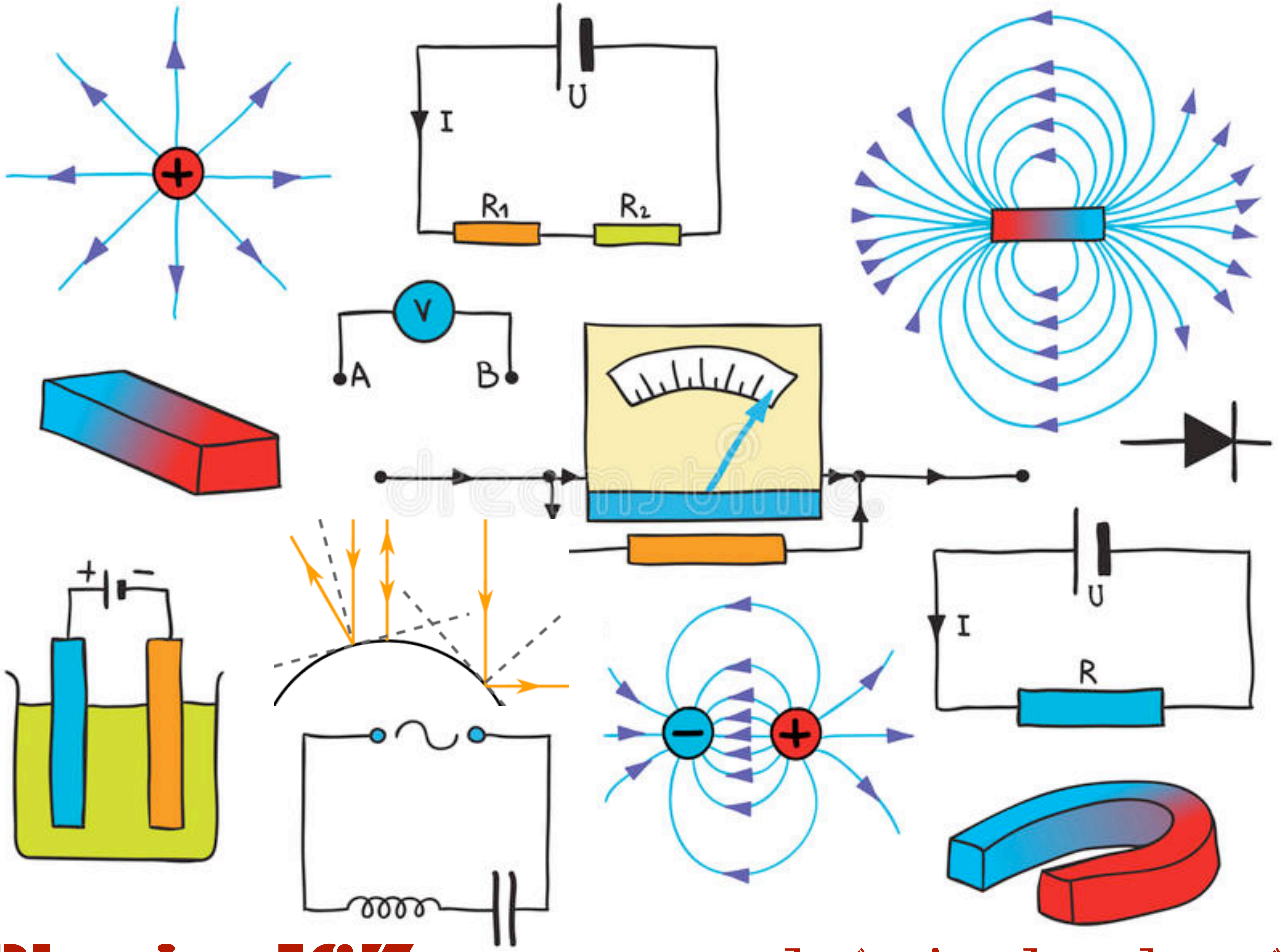


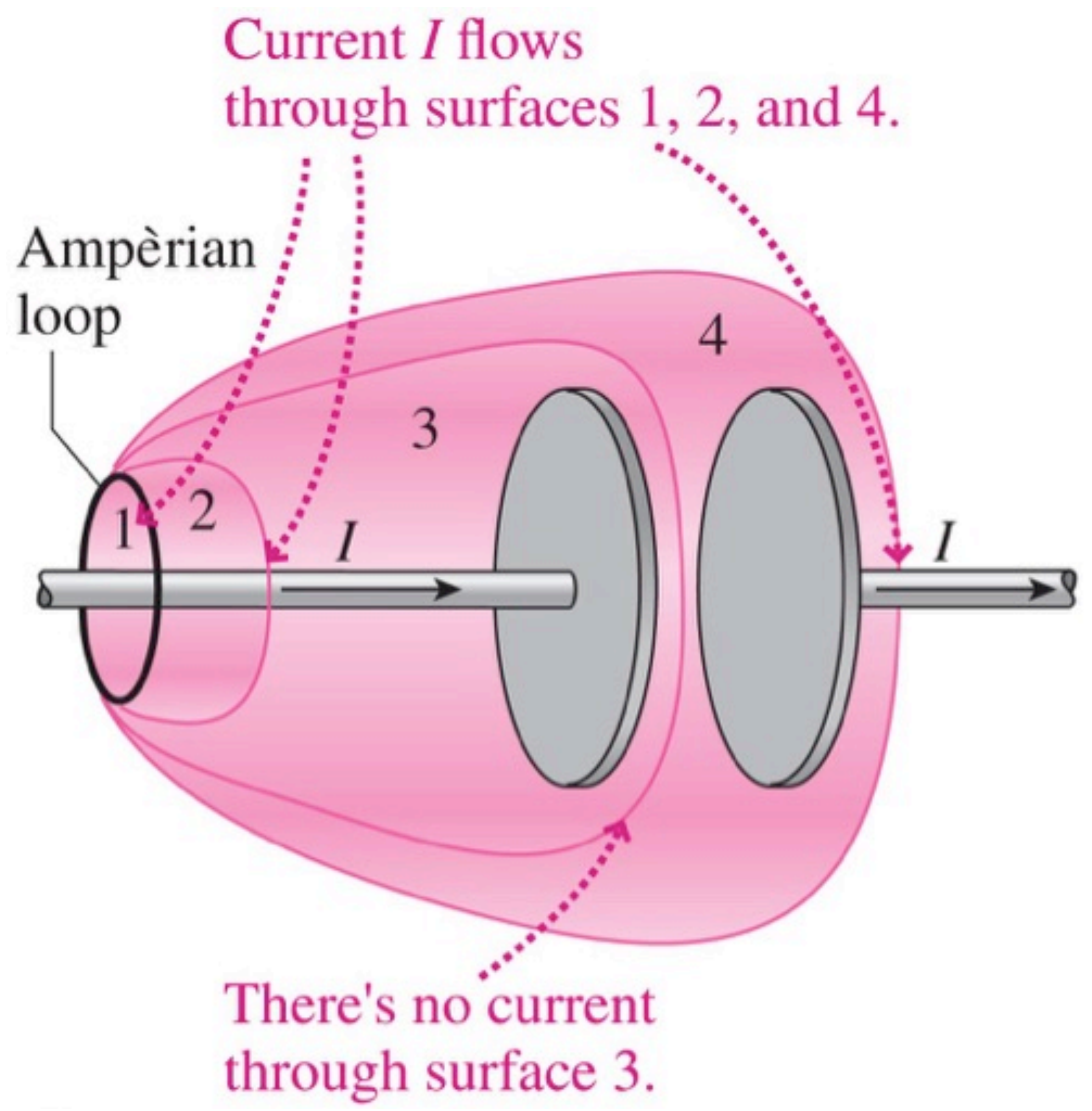
Physics 167

Luis Anchordoqui

Inconsistency in Ampere's Law

➢ Ampere's Law can be written

$$\sum_{\substack{\text{closed}\\\text{path}}} B_{\parallel}\ \Delta s = \mu_0 I_{\text{encl}} = \mu_0 \sum_{\text{surface}} j\ \Delta A$$



where we made explicit relation between current and current density

$$I = \sum_{\text{surface}} j\ \Delta A$$

➢ Note that surface over which sum is evaluated can be **any** open surface bounded by closed Amperian loop

