in the $+x$ direction, then from Eq. 2.1 the force on the proton is

$$F_x = qE_x = +eE_x$$

and the acceleration of the proton is

$$a_x = \frac{F_x}{m_\mathrm{p}} = \frac{eE_x}{m_\mathrm{p}}$$

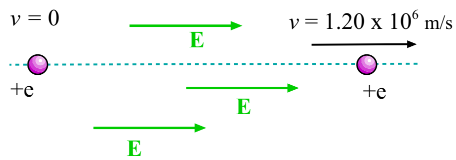

Figure 2.11: Proton is accelerated by E field in Example 5.

Use $m_{\rm p} = 1.67 \times 10^{-27}\,\mathrm{kg}$ and get:

$$a_x = \frac{(1.60 \times 10^{-19}\,\mathrm{C})(640\,\frac{\mathrm{N}}{\mathrm{C}})}{(1.67 \times 10^{-27}\,\mathrm{kg})} = 6.13 \times 10^{10}\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

(b) We have the (constant) acceleration of the proton and its initial and final speeds so using one of our equations from kinematics we can find the distance it traveled:

$$v_x^2 = v_{0x}^2 + 2a_x x \qquad \Longrightarrow \qquad x = \frac{(v_x^2 - v_{0x}^2)}{2a_x}$$

Plug in the numbers:

$$x = \frac{(1.20 \times 10^6\,\frac{\mathrm{m}}{\mathrm{s}})^2 - 0}{2(6.13 \times 10^{10}\,\frac{\mathrm{m}}{\mathrm{s}^2})} = 11.7\,\mathrm{m}$$

(c) Use the definition of kinetic energy, $\mathrm{KE} = \frac{1}{2}mv^2$ and get:

$$\mathrm{KE} = \tfrac{1}{2}(1.67 \times 10^{-27}\,\mathrm{kg})(1.20 \times 10^6)^2 = 1.20 \times 10^{-15}\,\mathrm{J}$$

2.2.2 Finding the Electric Field

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6. Three point charges are aligned along the x-axis as shown in Fig. 2.12. Find the electric field at the position $x = +2.0\,\mathrm{m}$, $y = 0$. [SF7 15-49]

The point at which we want to calculate the E field, $(2.0\,\mathrm{m}, 0)$, lies to the right of all the charges. At that point, the field due to the $-4.0\,\mathrm{nC}$ charge must point to the *left* since it is a negative charge. That charge lies at a distance of $2.50\,\mathrm{m}$ from So x-component of its contribution is

$$E_{1,x} = k\frac{|q_1|}{r_1^2} = (8.99 \times 10^9\,\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}})\frac{(4.0 \times 10^{-9}\,\mathrm{C})}{(2.50\,\mathrm{m})^2} = -5.75\,\frac{\mathrm{N}}{\mathrm{C}}$$

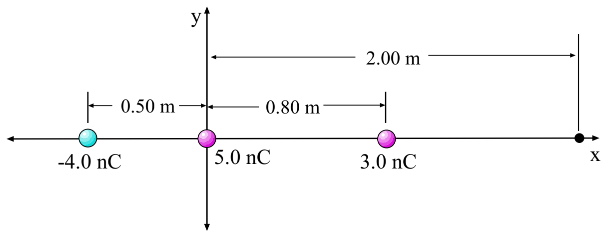

Figure 2.12: Configuration of charges for Example 6.

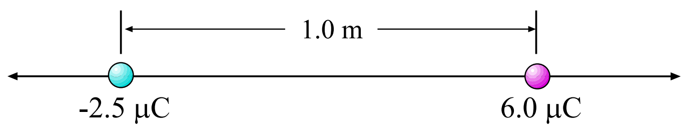

Figure 2.13: Configuration of charges for Example 7.

The field due to the charge at the origin must point to the *right* since it is a positive charge. The x-component of its contribution is

$$E_{2,x} = k\frac{|q_2|}{r_2^2} = (8.99 \times 10^9 \, \tfrac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}) \frac{(5.0 \times 10^{-9}\,\mathrm{C})}{(2.00\,\mathrm{m})^2} = +11.2\,\tfrac{\mathrm{N}}{\mathrm{C}}$$

Finally, the field due to the 3.0 nC charge must also point to the *right* since it is a positive charge. This charge's distance from our "observation" point is 1.20 m, so the x-component of its contribution is

$$E_{3,x} = k\frac{|q_3|}{r_3^2} = (8.99 \times 10^9 \, \tfrac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}) \frac{(3.0 \times 10^{-9}\,\mathrm{C})}{(1.20\,\mathrm{m})^2} = +18.7\,\tfrac{\mathrm{N}}{\mathrm{C}}$$

