Chapter 1

Electric Charge and Coulomb's Law

1.1 The Important Stuff

1.1.1 Electric Charge

In the latter part of the 18th century it was realized that any sample of matter has a property which is as fundamental as its mass. This property is the **electric charge** of the sample. Electric charge can be detected because it gives rise to electric forces. The reason that we don't see electric phenomena more often than we do is that electric charges come in two types —positive and negative— and usually the two types occur in equal numbers so that they add to give zero net charge. But when we can separate positive and negative charges we observe electric forces on a large scale.

In the SI system, electric charge is measured in **Coulombs**. Throughout our study of electromagnetism we will derive other electrical units based on the Coulomb and the units already encountered in mechanics.

After decades of study of the electrical properties of matter, it was found that the fundamental charges in nature occur in integer multiples of the **elementary charge** e,

$$e = 1.602 \times 10^{-19}\,\mathrm{C} \tag{1.1}$$

In discussing this property of charge we often say that electric charge is **quantized**.

In the atom, the nucleus has a charge which is a multiple of $+e$ while the orbiting electrons each have a charge of $-e$. The charge of the nucleus comes from the constituent protons, each of which has a charge of $+e$; the neutrons in the nucleus have no charge.

1.1.2 Some Facts About Electric Charge

Electric charges can be separated by rubbing, as when you rub a plastic rod with some roadkill; see Fig. 1.1. Then one of the objects will obtain a positive charge and the other

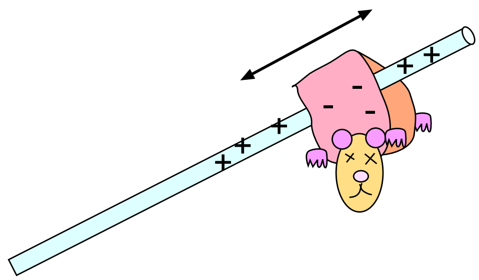

Figure 1.1: Roadkill: Good for separating charges and mighty good eatin'.

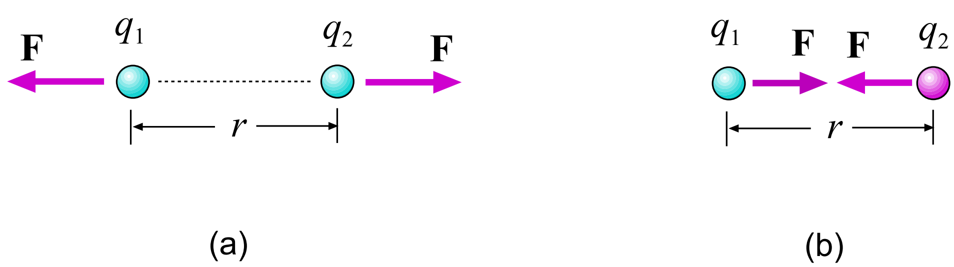

Figure 1.2: (a) Charges q_1 and q_2 have the same sign; the mutual force is repulsive. (b) Charges q_1 and q_2 have opposite signs; the mutual force is attractive.

a negative charge. This occurs because the negatively–charged electrons are *removed* from one object and deposited on the other.

It has been found that in an isolated system the *total* amount of charge stays the same, i.e. total electric charge is *conserved*.

It is also found that electric charges of the same sign (i.e. both positive or both negative) will *repel* and electric charges of opposite sign (i.e. one positive and one negative) will *attract*.

In understanding the behavior of charged objects it is important to understand how charges can move through them. To this end we distinguish objects as being either **conductors** or **insulators**. Excess charge can move freely through a conductor and since like charges repel one another, the charges on a charged conductor will generally move around to space themselves out as much as possible.

In contrast, for insulators excess charge cannot move freely and generally will stay where it is placed.

1.1.3 Coulomb's Law

The force between two small (point) charges is directed along the line which joins the two charges and is repulsive for two charges of the same sign, attractive for two charges of the opposite sign. (See Fig. 1.2. It is proportional to the size of either one of the two charges; finally, it gets weaker as the distance between the charges increases. But the force is not inversely proportional to the distance, it is inversely proportional to the *square* of the distance.

The law for the magnitude of the electric force between two small charges q_1 and q_2 separated by a distance r is

$$F = k\frac{|q_1 q_2|}{r^2} \qquad \text{where} \qquad k = 8.99 \times 10^9\ \tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}} \tag{1.2}$$

This is usually called **Coulomb's law**.