➢ Figure shows a schematic of a parallel plate capacitor being charged 4 possible surfaces are shown bounded by a single Amperian loop

➢ For each of these surfaces the flux of J must give the same current

➢ As indicated, surfaces 1, 2 and 4 are "pierced" by the current I

➢ However, as can be seen, no current passes through surface 3

What to do about this inconsistency ?

Fixing Ampere's Law

➢ Maxwell realized existence of a Displacement Current "flowing" between plates of capacitor, passing through surface 3

➢ Displacement current through surface 3 must be equal to "normal" (conduction) current passing through surface 1

➢ The conduction current through surface 1 can be written as

$$I_d = \sum_{\text{surface}} j\,\Delta A = \frac{\Delta q}{\Delta t} = \epsilon_0 \frac{\Delta \Phi_E}{\Delta t}$$

➢ Including the displacement current inconsistency is removed and Ampere's Law becomes

$$\sum_{\substack{\text{closed}\\\text{path}}} B_{\parallel}\,\Delta s = \mu_0 I_{\text{encl}} + \mu_0 \epsilon_0 \frac{\Delta \Phi_E}{\Delta t}$$

Maxwell's Equations

$$\sum_{\substack{\text{closed}\\\text{surface}}} E_\perp \, \Delta A = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Gauss Law

$$\sum_{\substack{\text{closed}\\\text{surface}}} B_\perp \, \Delta A = 0$$

No magnetic monopoles

$$\sum_{\substack{\text{closed}\\\text{path}}} B_\parallel \, \Delta s = \mu_0 I_{\text{encl}} + \mu_0 \epsilon_0 \frac{\Delta \Phi_E}{\Delta t}$$

Ampere-Maxwell Law

$$\sum E_\parallel \, \Delta s = \mathcal{E} = -\frac{\Delta \Phi_M}{\Delta t}$$

Faraday-Lenz Law

Lorentz Force

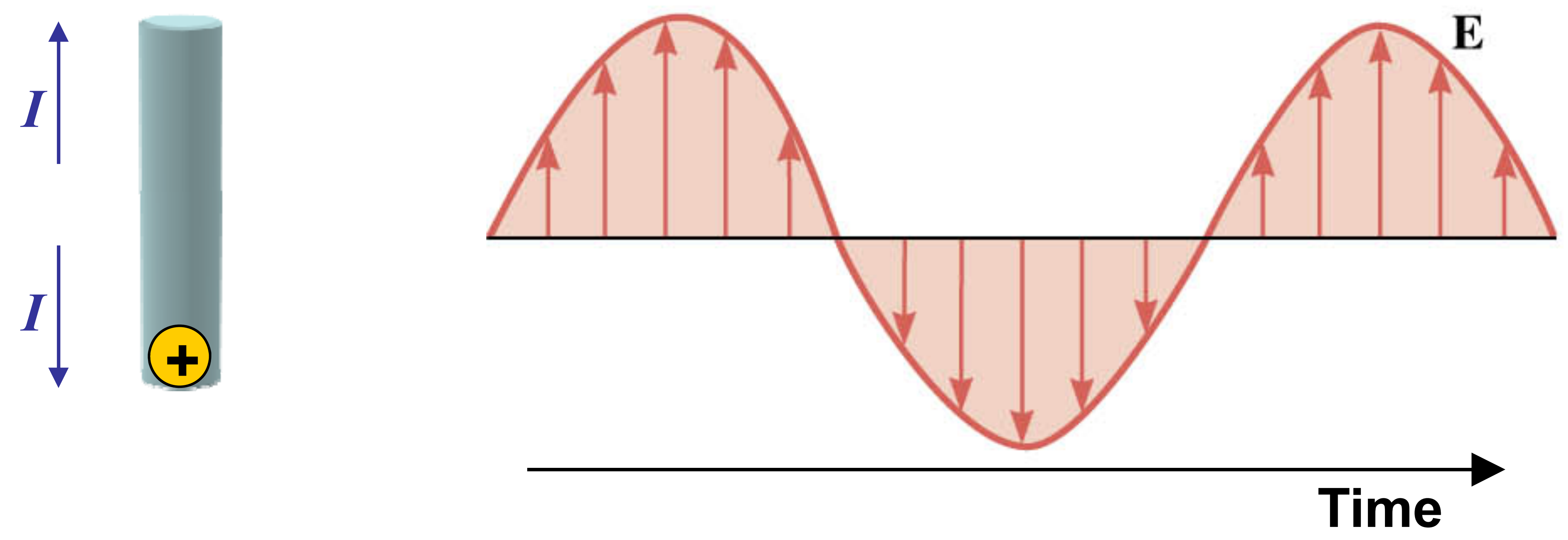
$$\vec{F} = q_0 \vec{E} + q_0 (\vec{v} \times \vec{B})$$

- Variation of magnetic flux creates electric field and variation of electric flux creates magnetic field

Electromagnetics Waves

➢ Take a single positive charge and wiggle it up and down

➢ Charge changes position as a función of time



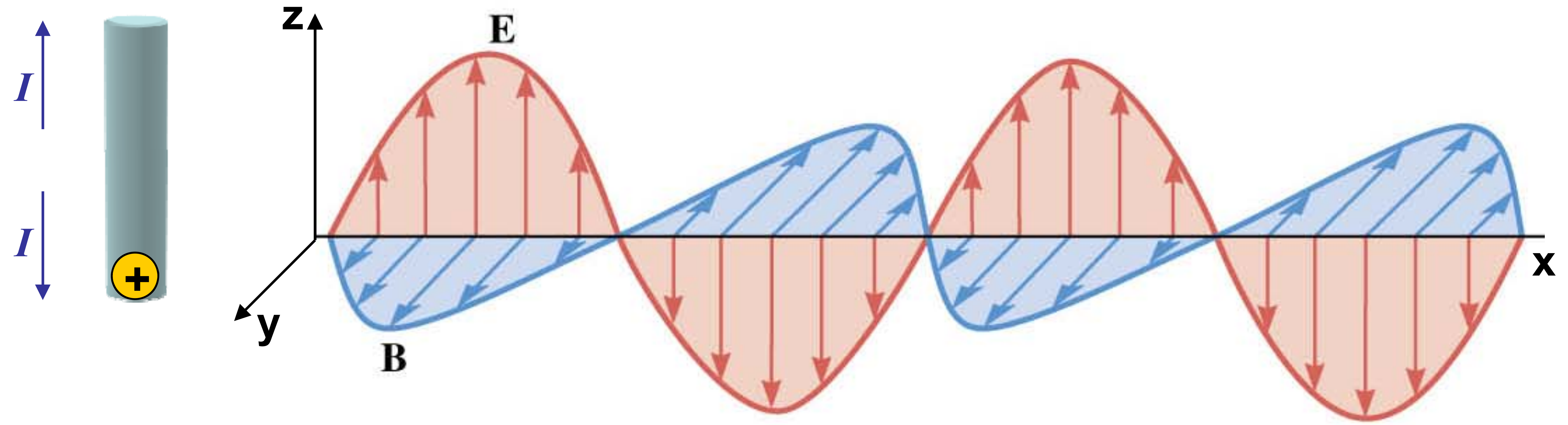
➢ Thus, electric field it creates changes in time

