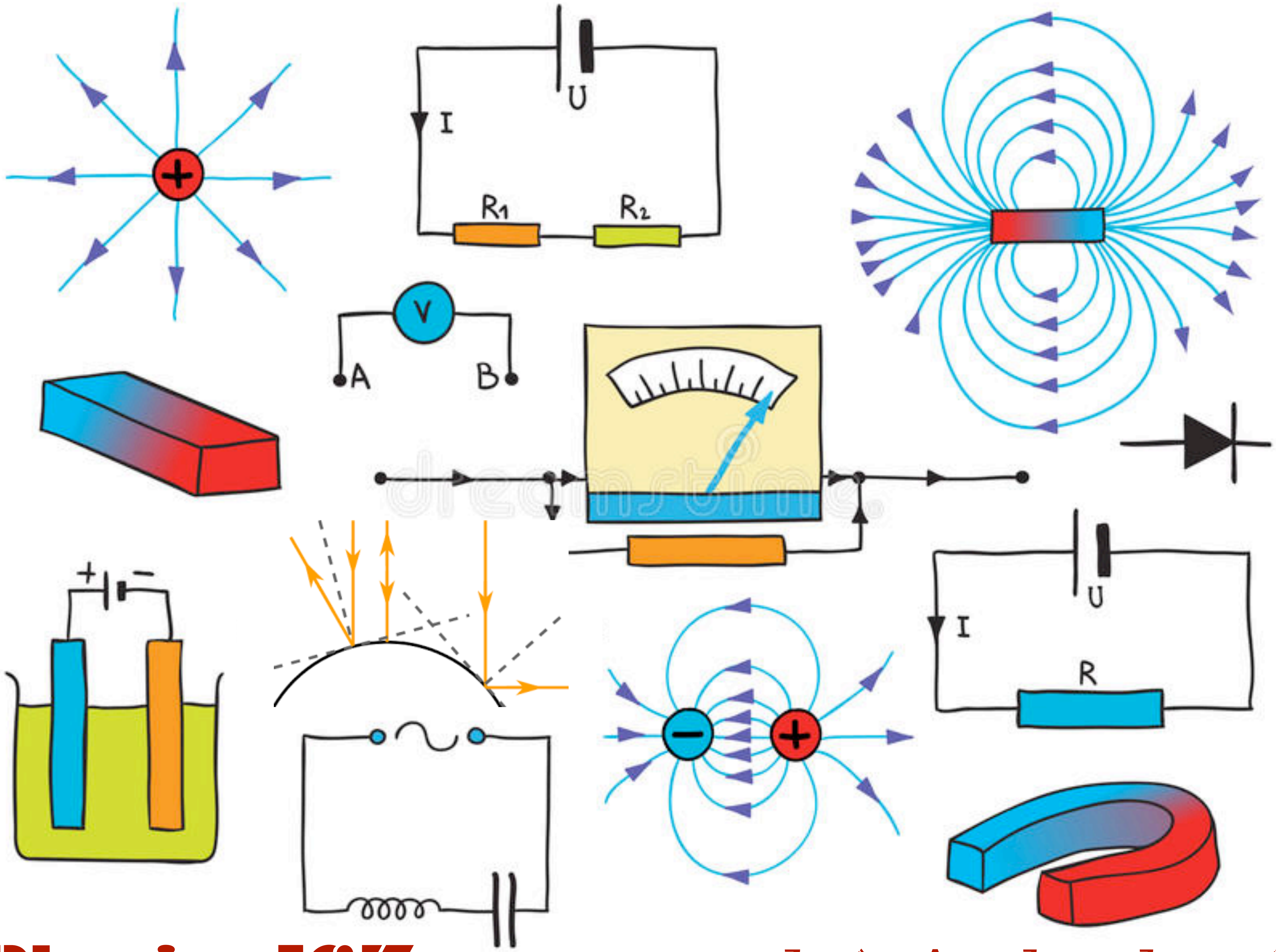


Physics 167

Luís Anchordoqui

Electric Field

- ➢ The earth exerts a force on the moon and vice versa, even though they are 240,000 miles apart
- ➢ Likewise, two charged objects located far apart exert forces on each other too

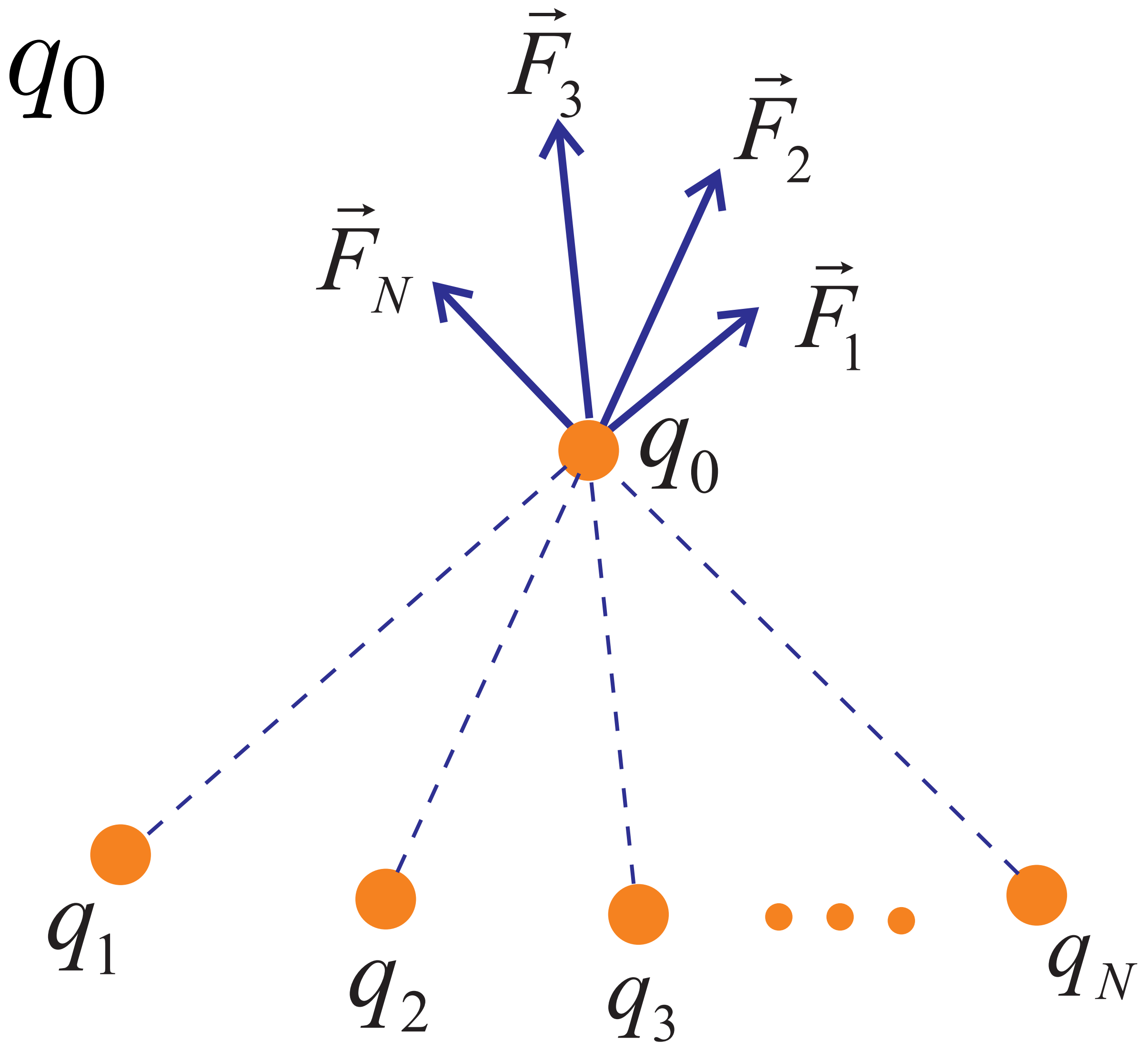
How can they do this if they are not in physical contact?

- ➢ In the case of the earth/moon system, we say that the earth fills all space with a gravitational field, and the moon feels the effect of this field
- ➢ Masses feel forces in gravitational fields
- ➢ Similarly, a charge creates an electric field that fills all space
- ➢ Any other charge in that field will feel a force
- ➢ Stationary charges create electric fields that fill all space
- ➢ Other charges will feel forces in these electric fields

Think of the electric field as a real physical entity!

Electric Field

➢ When we solved the Coulomb Law problems we added up the (vector) forces from charges $q_1, q_2, \ldots q_N$ acting on a certain charge q_0



$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N$$

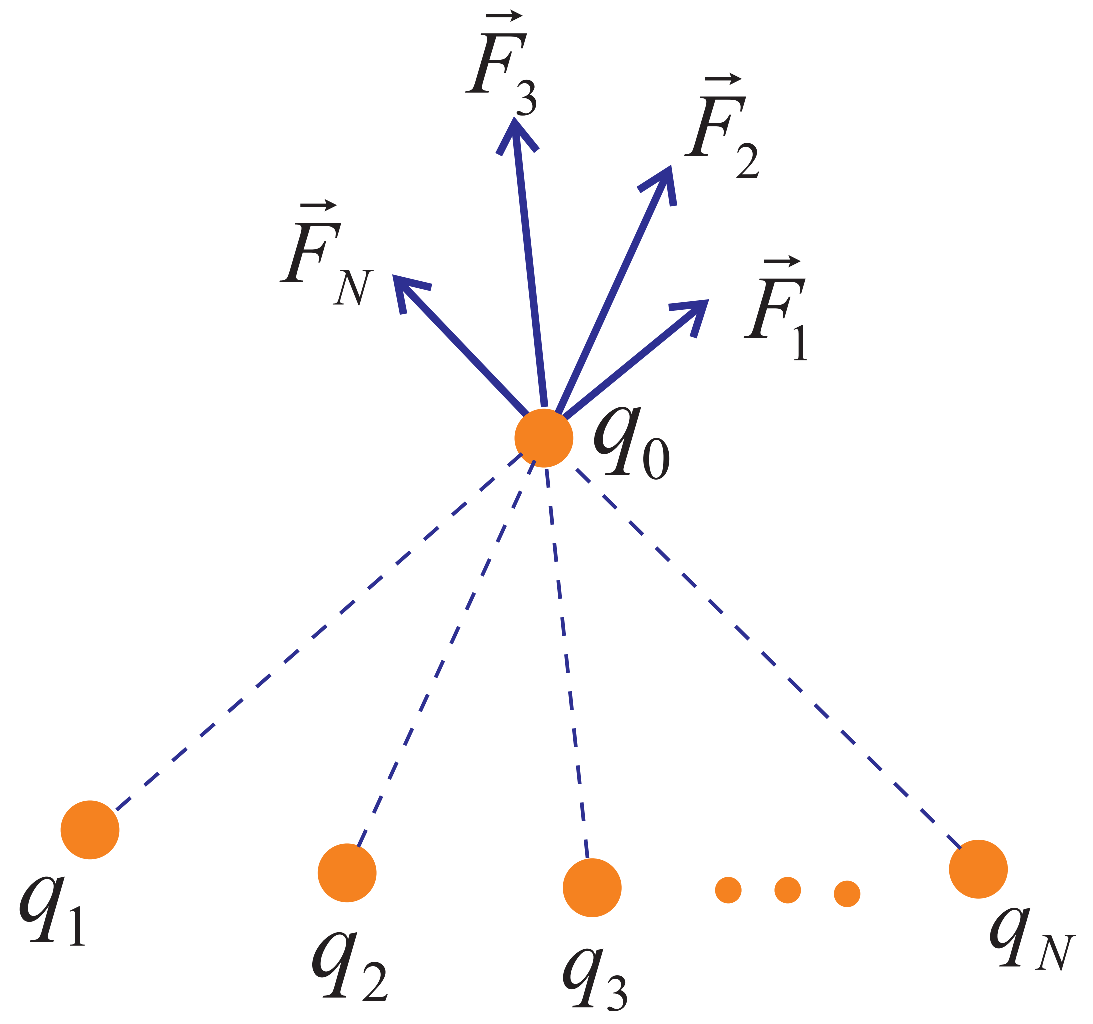
➢Now ☛ each one of these individual forces (and hence the sum of those forces) is proportional to the charge q_0

➢If in each of those problems we divided the net force by the charge q_0 we would get a force per unit charge at the location of q_0

➢This quantity (which is a vector, since force is a vector) would depend on the values and locations of the charges $q_1, q_2, \ldots q_N$

Electric Field

➢ So ☛ a given configuration of charges $q_1, q_2, \ldots q_N$ gives rise to an electric field



$$\vec{E} = \frac{\vec{F}}{q_0}$$

Units?