The constant k will come up often in our examples but later on it will be easier to work with the constant ϵ_0, which is related to k by

$$k = \frac{1}{4\pi\epsilon_0}$$

so that ϵ_0 has the value

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \tfrac{\mathrm{C^2}}{\mathrm{N\cdot m^2}} \tag{1.3}$$

The electric force given by Coulomb's law is similar to Newton's law for the gravitational force (from first semeseter) in that both are **inverse–square** laws; the force is inversely proportional to the *square* of the distance between the particles.

If we plug some easy numbers into Eq. 1.2 we find that if two 1.0 C charges are separated by a meter, then each one experiences a repulsive force of about 9.0×10^9 N, which is an *enormous* force. In this sense, 1 C is a *huge* amount of charge; typically the charges which one would encounter in real life are of the order of μC (10^{-6} C) or nC (10^{-9} C).

When a charge Q is in the vicinity of several other charges (q_1, q_2, etc.) the net force on Q is found by adding up the individual forces from the other charges. Of course, this is a *vector* sum of the forces.

1.2 Worked Examples

1.2.1 Electric Charge

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1. How many electrons must you have to get a total charge of -1.0 C? How many moles of electrons is this?

Since each electron has a charge of -1.6×10^{-19} C, the number of electrons required is

$$N = \frac{(-1.0\,\mathrm{C})}{(-1.6 \times 10^{-19}\,\mathrm{C})} = 6.2 \times 10^{18}$$

A *mole* of any kind of particle is $N_{\mathrm{Avo}} = 6.02 \times 10^{23}$ (Avogadro's number) of those particles. Here we have 6.2×10^{18} electrons and that is

$$n = \frac{N}{N_{\mathrm{Avo}}} = \frac{(6.2 \times 10^{18})}{(6.02 \times 10^{23})} = 1.04 \times 10^{-5}\,\mathrm{moles}$$

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2. A metal sphere has a charge of $+8.0\,\mu\mathrm{C}$. What is the net charge after 6.0×10^{13} electrons have been placed on it? [CJ6 15-2]

The total charge of 6.0×10^{13} electrons is

$$Q_{\mathrm{elec}} = (6.0 \times 10^{13})(-e) = (6.0 \times 10^{13})(-1.60 \times 10^{-19}\,\mathrm{C}) = -9.6 \times 10^{-6}\,\mathrm{C} = -9.6\,\mu\mathrm{C}$$

After this charge has been added to the metal sphere its total charge is

$$Q_{\mathrm{sph}} = +8.0\,\mu\mathrm{C} - 9.6\,\mu\mathrm{C} = -1.6\,\mu\mathrm{C}$$

1.2.2 Coulomb's Law

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3. A charge of 4.5×10^{-9} C is located 3.2 m from a charge of -2.8×10^{-9} C. Find the electrostatic force exerted by one charge on another. [SF7 15-1]

This will be a force of *attraction* between the two charges since they are of opposite signs. The magnitude of this force is given by Coulomb's law, Eq. 1.2,

$$\begin{aligned} F &= k\frac{|q_1 q_2|}{r^2} \\ &= (8.99 \times 10^9\,\tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})\frac{(4.5 \times 10^{-9}\,\mathrm{C})(2.8 \times 10^{-9}\,\mathrm{C})}{(3.2\,\mathrm{m})^2} = 1.1 \times 10^{-8}\,\mathrm{N} \end{aligned}$$

The charges will attract one another with a force of magnitude 1.1×10^{-8} N.

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4. An alpha particle (charge=$+2.0e$) is sent at high speed toward a gold nucleus (charge=$+79e$). What is the electrical force acting on the alpha particle when it is 2.0×10^{-14} m from the gold nucleus? [SF7 15-3]

Here both particles are *positively* charged so there is a force of *repulsion* between them. The magnitude of this force of repulsion is given by Coulomb's law,

$$\begin{aligned} F &= k\frac{|q_1 q_2|}{r^2} = k\frac{(2.0)e(79.0)e}{r^2} \\ &= (8.99 \times 10^9\ \tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})\frac{(2.0)(79.0)(1.6 \times 10^{-19}\ \mathrm{C})^2}{(2.0 \times 10^{-14}\ \mathrm{m})^2} = 91.1\ \mathrm{N} \end{aligned}$$

So the alpha particle experiences a (repulsive) force of 91 N from the gold nucleus.