➢ But since charge is moving, it constitutes a current

➢ **Current** points up when charge moves up, and current points down when charge moves down

➢ This current, like all currents, creates a **magnetic field**

➢ Direction of field is given by **RHR-2**



➢ By RHR-2, we see that when current points up, magnetic field points into screen,

and when current points down, magnetic field points out of screen

➢ Thus, we have a changing magnetic field and a changing electric field which are

oriented at right angles to each other!

➢ Here, electric field is in xz-plane, and magnetic field is in xy-plane

➢ Fields move out away from source (our accelerating charge)

Propagation of Electromagnetic (EM) Waves

➢ An EM wave is a transverse wave

Wave motion is at right angles to direction of propagation

Review

1. Stationary charges create electric fields
2. Moving charges (constant velocity) create magnetic fields
3. Accelerating charges create electromagnetic waves

- EM waves don't need a medium to travel through
- They can propagate through a vacuum
- How fast do EM waves travel?
- Maxwell's equations predict that E- and B- fields propagate through space at speed of light

$$v \equiv c = \frac{E}{B} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{1}{\sqrt{(8.85 \times 10^{-12}\ \mathrm{C^2 \cdot m^2/N})(4\pi \times 10^{-7}\ \mathrm{N \cdot s^2/C^2})}} = 3 \times 10^8\ \mathrm{m/s}$$

- E and B are magnitudes of electric and magnetic fields at same point in space

Speed of Light

Very Fast!.... but finite

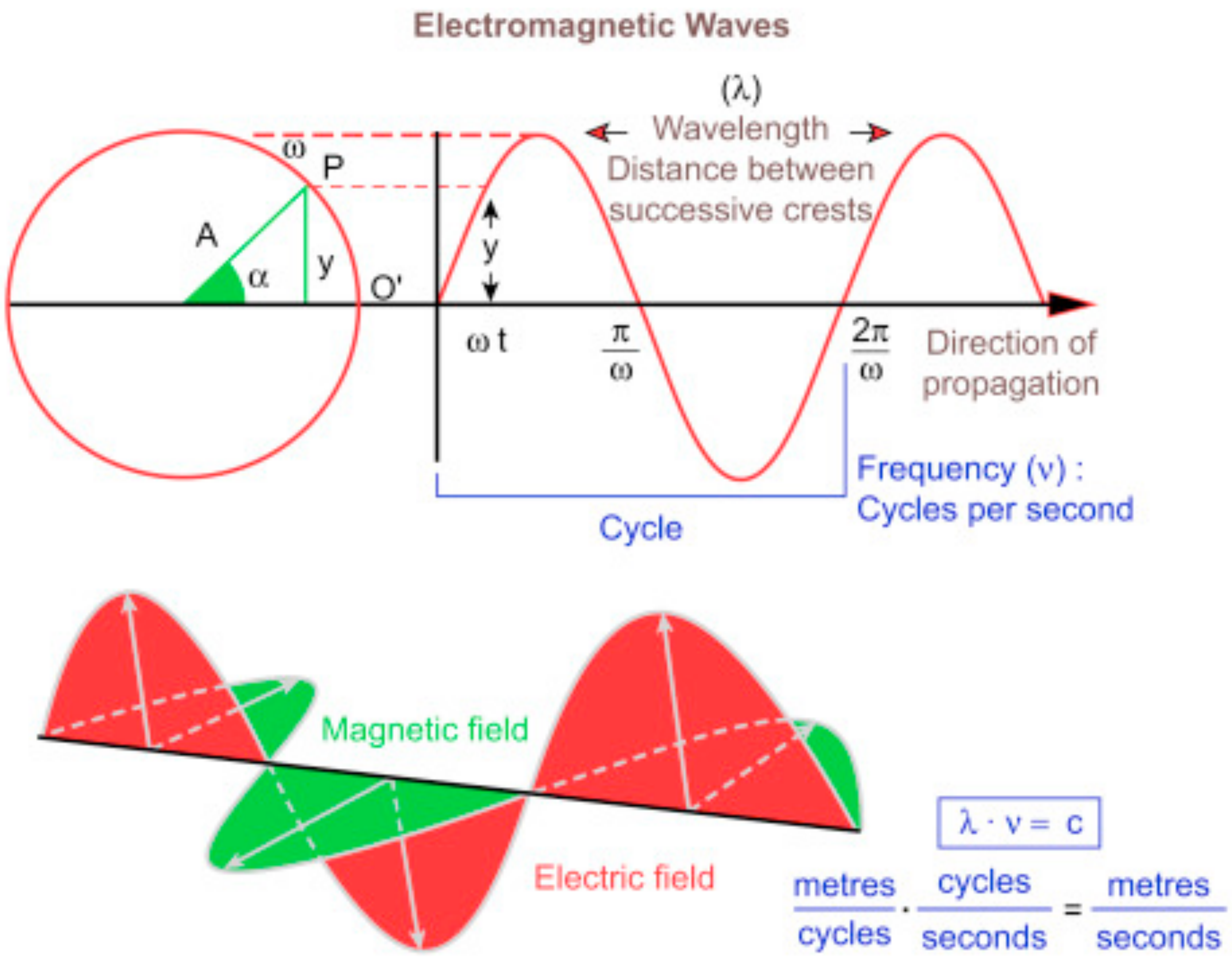
$c = 3.00 \times 10^8$ m/s

Moon to Earth ⟶ 1.3 seconds
Sun to Earth ⟶ 8 minutes

Distant stars and other astronomical objects are so far away that astronomers use a unit of distance called **light year (ly)**