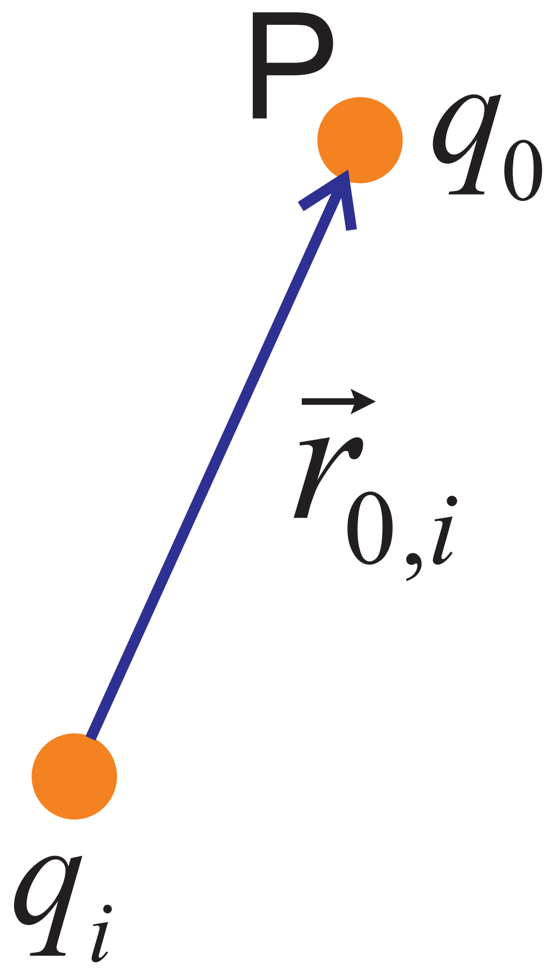
$$\left[\frac{\text{Force}}{\text{Charge}}\right] = \left[\frac{\text{N}}{\text{C}}\right]$$

q_0 ☛ **small positive probe charge**

but ... how small is small?

➢ When we use this equation we mean that after we put q_0 in place all the little charges $q_1, q_2, \ldots q_N$ are in the same places they were when we deduced the value of E from their values and positions!

(i) $\vec{E}$-field due to a single charge q_i



From definitions of Coulomb's Law

force experienced at location of q_0 (point P)

$$\vec{F}_{0,i} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_0 q_i}{r_{0,i}^2} \cdot \hat{r}_{0,i}$$

$\hat{r}_{0,i}$ ☛ unit vector along direction from charge q_i to q_0

Recall $\vec{E} = \frac{\vec{F}}{q_0}$ $\therefore$ $\vec{E}$ -field due to q_i at point P

$$\vec{E}_i = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_i}{r_i^2} \cdot \hat{r}_i$$

$\vec{r}_i$ ☛ vector pointing from q_i to point P

$\hat{r}_i$ ☛ unit vector pointing from q_i to point P

Note:

(1) $\vec{E}$ -field is a vector

(2) Direction of $\vec{E}$-field depends on both position of q_i and sign of P

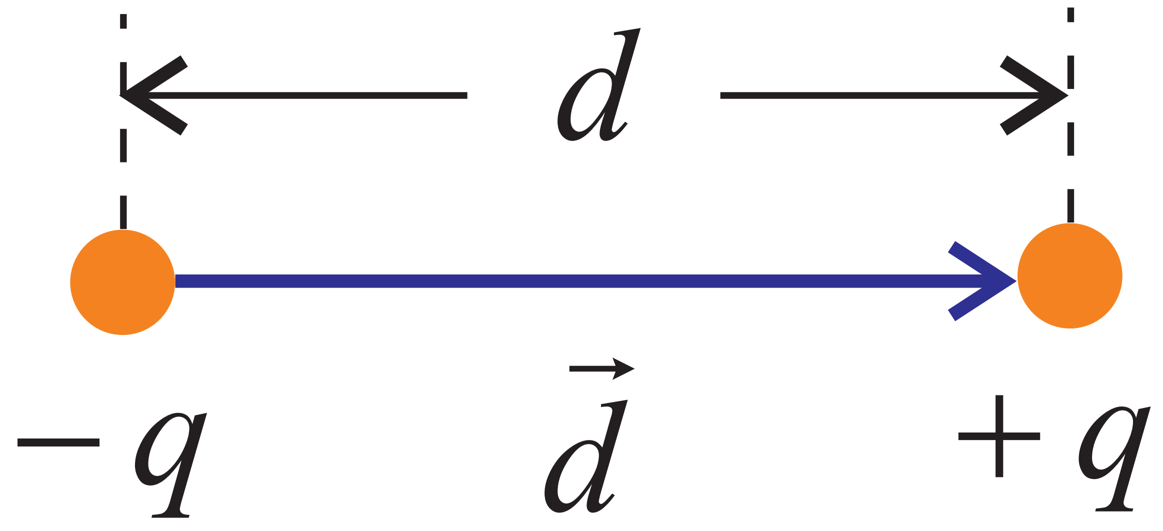
(ii) $\vec{E}$-field due to system of charges:

Principle of Superposition

In a system with N charges ☛ total $\vec{E}$-field due to all charges vector sum of $\vec{E}$-field due to individual charges

i.e. ☛ $$\vec{E} = \sum_i \vec{E}_i = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$