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5. Two identical conducting spheres are placed with their centers $0.30\ \mathrm{m}$ apart. One is given a charge of $12 \times 10^{-9}\ \mathrm{C}$, the other a charge of $-18 \times 10^{-9}\ \mathrm{C}$. (a) Find the electrostatic force exerted on one sphere by the other. (b) The spheres are connected by a conducting wire. Find the electrostatic force between the two after equilibrium is reached. [SF7 15-9]

(a) Use Coulomb's law to find the magnitude of the force, which in this case is attractive since the spheres are oppositely charged:

$$F = k\frac{|q_1 q_2|}{r^2} = (8.99 \times 10^9\ \tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})\frac{(12 \times 10^{-9}\ \mathrm{C})(18 \times 10^{-9}\ \mathrm{C})}{(0.30\ \mathrm{m})^2} = 2.16 \times 10^{-5}\ \mathrm{N}$$

(b) When the (conducting) spheres are connect by a (thin!) conducting wire, the electric charges are free to move between the spheres. The total charge on both spheres is

$$Q_{\mathrm{Tot}} = 12 \times 10^{-9}\ \mathrm{C} - 18 \times 10^{-9}\ \mathrm{C} = -6.0 \times 10^{-9}\ \mathrm{C}$$

and when this charge is free to move between the spheres it will attain an equilibrium when both spheres have the *same* charge. So after the spheres are connected the charge of each is

$$Q = Q_{\mathrm{Tot}}/2 = -3.0 \times 10^{-9}\ \mathrm{C}$$

This is shown in Fig. 1.3.

With the new charges on the spheres, use Coulomb's law to get the magnitude of the force on each:

$$F = k\frac{|q_1 q_2|}{r^2} = (8.99 \times 10^9\ \tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})\frac{(3.0 \times 10^{-9}\ \mathrm{C})(3.0 \times 10^{-9}\ \mathrm{C})}{(0.30\ \mathrm{m})^2} = 8.99 \times 10^{-7}\ \mathrm{N}$$

and now the force is *repulsive* since both spheres are both negatively charged.

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6. Three charges are arranged as shown in Fig. 1.4. Find the magnitude and

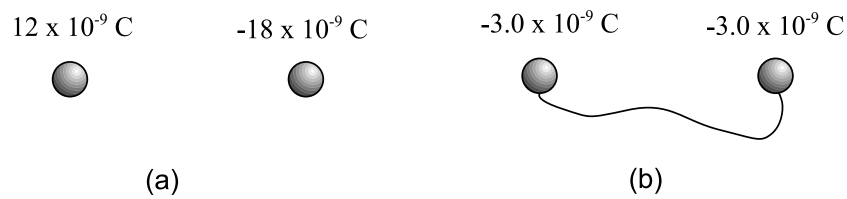

Figure 1.3: (a) Conducting spheres are given different charges. (b) Charges on the spheres after being joined by a conducting wire.

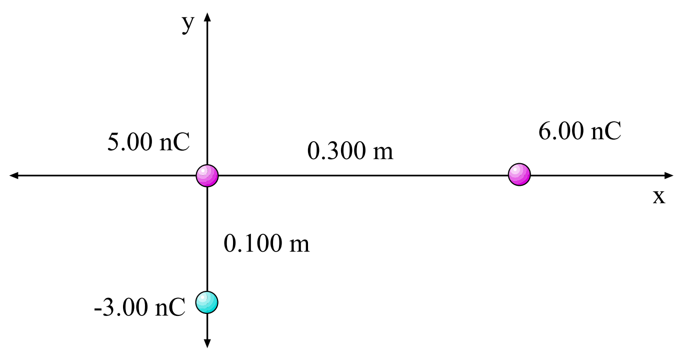

Figure 1.4: Charges in Example 6

direction of the electrostatic force on the charge at the origin. [SF7 15-11]

Let's call the $6.00\,\mathrm{nC}$ charge q_1 and the $-3.00\,\mathrm{nC}$ charge q_2. (The charge at the origin is $Q = +5.00\,\mathrm{nC}$.)

The force from q_1 is *repulsive* and points to the right. The force from q_2 is *attractive* and points downward, as shown in Fig. 1.5. We need to find the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ and then add those two force vectors.

From Coulomb's law we get the magnitude of $\mathbf{F}_1$; since charge q_1 is at a distance $r_1 =$

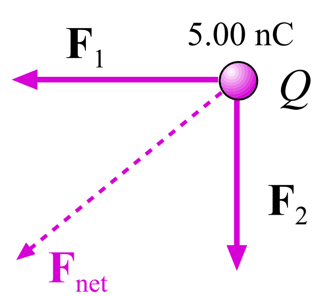

Figure 1.5: Forces on Q in Example 6

0.300 m from Q,

$$\begin{aligned} F_1 &= k\frac{|Qq_1|}{r_1^2} \\ &= (8.99 \times 10^9\ \tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})\frac{(5.00 \times 10^{-9}\ \mathrm{C})(6.00 \times 10^{-9}\ \mathrm{C})}{(0.300\ \mathrm{m})^2} = 3.00 \times 10^{-6}\ \mathrm{N} \end{aligned}$$