1 ly = distance light travels in 1 year = 9.5×10^{15} m

➢ Like any wave, EM waves have a frequency, a period and an amplitude



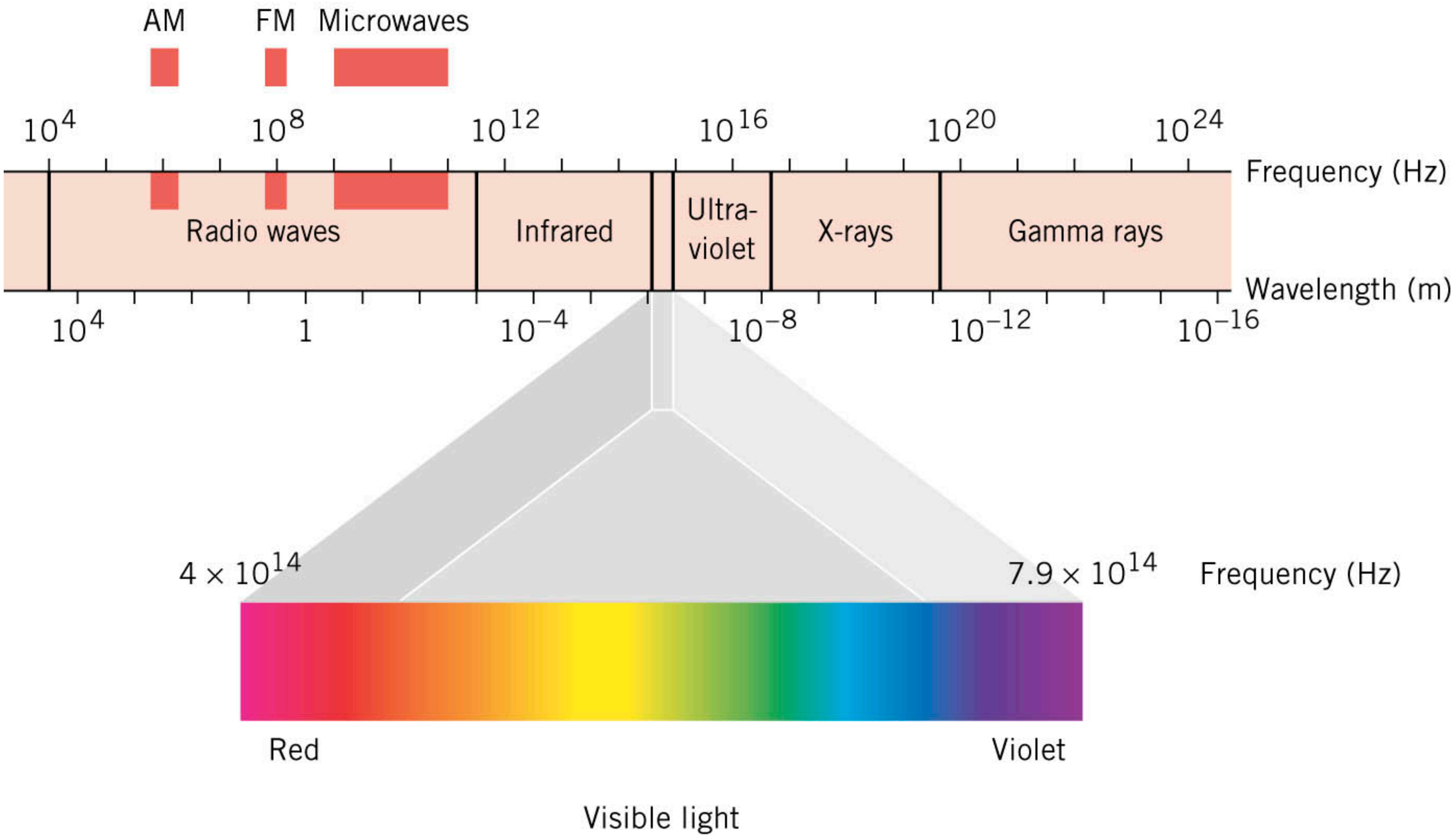
$$c = f\lambda$$

Speed of light (c), Frequency (Hz) (f), Wavelength (m) (λ)

$$f = \frac{c}{\lambda}$$

Higher frequencies mean shorter wavelengths!

Electromagnetic Spectrum



Energy Carried by EM Waves

An EM wave consists of both an electric and magnetic field, and energy is contained in both fields

energy density

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

Using $E = cB$

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{\epsilon_0\mu_0 E^2}{\mu_0} = \epsilon_0 E^2$$

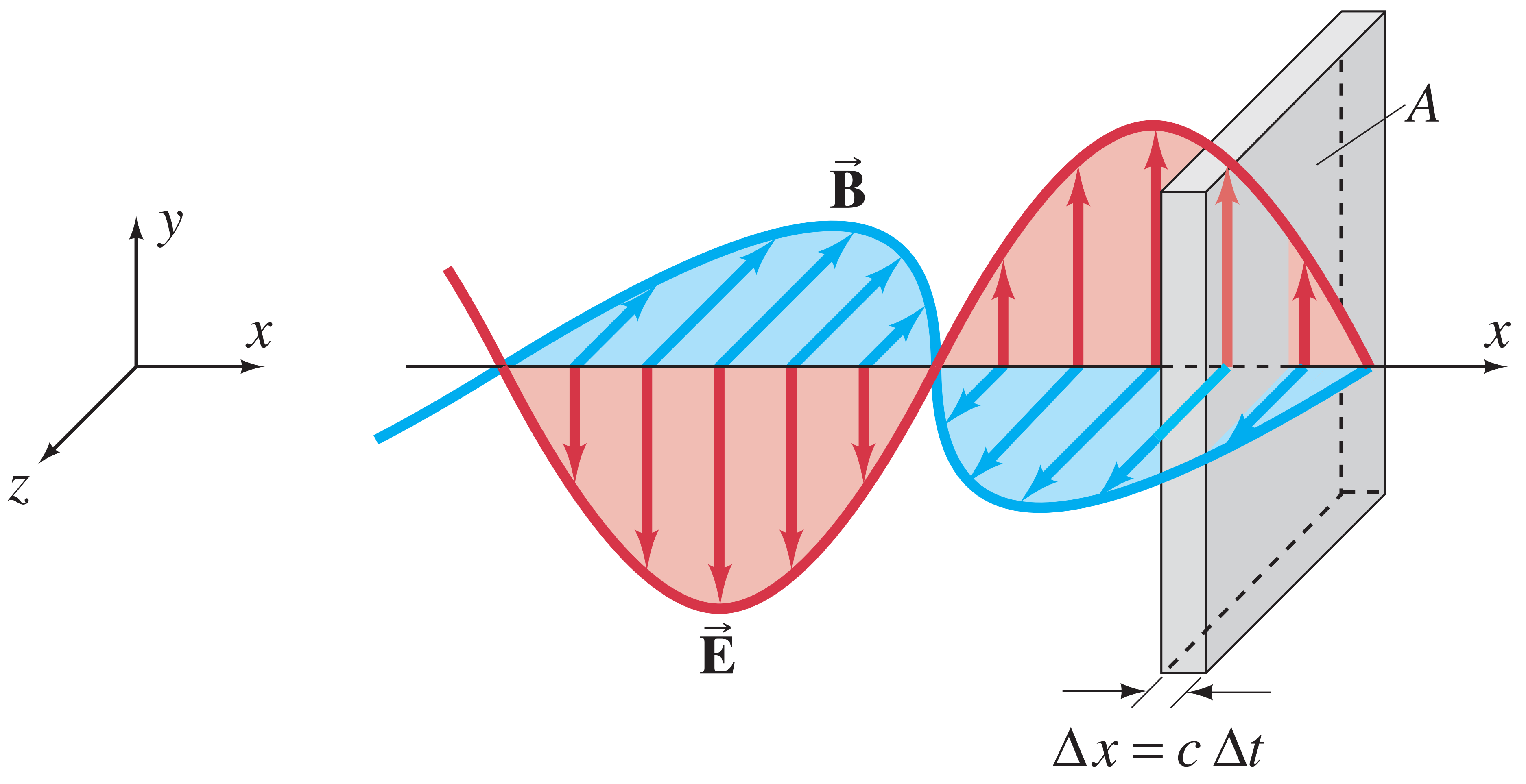
$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\epsilon_0 c^2 B^2 = \frac{B^2}{\mu_0}$$

$$u = \epsilon_0 E^2 = \epsilon_0 EcB = \frac{\epsilon_0 EB}{\sqrt{\epsilon_0\mu_0}} = \sqrt{\frac{\epsilon_0}{\mu_0}}EB$$