(iii) Electric Dipole

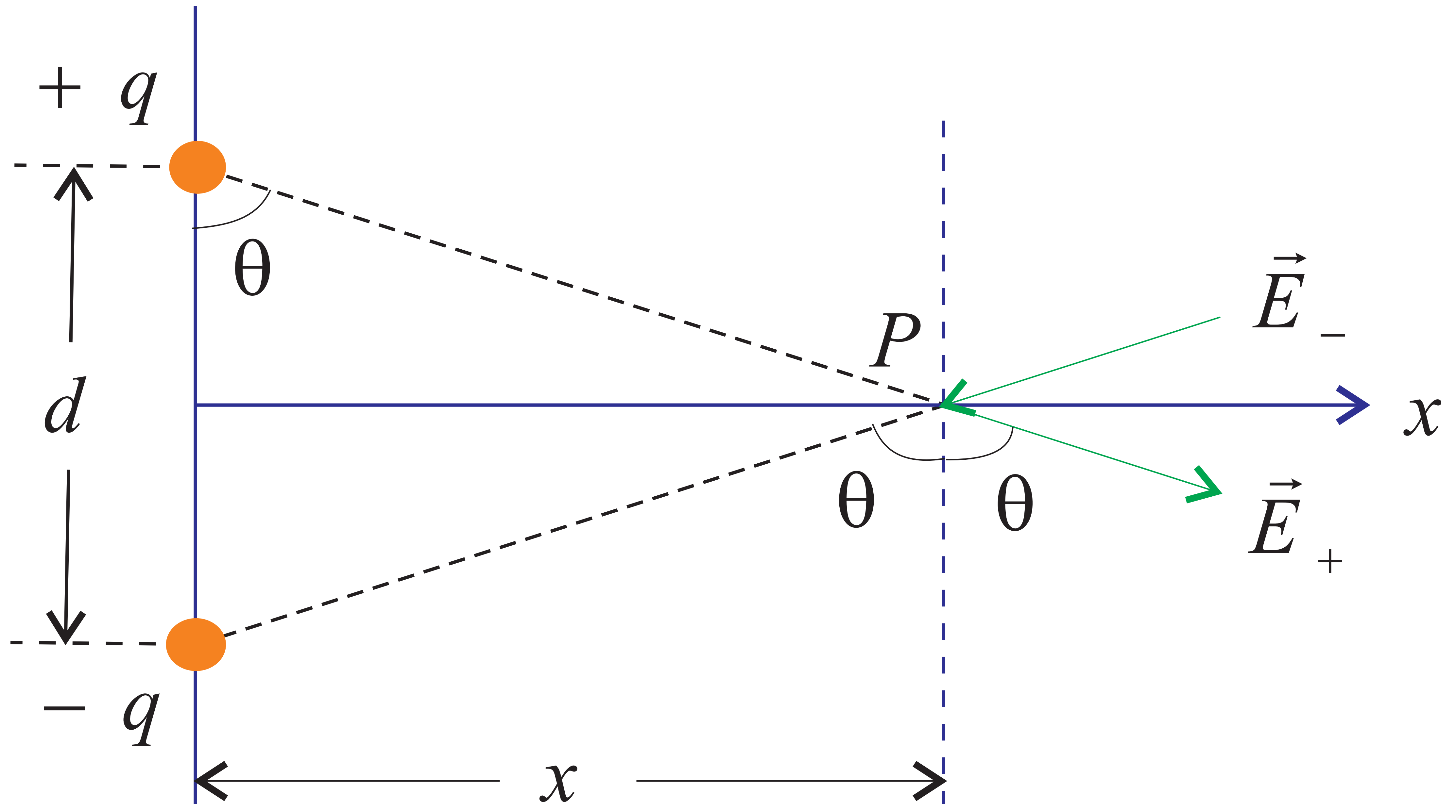


System of equal and opposite charges separated by a distance d

Electric Dipole Moment ☛ $$\vec{p} = q\vec{d} = qd\hat{d}$$

$$p = qd$$

Example: $\vec{E}$ due to dipole along x-axis

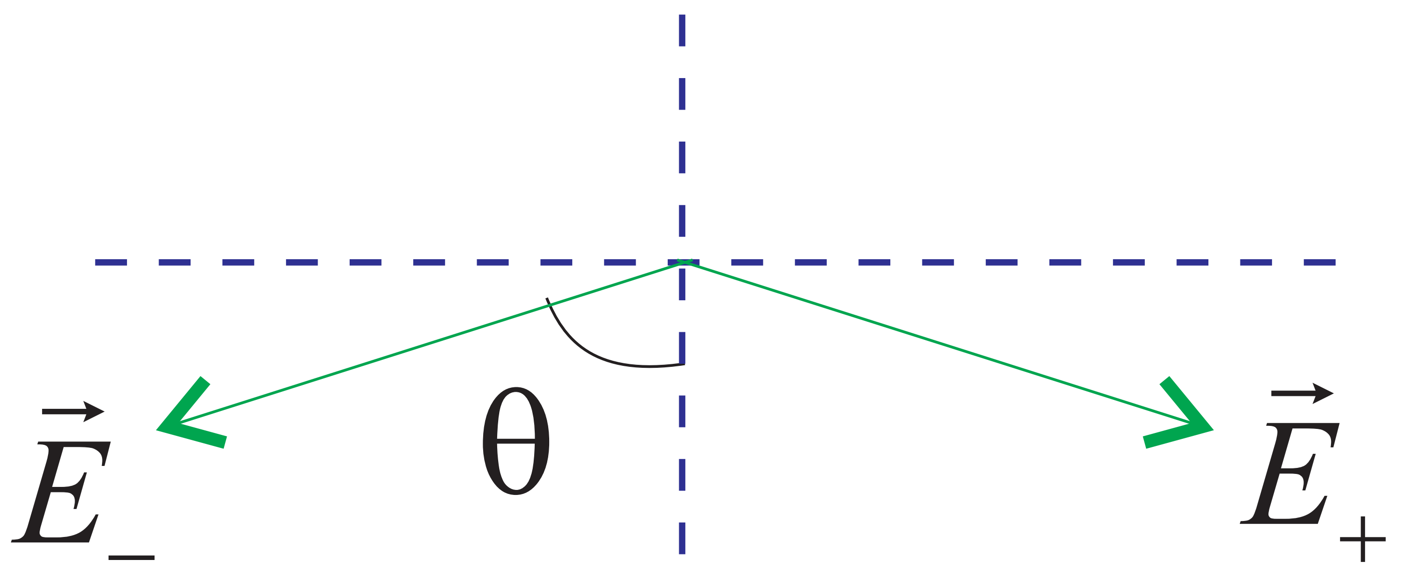


Consider point P at distance x along perpendicular axis of dipole $\vec{p}$

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

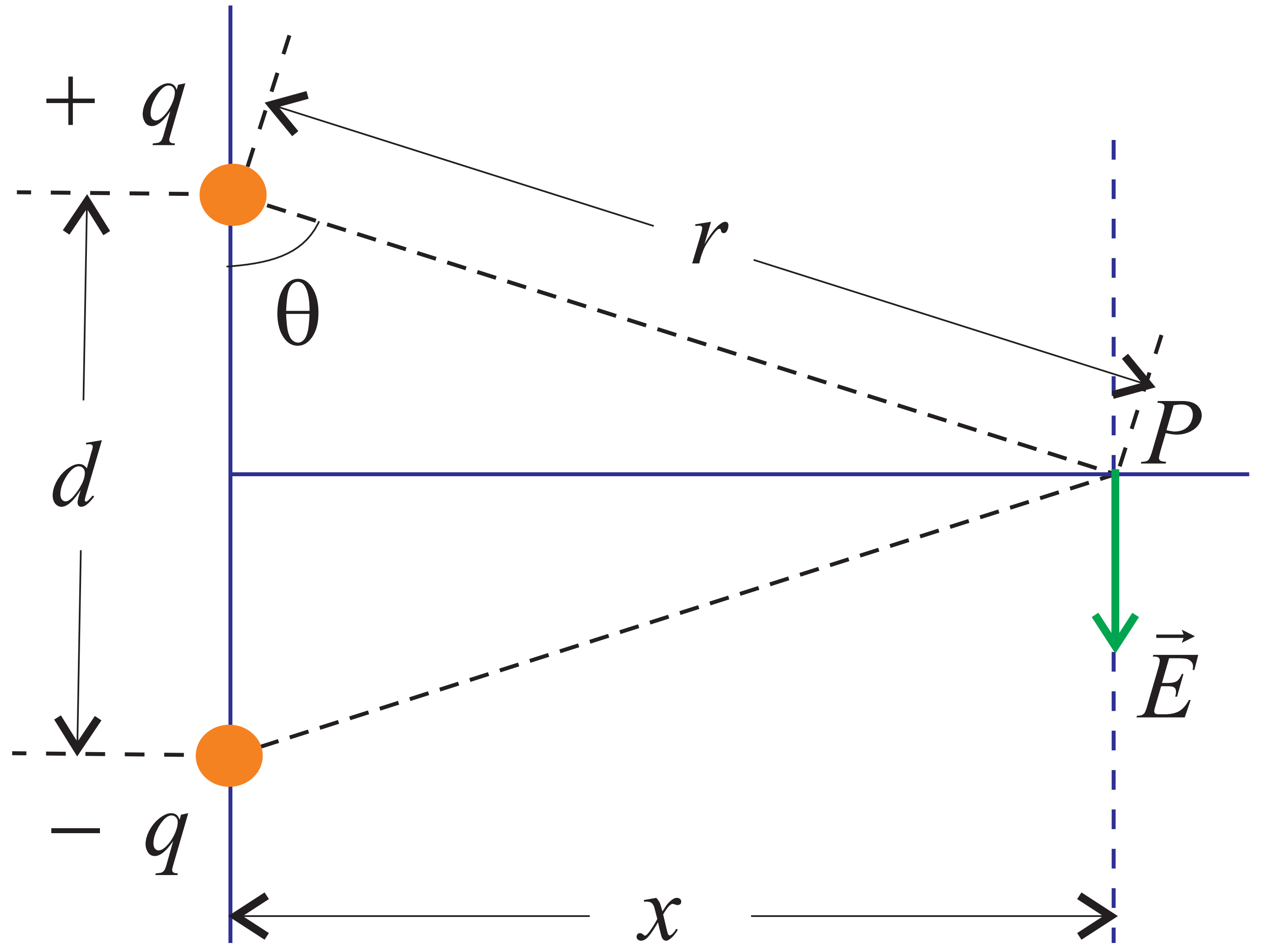
E-field due to $+q$ E-field due to $-q$

Notice: Horizontal $\vec{E}$ -field components of $\vec{E}_+$ and $\vec{E}_-$ cancel out



$\therefore$ Net $\vec{E}$ points along axis parallel but opposite to dipole moment vector

Magnitude of $\vec{E}$ -field $= 2E_+ \cos\theta$



$$\therefore E_- = 2\left(\underbrace{\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}}_{E_+ \text{ or } E_- \text{ magnitude}} \right) \cos\theta$$

But

$$r = \sqrt{\left(\frac{d}{2}\right)^2 + x^2}$$

$$\cos\theta = \frac{d/2}{r}$$

$$\therefore E = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{\left[x^2 + \left(\frac{d}{2}\right)^2\right]^{\frac{3}{2}}}$$

$$(p = qd)$$

Special case ☛ When $x \gg d$

$$\left[x^2 + \left(\frac{d}{2}\right)^2\right]^{\frac{3}{2}} = x^3 \left[1 + \left(\frac{d}{2x}\right)^2\right]^{\frac{3}{2}}$$

➢ Binomial Approximation

$$(1 + y)^n \approx 1 + ny \qquad \text{if} \quad y \ll 1$$

$$\vec{E} - \text{field of dipole} \simeq \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{x^3} \propto \frac{1}{x^3}$$

➢ Compare with $\frac{1}{r^2}$ $\vec{E}$ -field for single charge

➢ Result also valid for point P along any axis with respect to dipole

Electric Field Lines

To visualize electric field we can use a graphical tool called electric field lines

Conventions

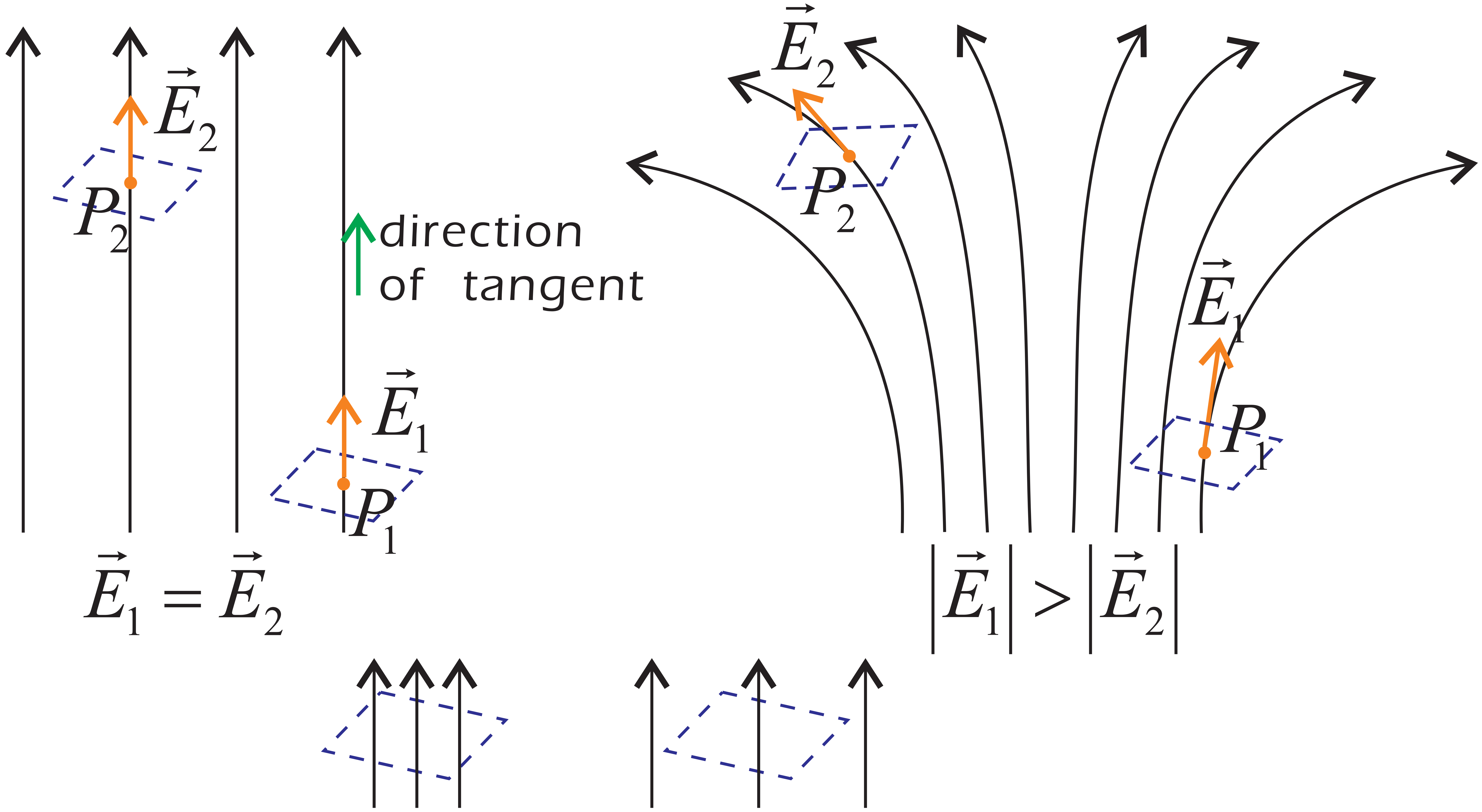
1. Start on positive charges and end on negative charges

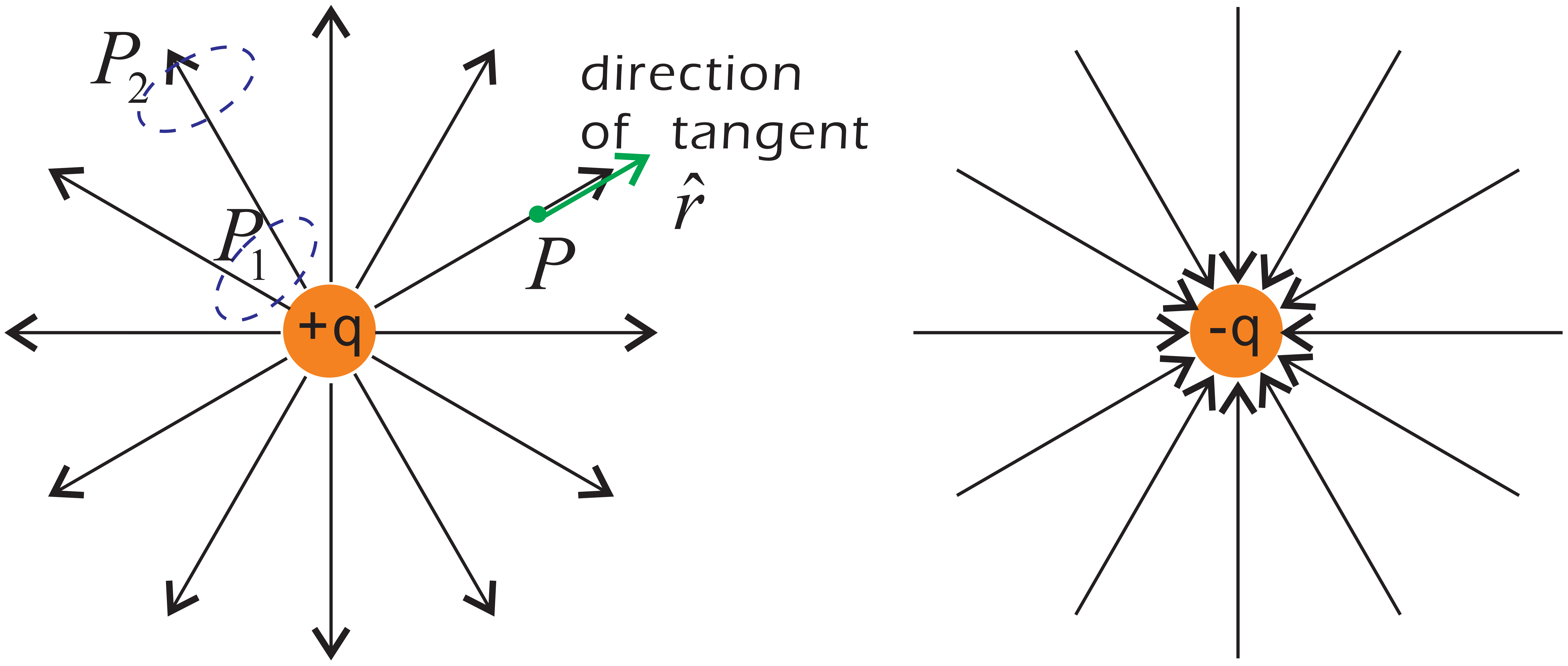
2. Direction of E-field at any point is given by tangent of E-field line