Likewise, the magnitude of $\mathbf{F}_2$ is

$$\begin{aligned} F_2 &= k\frac{|Qq_2|}{r_2^2} \\ &= (8.99 \times 10^9\ \tfrac{\mathrm{N\cdot m^2}}{\mathrm{C^2}})\frac{(5.00 \times 10^{-9}\ \mathrm{C})(3.00 \times 10^{-9}\ \mathrm{C})}{(0.100\ \mathrm{m})^2} = 1.35 \times 10^{-5}\ \mathrm{N} \end{aligned}$$

Then the *total* force on Q has the components

$$F_x = -3.00 \times 10^{-6}\ \mathrm{N} \qquad F_y = -1.35 \times 10^{-5}\ \mathrm{N}$$

What is the magnitude and direction of this vector? Its magnitude is

$$F = \sqrt{(-3.00 \times 10^{-6}\ \mathrm{N})^2 + (-1.35 \times 10^{-5}\ \mathrm{N})^2} = 1.38 \times 10^{-5}\ \mathrm{N}$$

and the direction we can find from

$$\theta = \tan^{-1}\frac{(-1.35 \times 10^{-5})}{(-3.00 \times 10^{-6})} = 77.5^\circ - 180^\circ = -103^\circ$$

(Note, we subtract 180° from the simple answer because the direction of the force is in the third quadrant.)

The net force on Q has magnitude 1.38×10^{-5} N and points at an angle of -103° from the $+x$ axis.

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7. Two small metallic spheres, each of mass 0.20 g are suspended as pendulums by light strings from a common point as shown in Fig. 1.6. The spheres are given the same electric charge and it is found that they come to equilibrium when each string is at an angle of 5.0° with the vertical. If each string is 30.0 cm long, what is the magnitude of the charge on each sphere? [SF7 15-15]

From simple trig we can calculate the distance between the two spheres. If this distance is x, then

$$x = 2(30.0\ \mathrm{m})\sin 5.0^\circ = 5.23\ \mathrm{cm} = 5.23 \times 10^{-2}\ \mathrm{m}$$

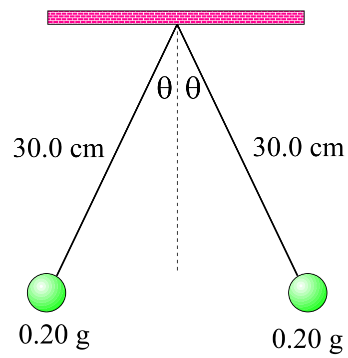

Figure 1.6: Suspended charged spheres in Example 7

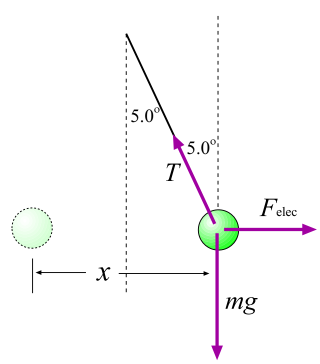

Figure 1.7: Forces acting on a charged sphere in Example 7

Now consider the forces acting on one of the spheres, say the one on the right. These are shown in Fig. 1.7, where we also note (for reference) the location of other sphere. The right sphere experiences a force of electric repulsion from the left sphere. The forces are the force of gravity (mg, downward), the tension of the string (magnitude T; it pulls at an angle 5.0° from the vertical) and the electric repulsive force. From Coulomb's law, the magnitude of the latter is

$$F_{\rm elec} = k\frac{q^2}{x^2}$$

where q is the magnitude of the charge on each sphere.

The sphere is in equilibrium, so the forces must sum to zero. The vertical forces cancel out, giving us:

$$T\cos 5.0^\circ = mg \qquad \Longrightarrow \qquad T = \frac{mg}{\cos 5.0^\circ} = \frac{(2.00\times 10^{-4}\,\mathrm{kg})(9.80\,\frac{\mathrm{m}}{\mathrm{s}^2})}{\cos 5.0^\circ} = 1.97\times 10^{-3}\,\mathrm{N}$$

The horizontal forces cancel out and this gives:

$$T\sin 5.0^\circ = F_{\rm elec} = k\frac{q^2}{x^2}$$

which lets us solve for q:

$$q^2 = \frac{Tx^2\sin 5.0^\circ}{k} = \frac{(1.97\times 10^{-3}\,\mathrm{N})(5.23\times 10^{-2})^2\sin 5.0^\circ}{(8.99\times 10^{9}\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C}^2})} = 5.22\times 10^{-17}\,\mathrm{C}^2$$

so then

$$q = 7.2\times 10^{-9}\,\mathrm{C} = 7.2\,\mathrm{nC}$$