Poynting Vector

➢ Energy a wave transports is

$$\Delta U = u\,\Delta V = (u)(A\,\Delta x) = (\epsilon_0 E^2)(Ac\,\Delta t)$$



➢ Rate of energy flow per unit area is P

$$S = \frac{\Delta U}{A\,\Delta t} = \frac{(\epsilon_0 E^2)(Ac\,\Delta t)}{A\,\Delta t} = \epsilon_0 c E^2$$

$$S = \epsilon_0 c E^2 = \frac{c}{\mu_0} B^2 = \frac{EB}{\mu_0}$$

➢ Poynting vector ☛ $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$

➢ Plane transverse electromagnetic waves ☛ fields $\vec{E}$ and $\vec{B}$ are perpendicular

$$|\vec{S}| = \frac{1}{\mu_0} EB$$

➢ I ☛ intensity of wave defined as time-average of S

$$I = \overline{S} = \frac{1}{2}\epsilon_0 c E_0^2 = \frac{1}{2}\frac{c}{\mu_0}B_0^2 = \frac{E_0 B_0}{2\mu_0}$$

➢ E and B are sinusoidal ☛ $\overline{E^2} = E_0^2/2$ and $\overline{B^2} = B_0/2$ just as for electric currents and voltages

$$I = \overline{S} = \frac{E_{\rm rms} B_{\rm rms}}{\mu_0}$$

$$E_{\rm rms} = \sqrt{\overline{E^2}} = E_0/\sqrt{2} \qquad B_{\rm rms} = \sqrt{\overline{B^2}} = B_0/\sqrt{2}$$

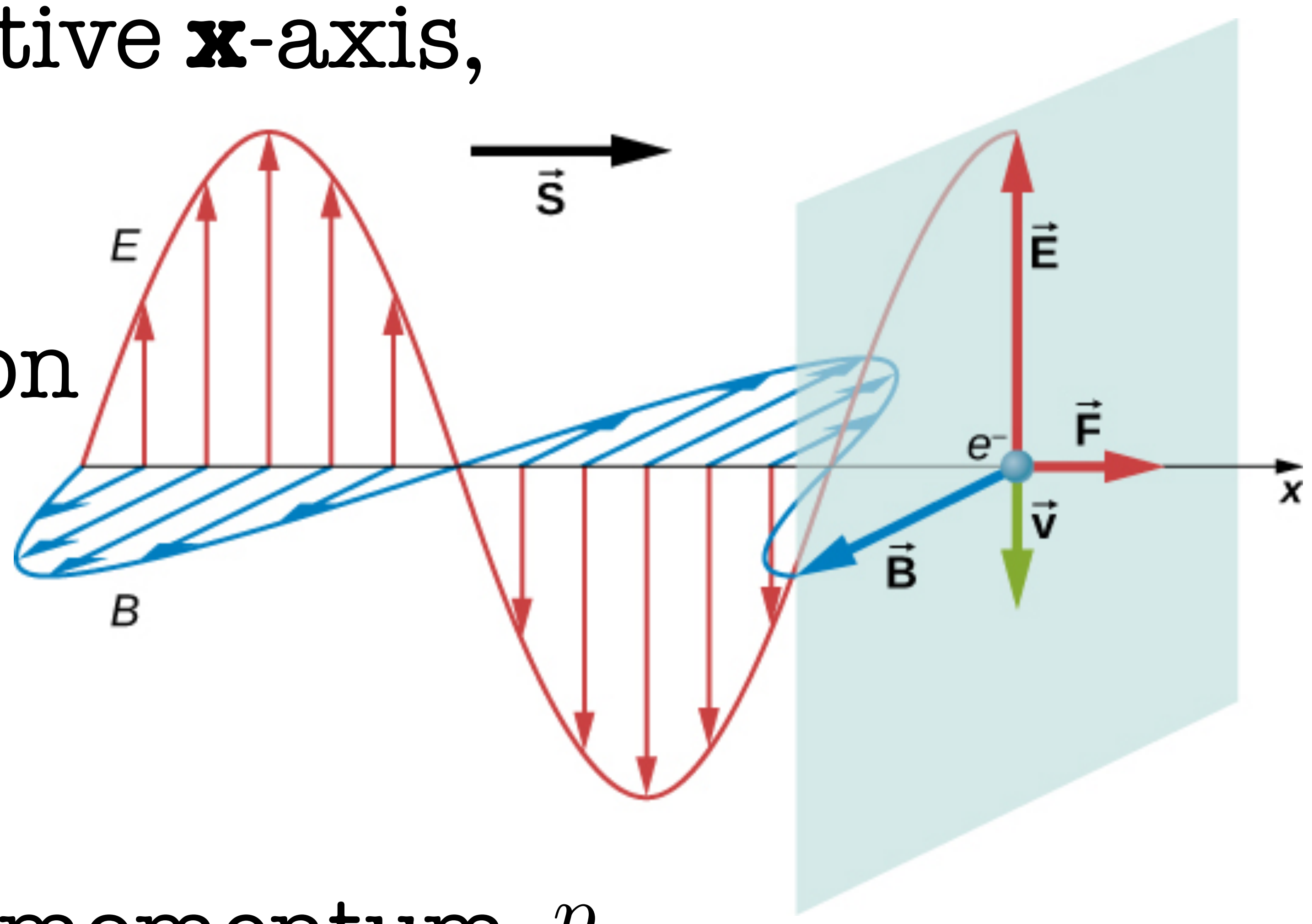
➢ Time-averaged energy density of wave is then

$$\overline{u} = \overline{u_E + u_B} = \epsilon_0 \overline{E^2} = \frac{\epsilon_0}{2}E_0^2 = \frac{1}{\mu_0}\overline{B^2} = \frac{B_0^2}{2\mu_0}$$

➢ Intensity is related to average energy density by

$$I = \overline{S} = c\overline{u}$$

- Material objects consist of charged particles
- An electromagnetic wave incident on the object exerts forces on the charged particles
- To understand direction of force for a very specific case, consider a plane electromagnetic wave incident on a metal
- When electric field is in direction of positive **y**-axis, electrons move in negative **y**-direction, with magnetic field in direction of positive **z**-axis
- By applying right-hand rule, and accounting for negative charge of electron, we can see that force on electron from magnetic field is in direction of positive **x**-axis, which is direction of wave propagation
- When $\vec{E}$ field reverses $\vec{B}$ field does too and force is again in same direction
- Force does work on the particles of the object increasing its energy



$$\Delta U = F\ \Delta x \Rightarrow F = \frac{\Delta U}{\Delta x}$$

- This force occurs because electromagnetic waves contain and transport momentum p
- Change in momentum Δp is estimated to be

$$F = \frac{\Delta p}{\Delta t} \Rightarrow \Delta p = F\Delta t = \frac{\Delta U}{\Delta x}\Delta t = \frac{\Delta U}{\Delta x/\Delta t} = \Delta U/c$$

Radiation Pressure

- ΔU is energy absorbed in a time Δt
- If the EM wave is completely reflected the momentum transferred is ☛ $\Delta p = 2\frac{\Delta U}{c}$
- Radiation pressure is given by