3. Magnitude of E-field at any point proportional to number of E-field lines
   per unit area perpendicular to lines

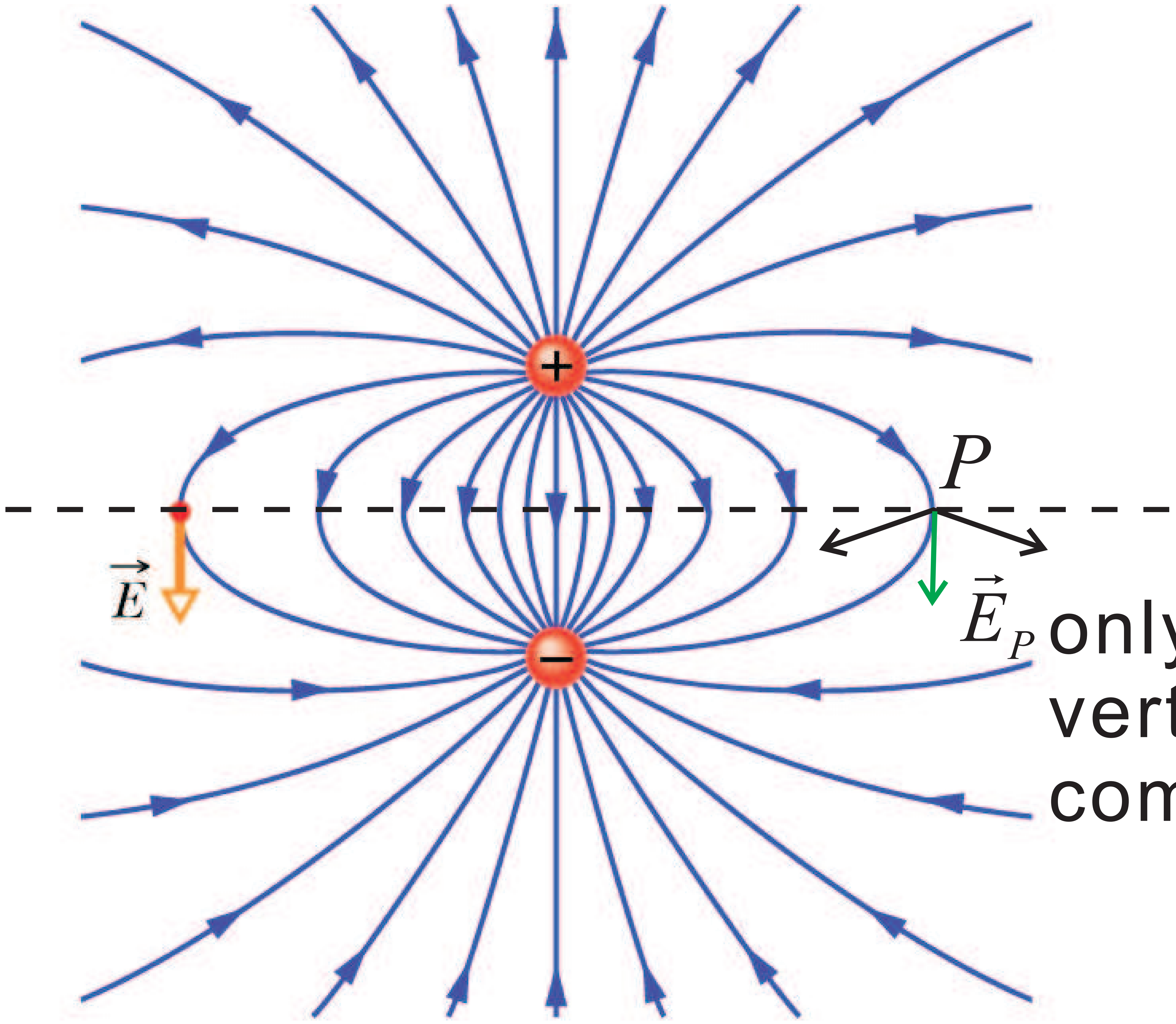
Uniform E-field

Non-uniform E-field

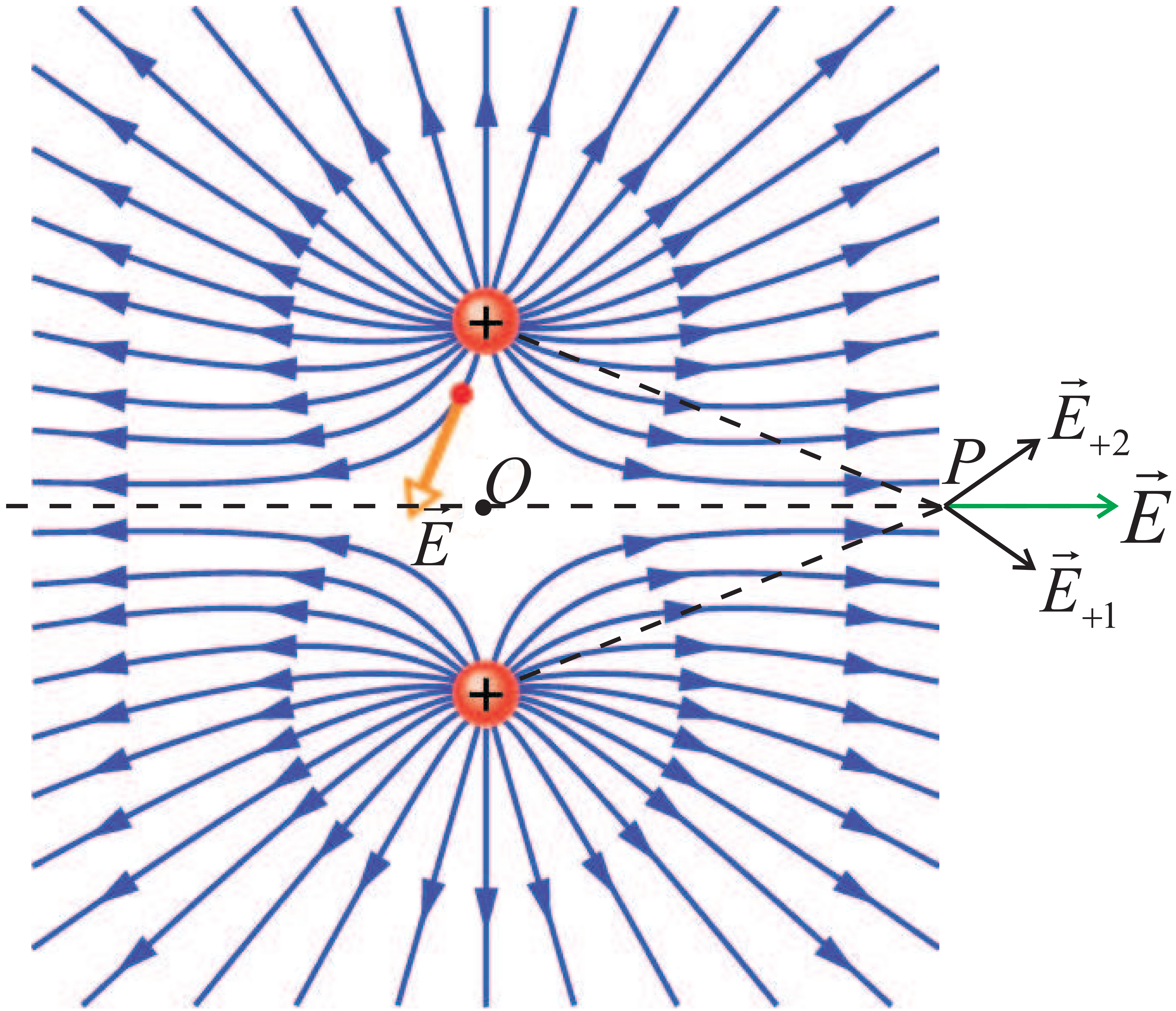




$$\left|\vec{E}_{P_1}\right| > \left|\vec{E}_{P_2}\right| \qquad \vec{E} = \frac{+q}{4\pi\epsilon_0 r^2}\hat{r}$$

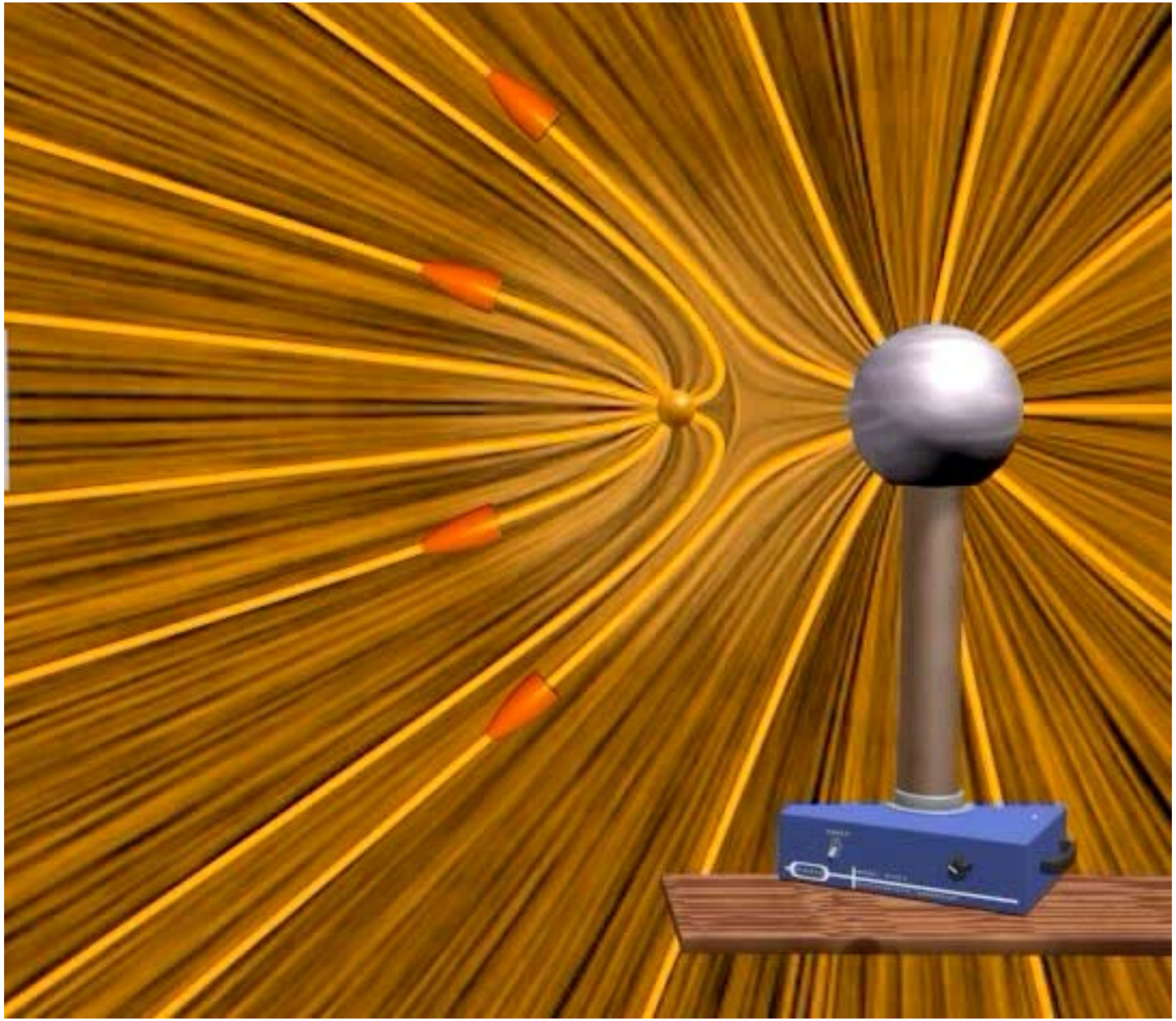


only has vertical component.