Add these up, and the total E field at the given point is

$$E_x = -5.75\,\tfrac{\mathrm{N}}{\mathrm{C}} + 11.2\,\tfrac{\mathrm{N}}{\mathrm{C}} + 18.7\,\tfrac{\mathrm{N}}{\mathrm{C}} = +24.1\,\tfrac{\mathrm{N}}{\mathrm{C}}$$

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7. In Fig. 2.13, determine the point (other than infinity) at which the total electric field is zero. [SF7 15-27]

For all points that we consider there will be a (vector) electric field due to the $-2.5\,\mu\mathrm{C}$ charge and one due to the $+6.0\,\mu\mathrm{C}$ charge; we want to find the point at which these vectors add to zero.

It would seem that this point should lie on the line joining the two charges, but do we need to consider points off this line? No, because at points off this axis the two field vectors will point toward or away from the individual charges and at points off the axis those vectors can't be parallel and so can't cancel. So we only need to think about points on the axis.

Could this point lie between the two charges? In that region, the field due to the $-2.5\,\mu\mathrm{C}$ charge will point *toward* that charge (i.e. to the left) and that due to the $+6.0\,\mu\mathrm{C}$ charge will point *away* from that charge (i.e. also to the left). Those vectors can't cancel regardless of their magnitudes, so the point can't lie between the two charges.

How about someplace to the right of both charges? In that region, the $+6.0\,\mu\mathrm{C}$ charge is always closer than the $-2.5\,\mu\mathrm{C}$ charge. That being the case, the field from the $+6.0\,\mu\mathrm{C}$ charge must always have the larger magnitude (charge is bigger and distance is smaller) so again the vectors can't cancel.

The point we want must lie to the left of both charges. In that region, the field due to the $-2.5\,\mu\mathrm{C}$ charge points to the right and that due to the $+6.0\,\mu\mathrm{C}$ charge points to the left. (Note that the $-2.5\,\mu\mathrm{C}$ charge is always *closer* and since it also has a smaller charge, there could be some place where the fields cancel.) If we consider a point which lies at a distance d to the left of the $-2.5\,\mu\mathrm{C}$ charge, then its distance from the $+6.0\,\mu\mathrm{C}$ charge will be $d + 1.0\,\mathrm{m}$, and using Eq. 2.3 the x component of the total field will be

$$E_{x,\mathrm{total}} \;=\; k\frac{(+2.5\,\mu\mathrm{C})}{d^2} - k\frac{(6.0\,\mu\mathrm{C})}{(d+1.0\,\mathrm{m})^2} \;=\; 0 \tag{2.6}$$

It is now just a *math* problem to solve for d. We're done with the *physics*.

First off, we can cancel the constant k in Eq. 2.6 as well as the "μC" units. One trick that will work (unless you've got any better ideas!) is to multiply both sides of Eq. 2.6 by $d^2(d+1.0\,\mathrm{m})^2$. That gives us:

$$d^2(d+1.0\,\mathrm{m})^2\frac{(+2.5)}{d^2} - d^2(d+1.0\,\mathrm{m})^2\frac{(6.0)}{(d+1.0\,\mathrm{m})^2} \;=\; 0$$

Cancel things and get:

$$(d+1.0\,\mathrm{m})^2(2.5) - d^2(6.0) \;=\; 0$$

which you might recognize as a quadratic equation, so that we *can* get an answer. Expand the square:

$$(2.5)(d^2 + (2.0\,\mathrm{m})d + 1.0\,\mathrm{m}^2) - (6.0)d^2 \;=\; 0$$

and ignoring the "m" units symbol for now, multiply and get:

$$2.5d^2 + 5.0d + 2.5 - 6.0d^2 \;=\; -3.5d^2 + 5.0d + 2.5 \;=\; 0$$

or, without the leading minus sign,

$$3.5d^2 - 5.0d - 2.5 \;=\; 0$$

Almost there! Use the quadratic formula to find:

$$\frac{+5.0 \pm \sqrt{25.0 + 35}}{7.0} = 1.8\,\mathrm{m}$$

Here we've considered only the "+" root since the other would give a negative value for d which we assumed was *positive*.

So the point we want is $1.8\,\mathrm{m}$ to the left of the $-2.5\,\mu\mathrm{C}$ charge.