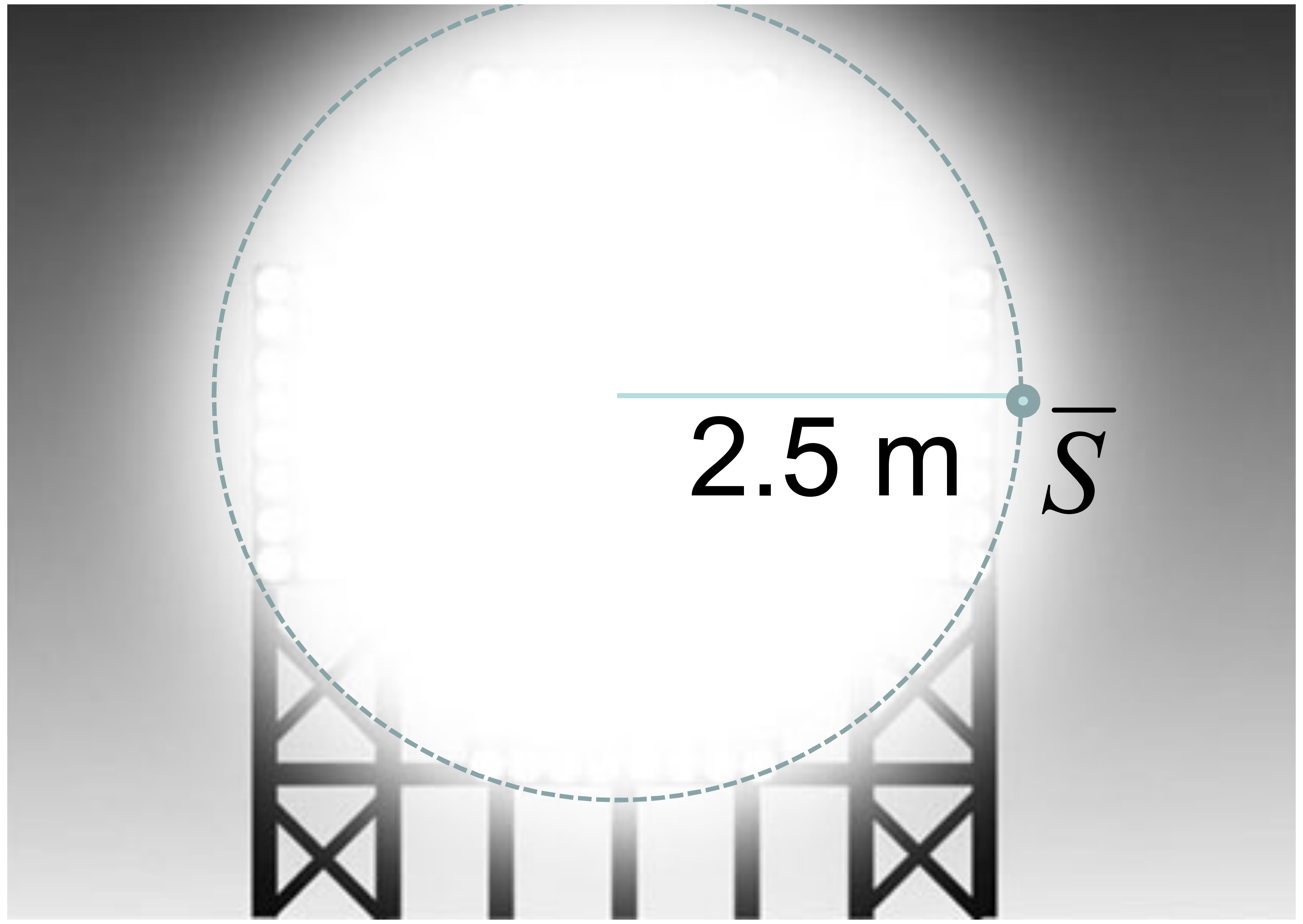
$$P_{\text{rad}} = \frac{F}{A} = \frac{\Delta p}{\Delta t}\frac{1}{A} = \frac{\Delta U}{\Delta t}\cdot\frac{1}{Ac} = SA\cdot\frac{1}{Ac} = \frac{S}{c}$$

$$P_{\text{rad}} = \frac{F}{A} = \frac{\Delta p}{\Delta t}\frac{1}{A} = 2\frac{\Delta U}{\Delta t}\cdot\frac{1}{Ac} = 2SA\cdot\frac{1}{Ac} = 2\frac{S}{c}$$

Solar Sail

Example

- ➢ A point light source is emitting light uniformly in all directions
- ➢ At a distance of 2.5 m from source, *rms* electric field strengh of light is 19.0 N/C
- ➢ Assuming that light does not reflect from anything in environment, determine average power of light emitted by source



- ➢ What do we know ☛ $E_{rms}\,;\,r$
- ➢ Average light intensity at imaginary spherical surface

$$\overline{S} = c\overline{u} = c\varepsilon_0 E_{rms}^2$$

- ➢ Power of source $\overline{P} = \overline{S}\cdot A = \overline{S}\cdot(4\pi r^2)$ $\qquad \overline{S} = \dfrac{\overline{P}}{4\pi r^2}$

$$\overline{P} = \overline{S}\cdot A = \overline{S}\cdot(4\pi r^2)$$

$$= c\varepsilon_0 E_{rms}^2\cdot(4\pi r^2)$$

$$= 75.3\,W$$

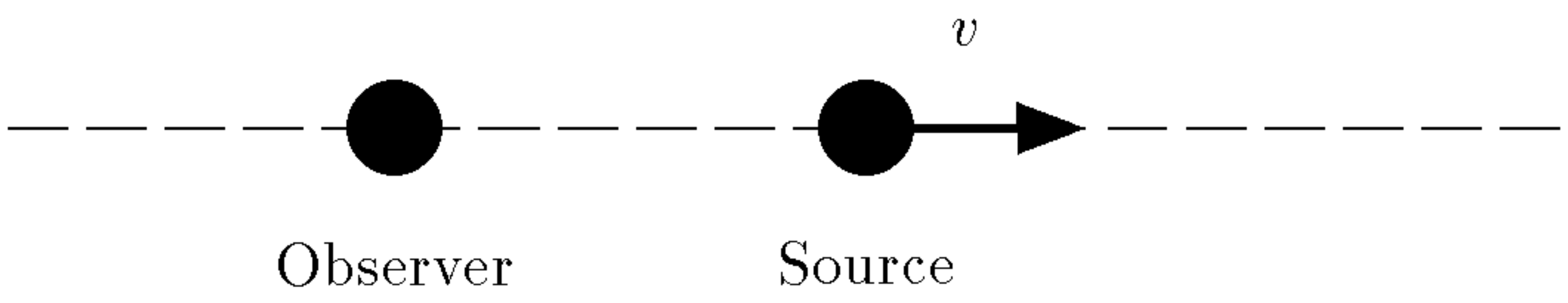
Doppler Effect (Sound Waves)