$$\vec{E}_{\text{at point } O} = 0$$

This is not a probe charge



Point Charge in E-field

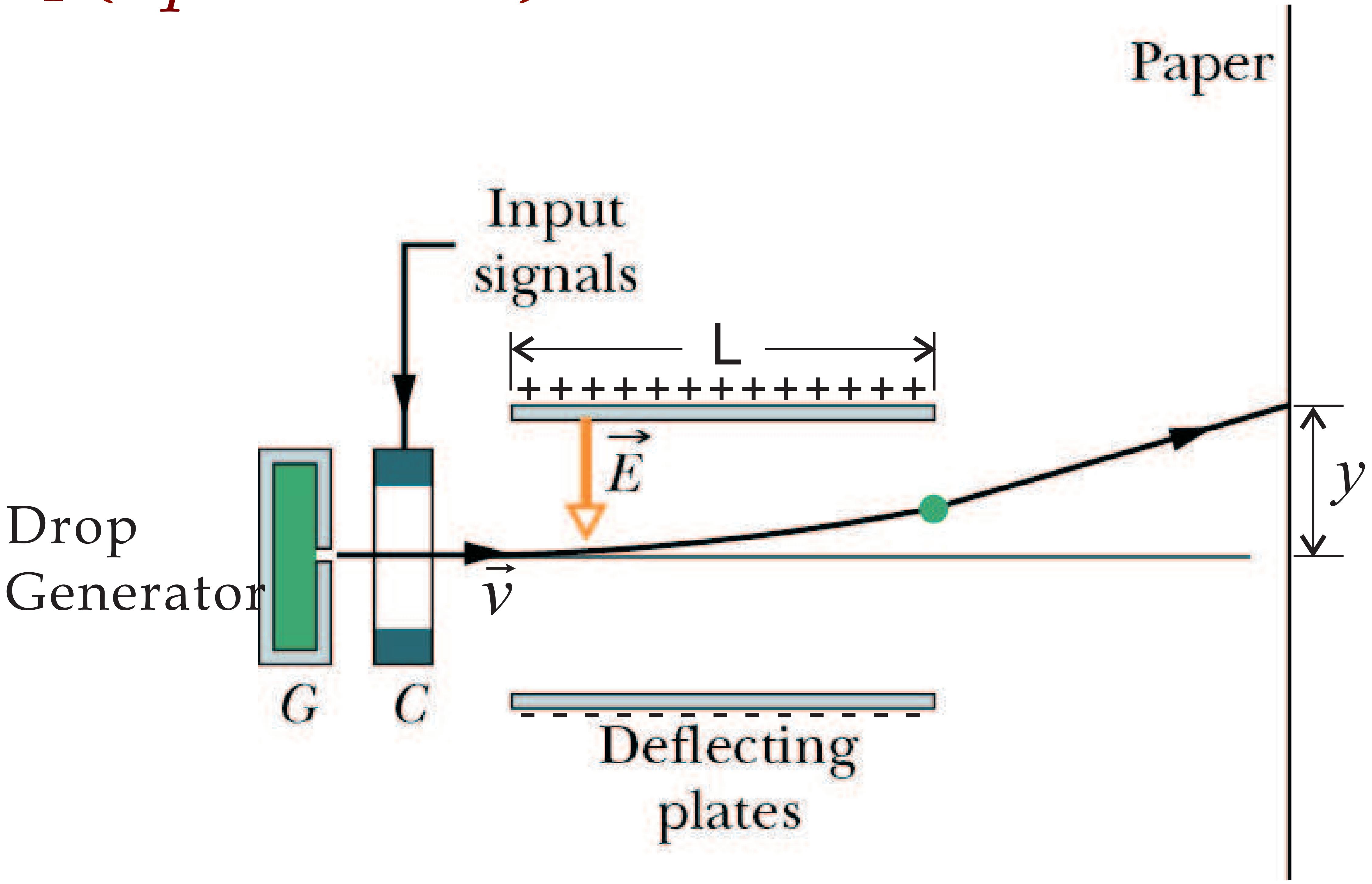
When we place a charge q in an E-field $\vec{E}$,force experienced by charge is

$$\vec{F} = q\vec{E} = m\vec{a}$$

Applications ☛ Ink-jet printer, TV cathode ray tube

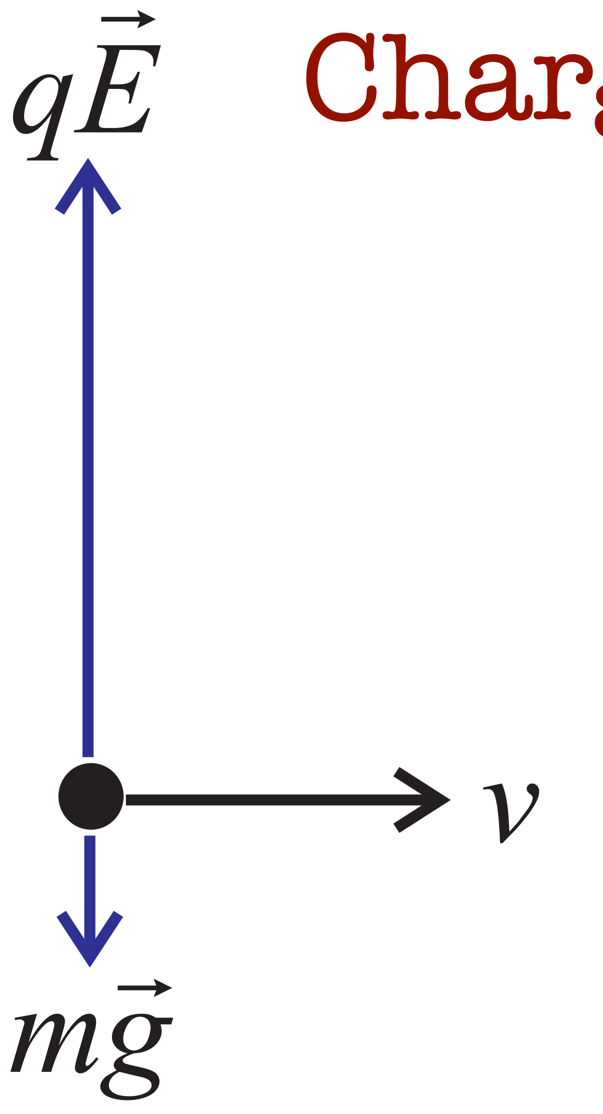
Example

Ink particle has mass m & charge q ($q < 0$ here)



Assume that mass of inkdrop is small, what's deflection of charge?

Solution ☛



Charge carried by inkdrop is negative ☛ $q < 0$

Note: $q\vec{E}$ points in opposite direction of $\vec{E}$

Horizontal motion ☛ Net force $= 0$

$$\therefore L = vt$$

Vertical motion ☛ $|q\vec{E}| \gg |m\vec{g}|$. q is negative

$\therefore$ Net force $= -|q|E = ma$ ☛ **Newton's 2nd Law**

$$\therefore a = -\frac{|q|E}{m}$$

Vertical distance travelled ☛ $y = \frac{1}{2}at^2$

Conductors and insulators

- ➢ Charges move through some materials more easily than others:

  - ✱ Charge moves easily: **Conductor**

    i.e. copper, silver, aluminum (metals)

  - ✱ Charge can't move: **Insulator**

    i.e. wood, paper, rubber, plastic,

- ➢ Charge can stick on the surface of insulators, but it doesn't really move

Electrical Wire

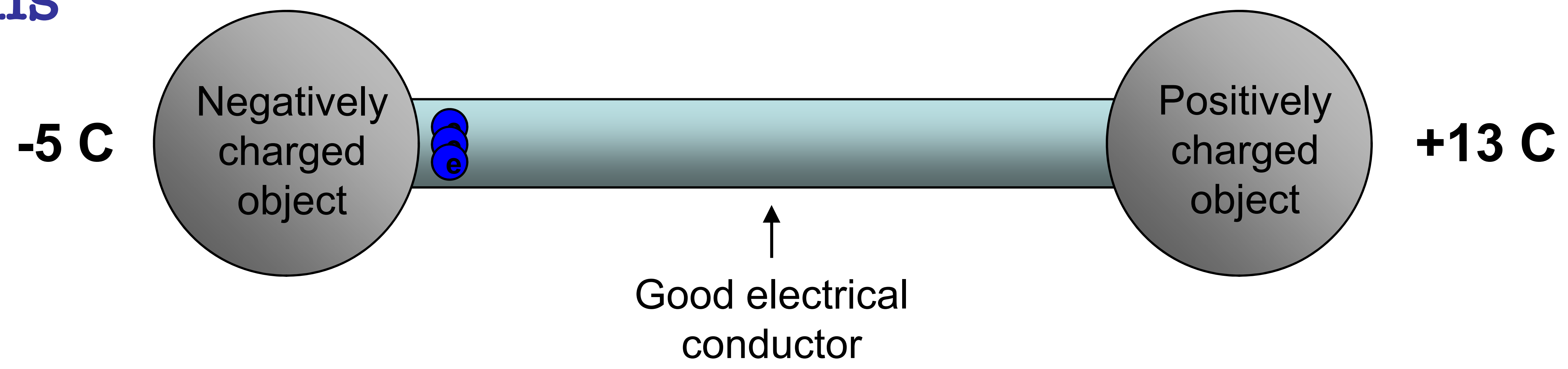
What determines whether a material is a good conductor o insulator?

☛ Ultimately, it's the **atomic structure**

➢ The outer most electrons (valence electrons) in an atom are more weakly bound to the nucleus

➢ They can "break free" and move through the material

➢ These are called **conduction electrons**



Let's say the object on the left starts out with a charge of -5 C, and the object on the right starts out with +13 C

Electrons will continue to flow until the charge on each object is....? EQUAL!

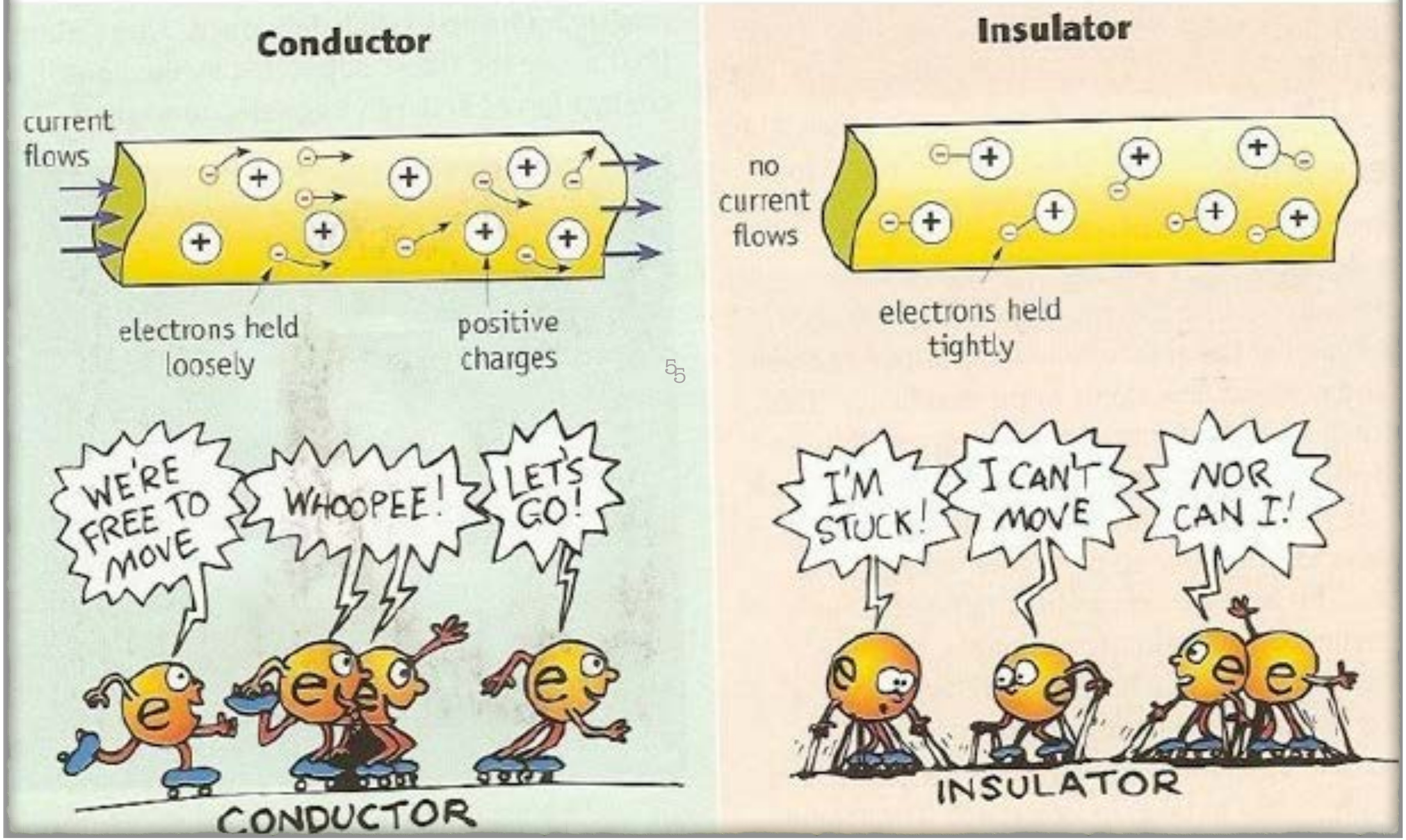
And, each must end up with a charge of +4C, since the total (+8C) must remain constant!

Electrons can "flow" through a good conductor

There would be no charge flow if the bridge above was an insulator

In a conductor, **electrons are free to flow**

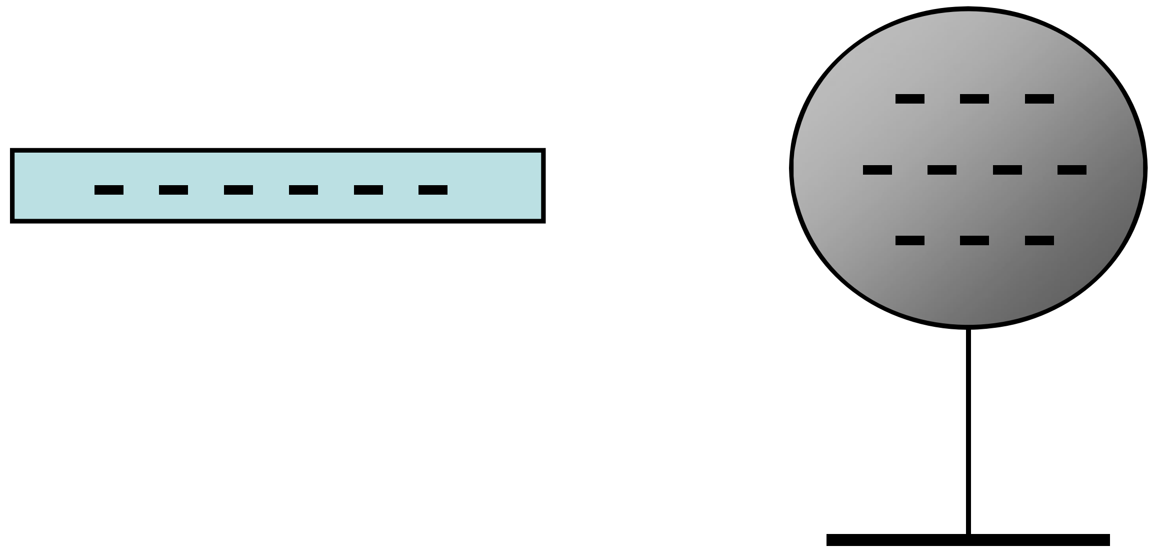
In an insulator, **electrons are fixed**



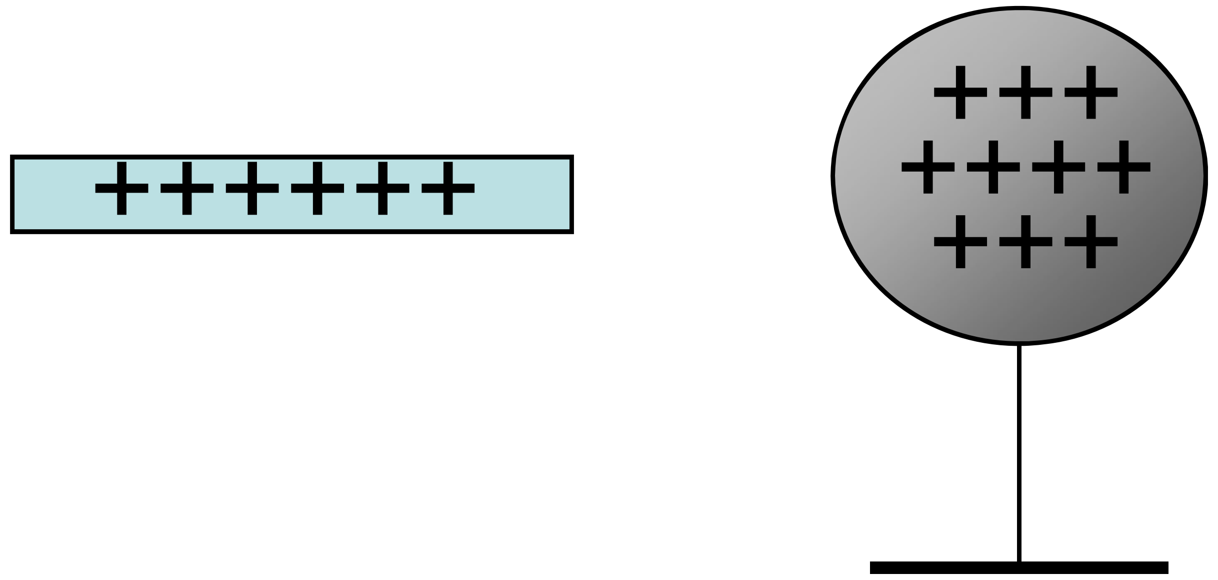
Charging an Object

Charging by Contact

- ➢ Touching a metal sphere with a negatively charged rod can give the sphere a negative charge



- ➢ Similarly, if we started with a positively charged rod:



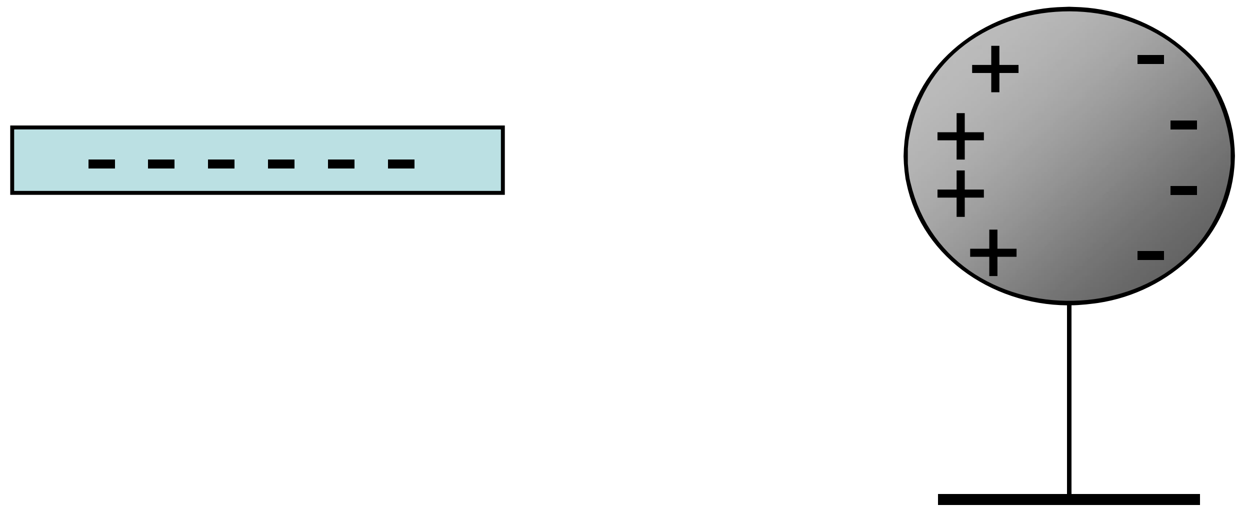
This is charging by contact

Charging an Object

Charging by Induction

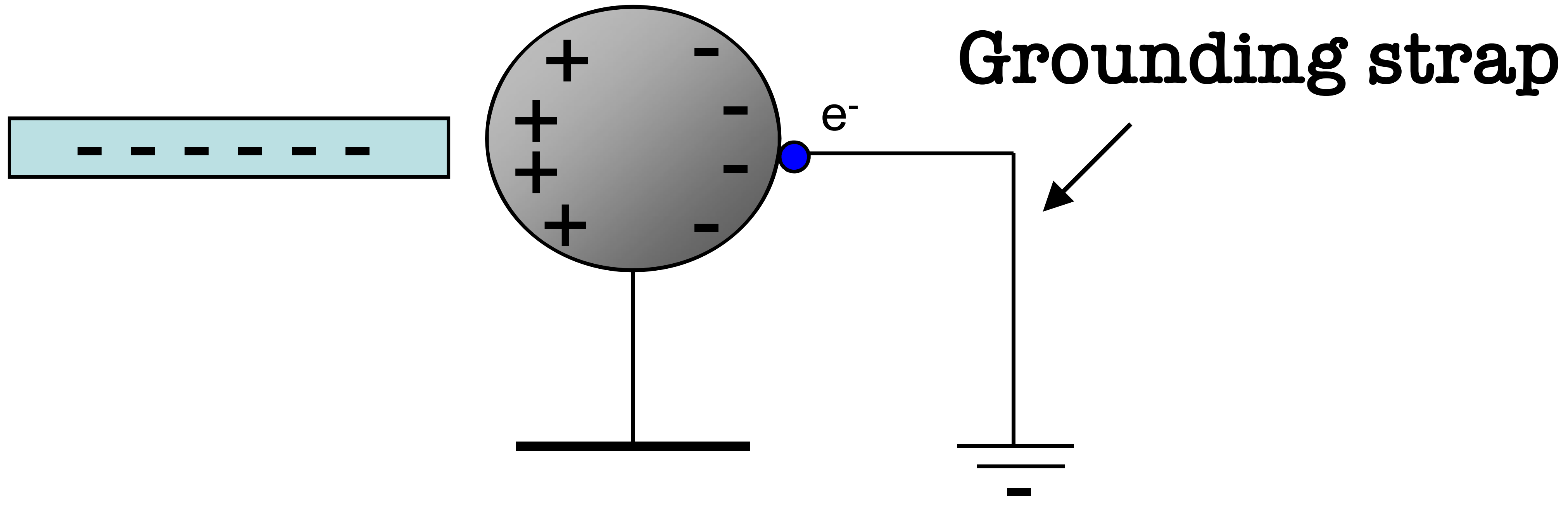
We can also charge a conductor without actually touching it

Bring a negatively charged rod close to the surface of an electrically neutral metal sphere