$$u \equiv \text{velocity of sound waves,}$$

$$v \equiv \text{recession velocity of the source,}$$

$$\Delta t_S \equiv \text{the period of the wave at the source,}$$

$$\Delta t_O \equiv \text{the period of the wave as observed.}$$



➢ Now consider the following sequence, as illustrated below

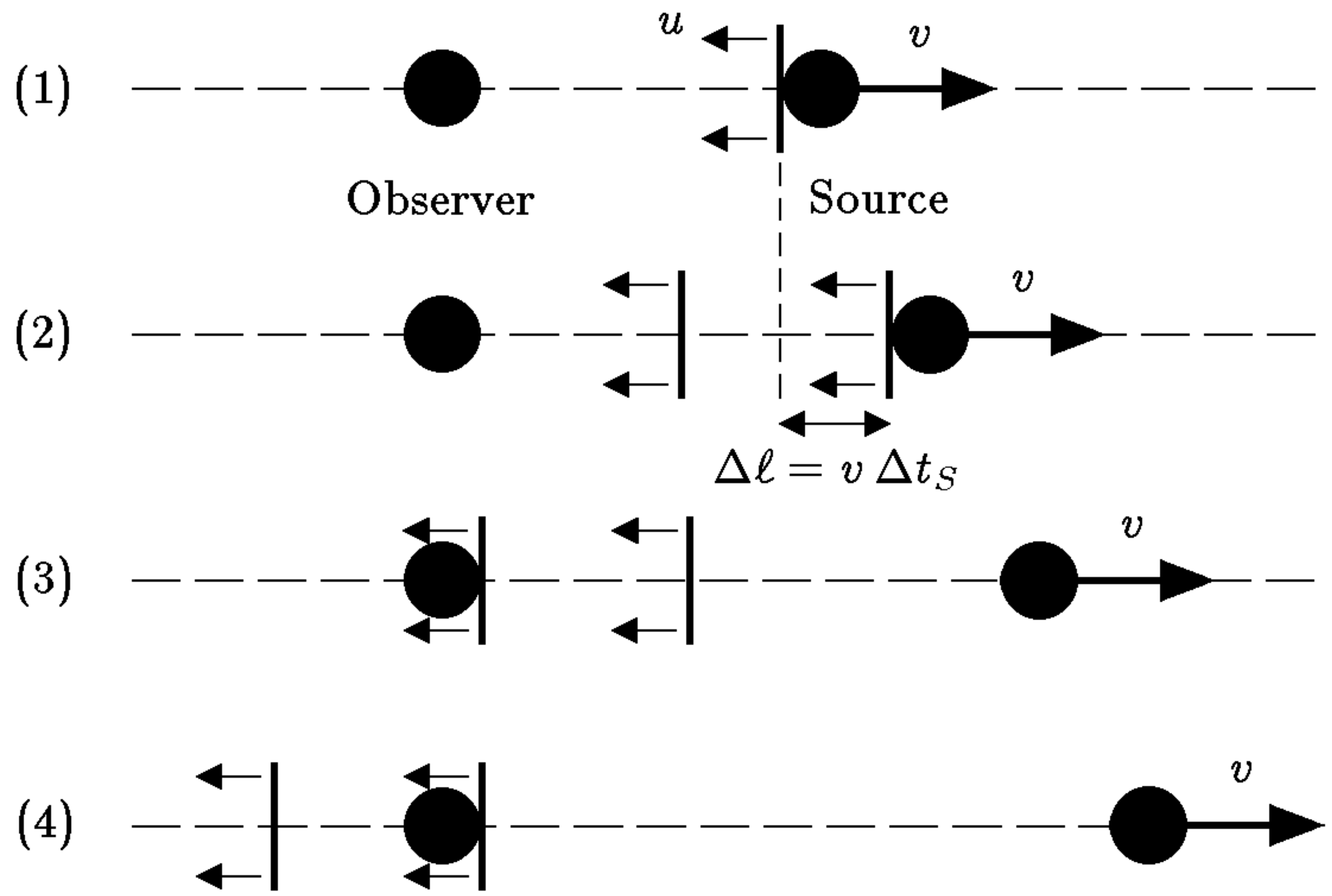
(1) Source emits a wave crest

(2) At a time Δt_S later, source emits a second wave crest ☛ during this time interval source has moved a distance $\Delta \ell = v \Delta t_S$ further away from observer

(3) Stationary observer receives first wave crest

(4) At some time Δt_O after (3), observer receives second wave crest

Our gol is to find Δt_O



- We are interested, however, only in time difference Δt_O between reception of first and second crests
- This time difference does not depend on distance between source and observer, since both crests have to travel this distance
- Second crest, however, has to travel an extra distance

$$\Delta \ell = v\,\Delta t_S$$

since source moves this distance between emission of two crests

- Extra time that it takes second crest to travel this distance is $\Delta \ell / u$, so time between reception of two crests is ☛

$$\begin{aligned} \Delta t_O &= \Delta t_S + \frac{\Delta \ell}{u} \\ &= \Delta t_S + \frac{v\,\Delta t_S}{u} \\ &= \left(1 + \frac{v}{u}\right) \Delta t_S \end{aligned} \qquad (\diamondsuit)$$

➢ Result is usually described in terms of **redshift** z, which is defined by statement that wavelength is increased by a factor of $(1 + z)$

➢ Since wavelength λ is related to period Δt by $\lambda = u\Delta t$, , we can write definition of redshift as

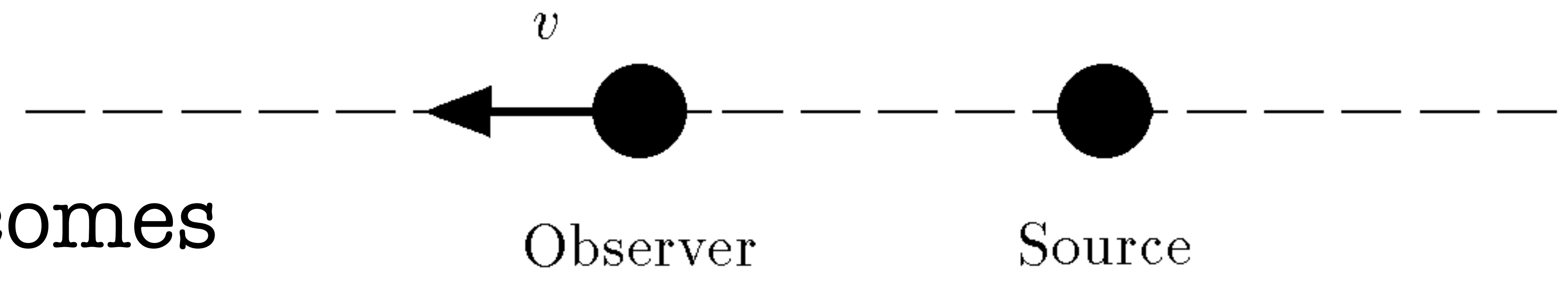
$$\frac{\lambda_O}{\lambda_S} = \frac{\Delta t_O}{\Delta t_S} \equiv 1 + z$$

where λ_S and λ_O are wavelength as measured at source and at observer respectively

➢ Combining this definition with Eq. (✧), we find that redshift for this case is given by

$$z = v/u \qquad \text{(Nonrelativistic, source moving)}$$

➢ Suppose now that source stands still, but observer is receding at a speed v



➢ In this case secuence becomes

(1') Source emits a wave crest

(2') At a time Δt_S later, source emits a second wave crest

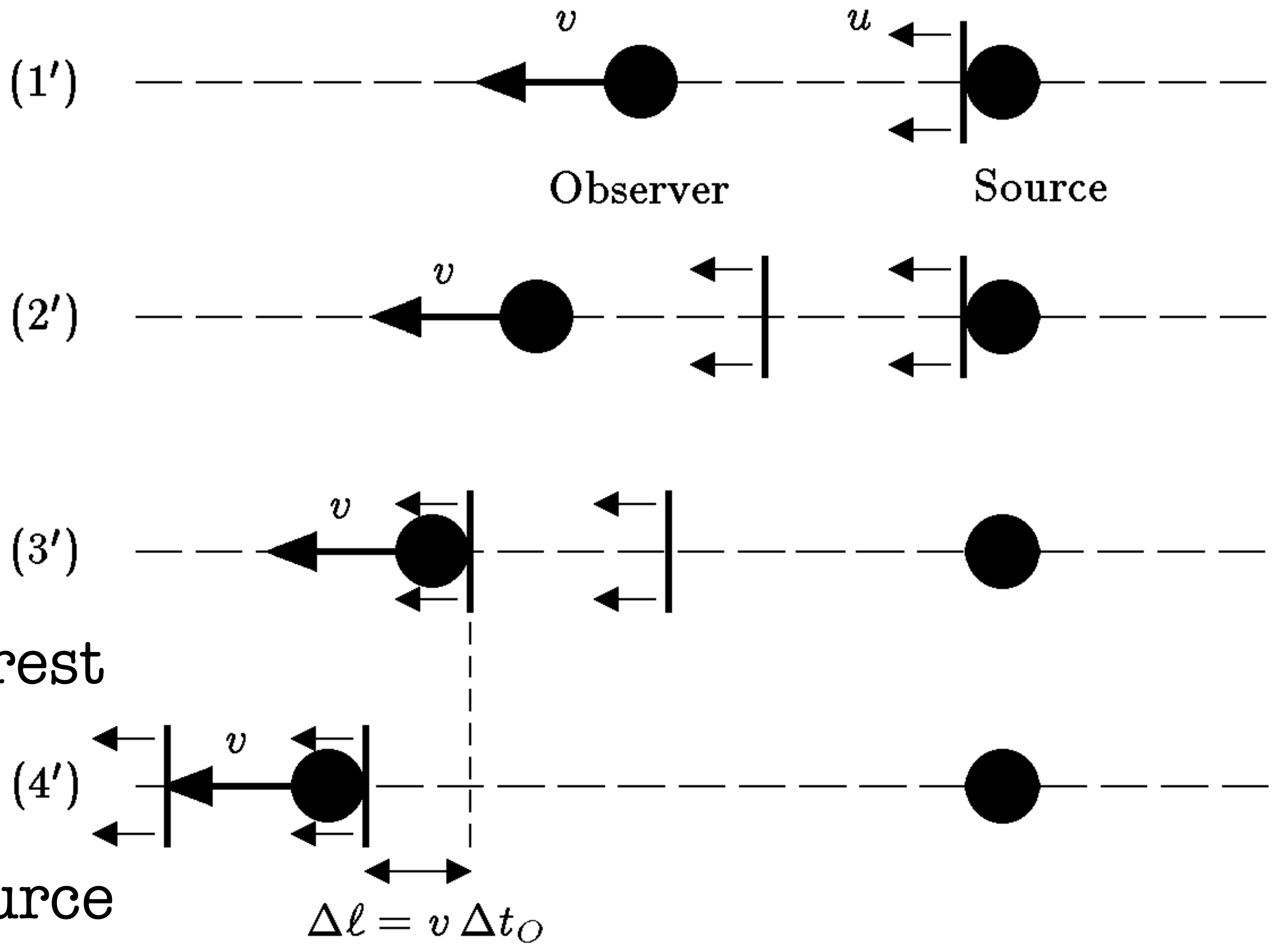
Source is standing still

(3') Moving observer receives first wave crest

(4') At a time Δt_O after (3'), observer receives second wave crest

During time interval between (3') and (4'),

observer has moved a distance $\Delta \ell = v \Delta t_O$ further from source



➢ Using same strategy as in first case, we note that in this case, second wave crest must travel an extra distance $\Delta \ell = v \Delta t_O$

Thus ☛ $$\Delta t_O = \Delta t_S + \frac{\Delta \ell}{u} = \Delta t_S + \frac{v \Delta t_O}{u}$$

➢ In this case Δt_O appears on both sides of equation, but we can easily solve Δt_O to find

$$\Delta t_O = \left(1 - \frac{v}{u}\right)^{-1} \Delta t_S$$

➢ Recalling definition of z

$$z = \frac{\Delta t_O}{\Delta t_S} - 1 = \frac{1}{1 - (v/u)} - 1$$

$$= \frac{v/u}{1 - (v/u)}$$ (Nonrelativistic, observer moving)

Summary

- ➢ Frequency ☛ $f = 1/\Delta t$
- ➢ Motion of Observer ☛ $f_o = f_s(1 \pm v_o/u)$ ☛ + towards source and – away from the source
- ➢ Motion of Source ☛ $f_o = f_s(1 \mp v_s/u)^{-1}$ ☛ – towards source and + away from source

$$f_o = f_s \frac{1 \pm v_o/u}{1 \mp v_s/u}$$

- ➢ Since $v_o/u < 1$ and $v_s/u < 1$ ☛ for $x < 1$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad \frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$$

Take $x^2 \ll 1$ negligible

$$f_o \approx f_s(1 \pm v_{\rm rel}/u) \Rightarrow v_{\rm rel} = v_o + v_s$$

Doppler Effect (Electromagnetic Waves)

When observer of a wave, or source of wave (or both) is moving,

observed wave frequency is different than that emitted by source

➢ EM waves also exhibit a Doppler effect

But ☛ 1- They do not require a medium through which to propagate, and ...

2- Only relative motion of source to observer is important,

since speed at which all EM waves move is same speed of light

➢ So how do we calculate shift in frequency?

➢ If EM wave, sources and observer all travel along same line, then

$$f_o = f_s\left(1 \pm \frac{v_{rel}}{c}\right)$$

f_o is observed frequency

f_s is frequency emitted by source

v_{rel} is relative velocity between observer and source

+ sign is used when object and source move **toward** each other

– sign is used when object and source move **away** from each other

This is valid for speeds $v_{rel} << c$

Example

- A distant galaxy emits light that has a wavelength of 500.7nm
- On earth, wavelength of this light is measured to be 503.7 nm

(a) Decide if galaxy is moving away from or toward earth

(b) Find speed of galaxy relative to earth

Solution

- We start with Doppler equation $f_o = f_s\left(1 \pm \frac{v_{rel}}{c}\right)$
- Light is shifted to longer wavelengths, which means smaller frequencies ☛ $f = c/\lambda$
- Thus ☛ $f_o < f_s$
- Which means that parentheses $\left(1 \pm \frac{v_{rel}}{c}\right)$ must be < 1
- Therefore, correct sign in parentheses is –sign ☛ galaxy is moving away from earth

(b) from Doppler equation $v_{rel} = c\left(1 - \frac{f_o}{f_s}\right)$ But ☛ $f = c/\lambda$

- Thus ☛ $v_{rel} = c\left(1 - \frac{\lambda_s}{\lambda_o}\right) = 3\times10^8\left(1 - \frac{500.7\text{ nm}}{503.7\text{ nm}}\right) = \boxed{1.8\times10^6\text{ m/s}}$


aNy