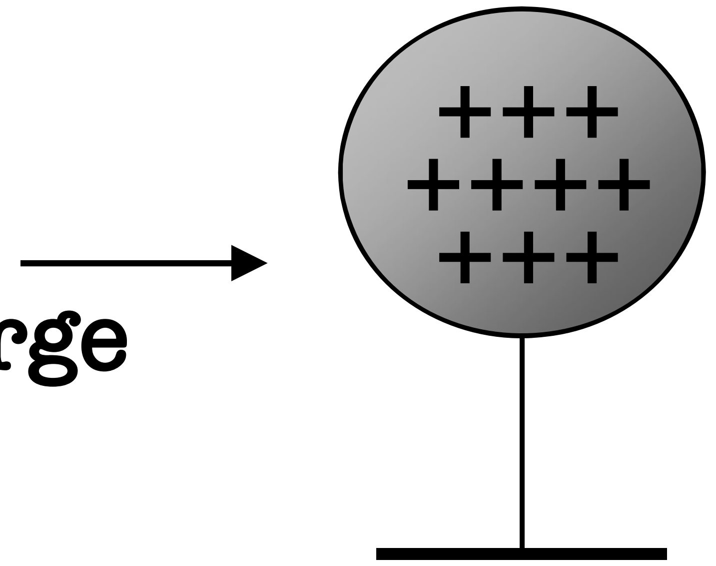


The free charges separate on the sphere’s surface

Now attach a metal wire between the sphere and ground



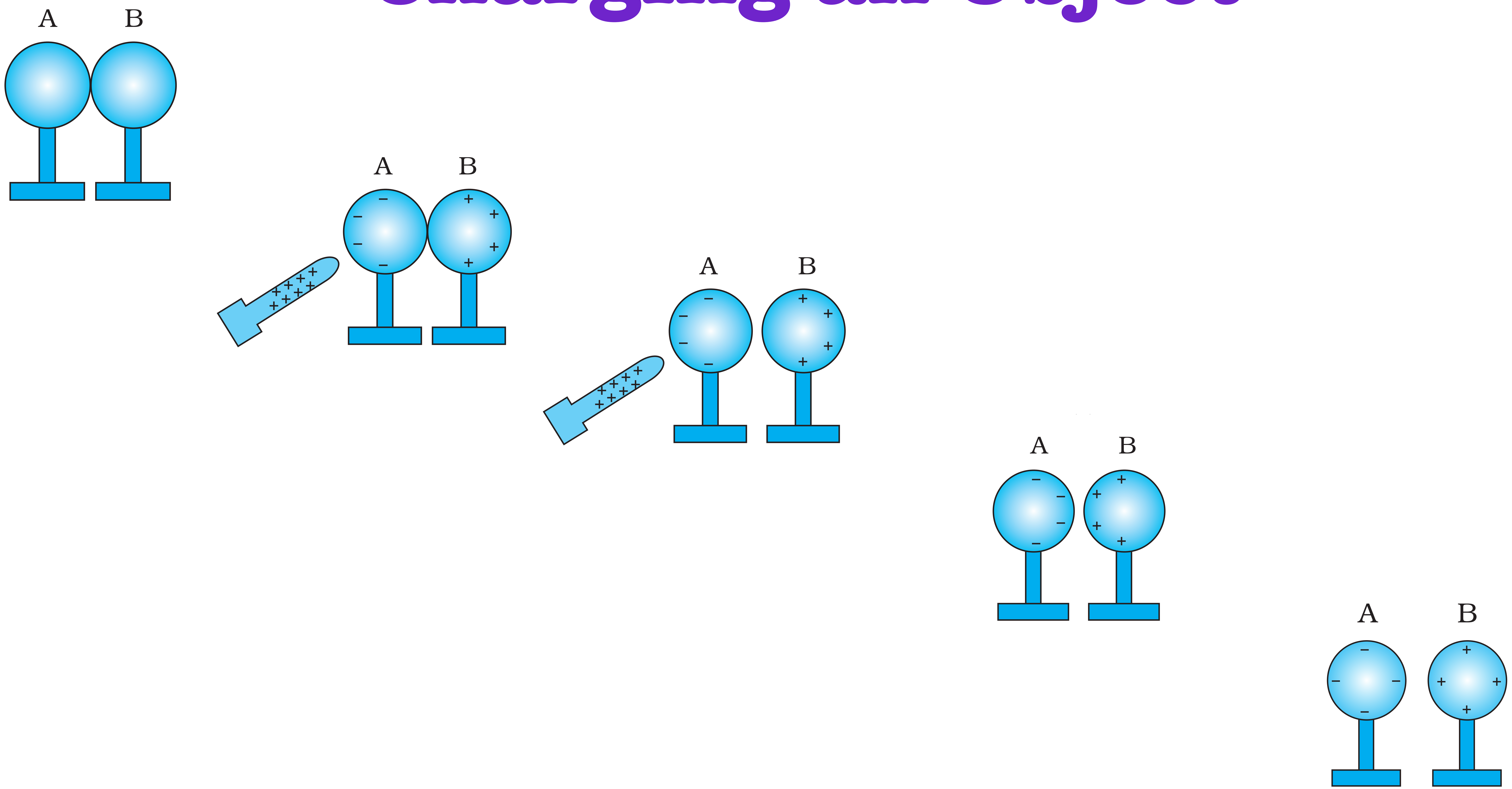
This leaves the sphere with a net positive charge



The electrons travel down the strap to ground

This is charging by Induction

Charging an Object

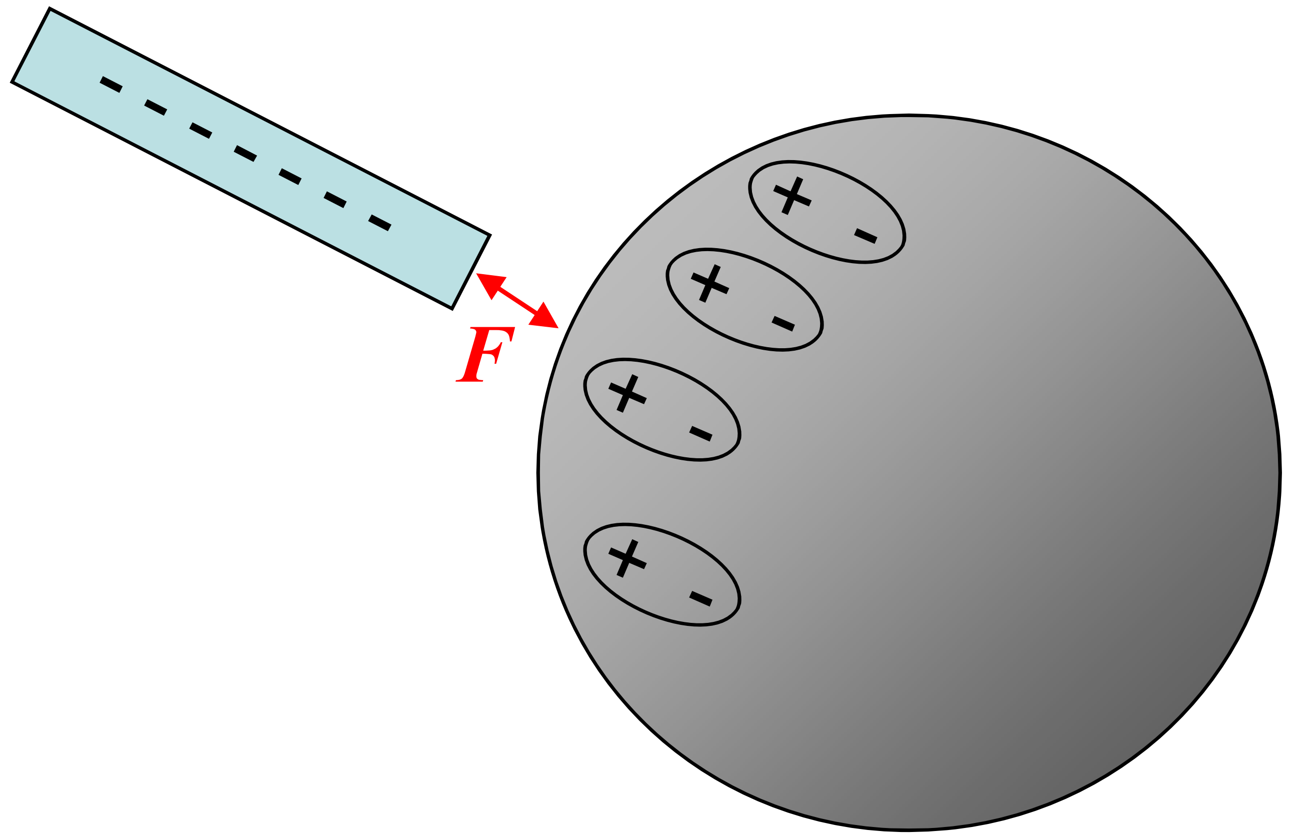


Another way by charging by Induction

➢ Charging by induction doesn't work for insulators, since the charge can't move through the material or down the grounding strap

But it does have an effect....

➢ Bring a negatively charged rod close to the surface of an insulating sphere



Even though the electrons can't move through the insulator, the positive and negative charge in each atom separates slightly and forms dipoles, since the positive protons in the atoms are attracted to the rod, and the negative electrons are repelled

This is called Polarization

Coulomb's Law and Electric Field

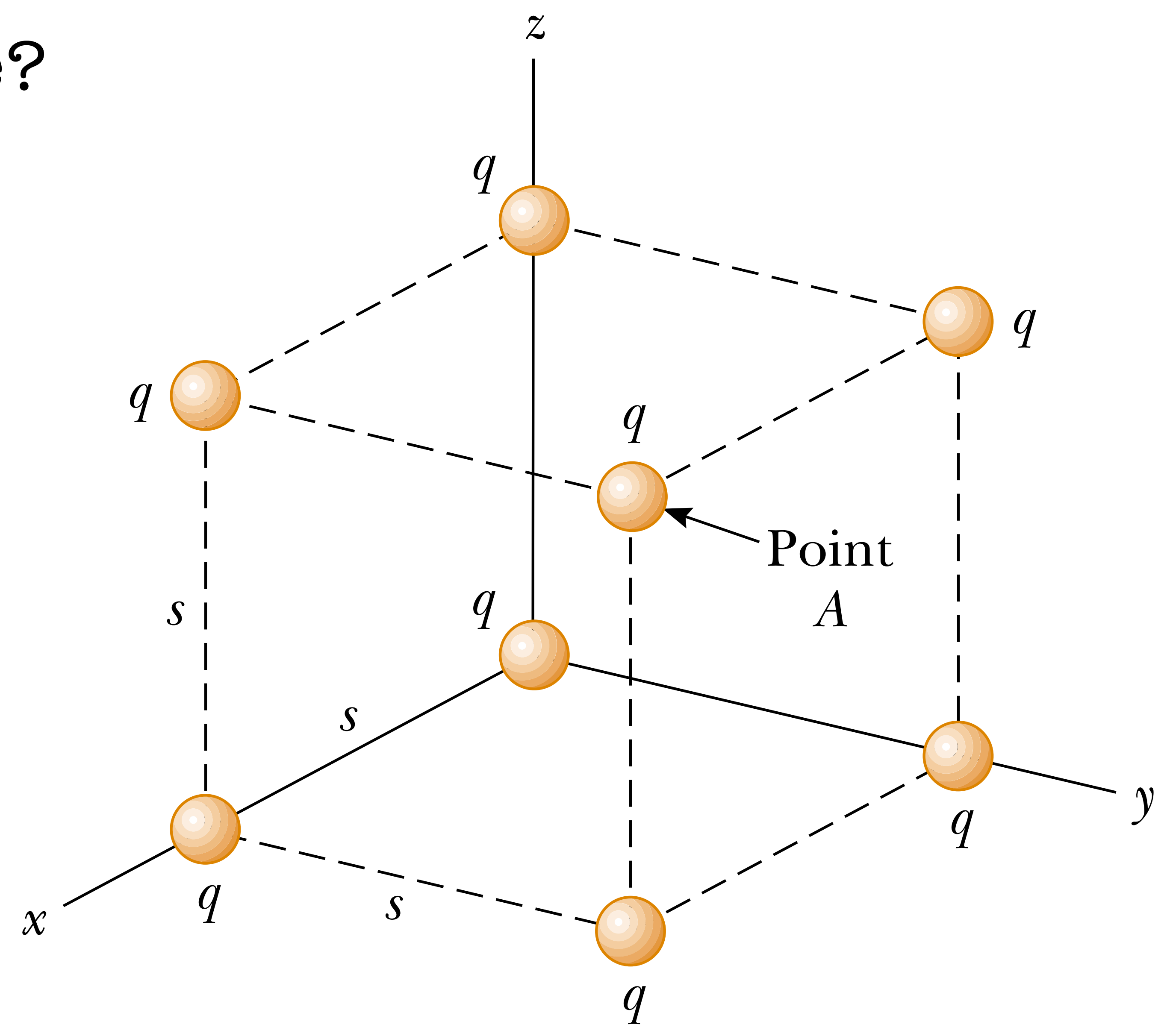
Eight point charges, each of magnitude q, are located on the corners of a cube of edge s.

(i) Determine the x, y, and z components of the resultant force exerted by the other charges on the charge located at point A.

(ii) What are the magnitude and direction of this resultant force?

(iii) Show that the magnitude of the electric field at the center of any face of the cube has a value of $2.18 \frac{1}{4\pi\epsilon_0} \frac{q}{s}$

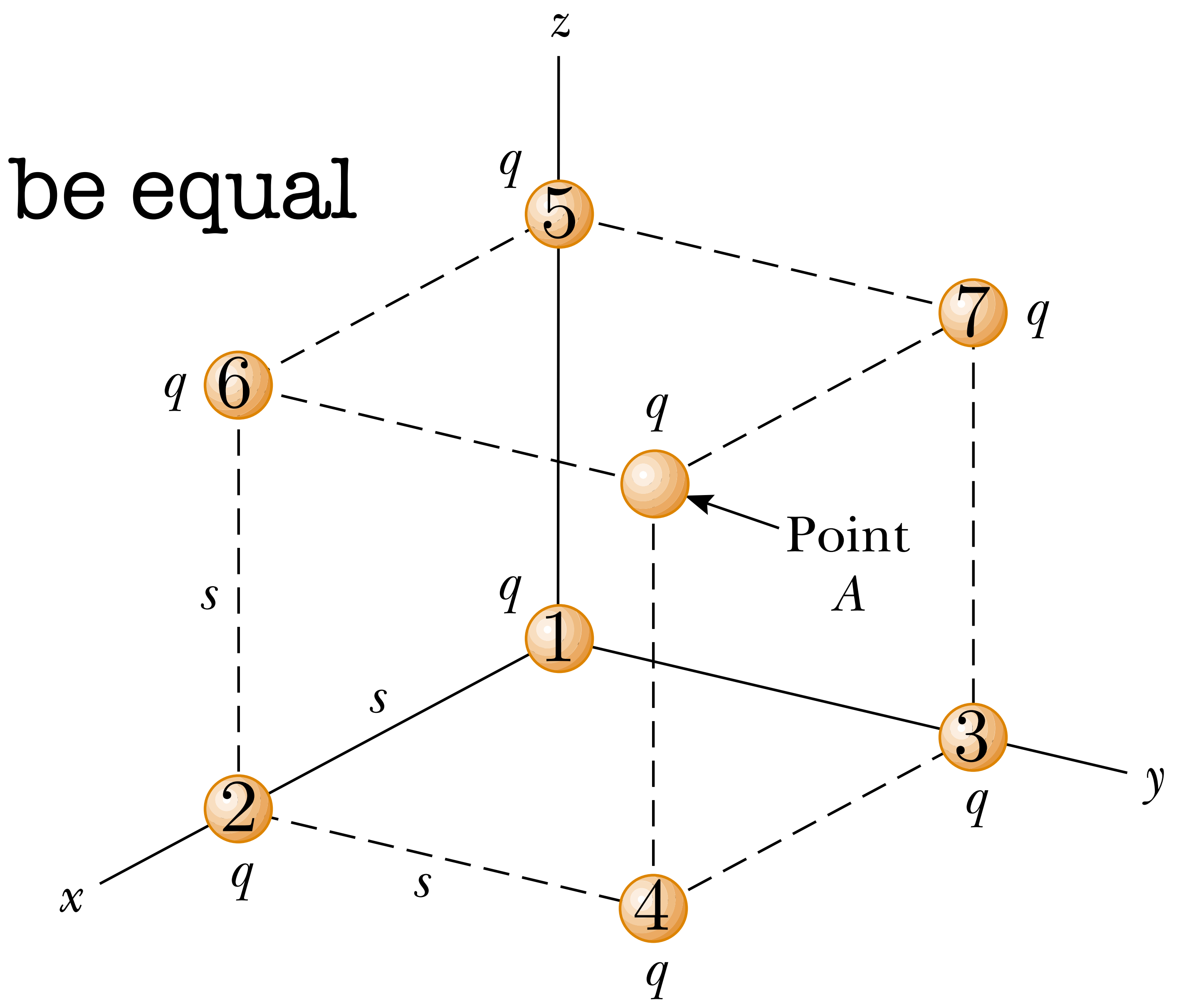
(iv) What is the direction of the electric field at the center of the top face of the cube?



Coulomb's Law and Electric Field

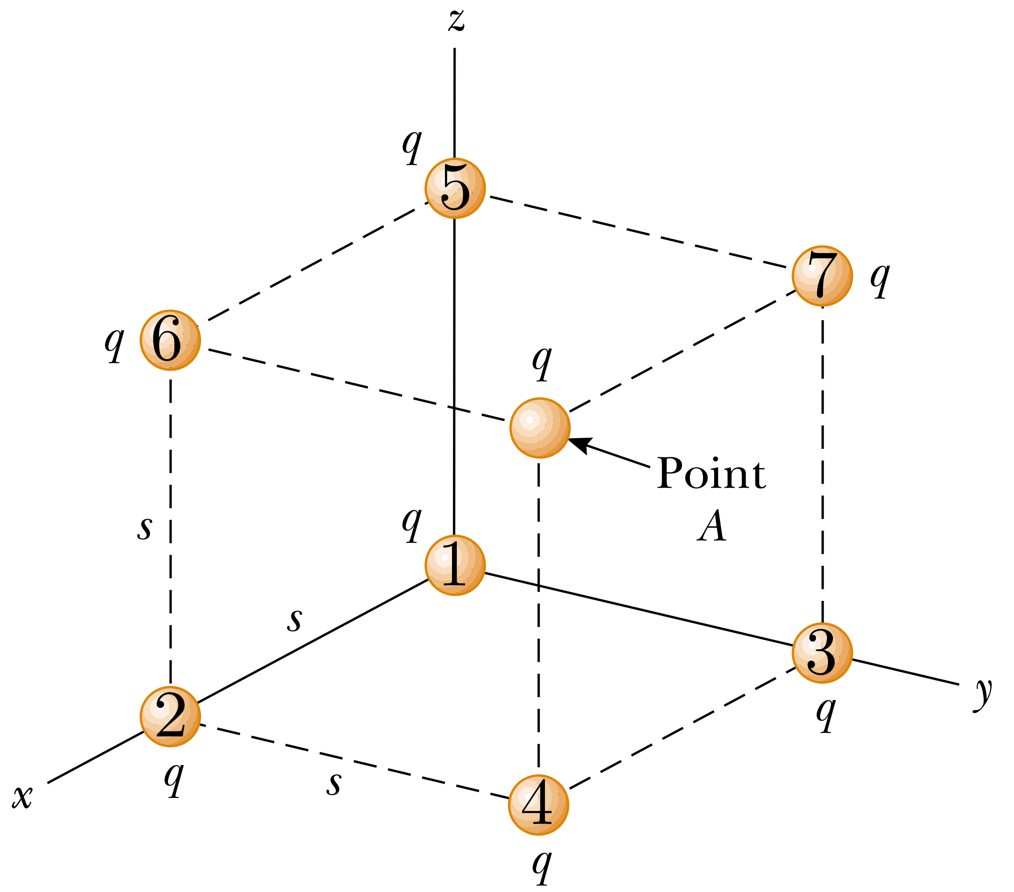
Solution (i)

- There are 7 terms that contribute
- There are 3 charges a distance s away (along sides), 3 a distance $\sqrt{2}s$ away (face diagonals), and one charge a distance $\sqrt{3}s$ away (body diagonal)
- By symmetry, the x, y and z components of the electric force must be equal
- Thus, we only need to calculate one component of the total force on the charge of ineterest
- We will choose the coordinate system as indicated in Figure, and calculate the y component of the force.



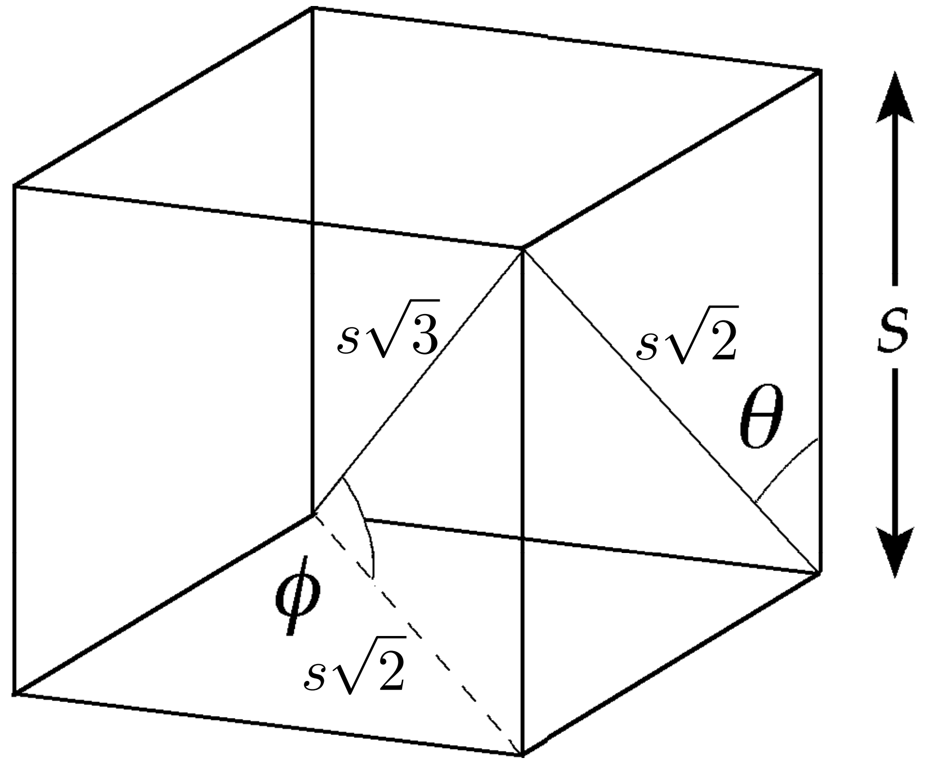
Coulomb's Law and Electric Field

- We can already see that several charges will not give a y component of the force at all, just from symmetry - charges 3, 4 and 7
- This leaves only charges 1, 2, 5, and 6 to deal with
- Charge 6 will give a force purely in the y direction: $F_{6,y} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}$

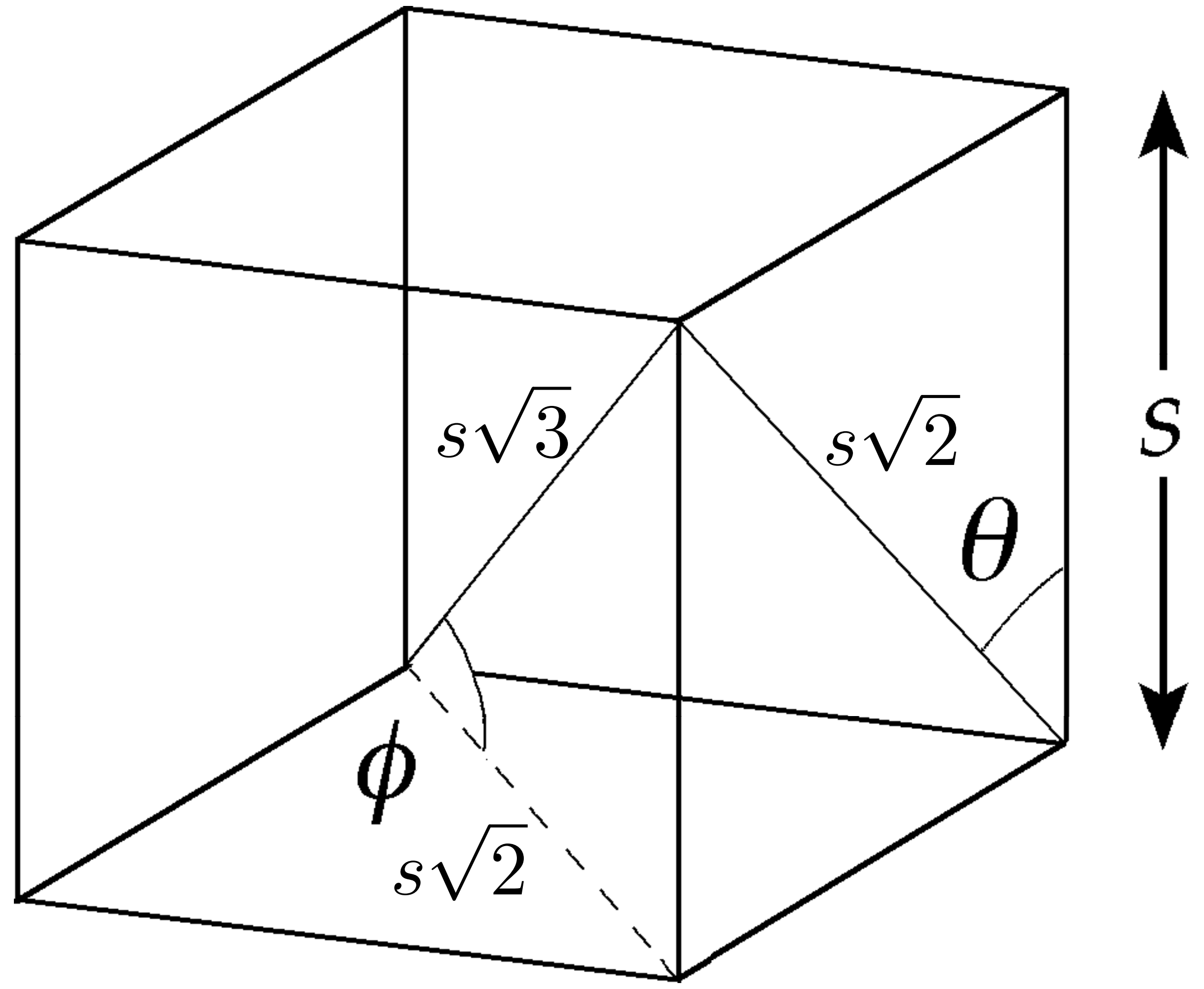


- Charge 5 and 2 are both a distance $s\sqrt{2}$ away, and a line connecting these charges with the charge of interest make an angle $\theta = 45^\circ$ with the y-axis in both cases
- Hence, noting that $\cos\theta = 1/\sqrt{2}$ we obtain

$$F_{2,y} = F_{5,y} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(s\sqrt{2})^2}\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{q^2}{2s^2}\frac{1}{\sqrt{2}}$$



Coulomb's Law and Electric Field



- Finally, we have charge 1 to deal with.
- It is a distance $\sqrt{3}$ away
- What is the y component of the force from charge 1?
- First, we can find the component of the force in the x-y plane $F_{1,x-y} = F_1 \cos\phi = F_1 \frac{\sqrt{2}}{\sqrt{3}}$
- Now, we can find the component of of the force along the y direction:

$$F_{1,y} = F_{1,x-y}\cos\theta = F_{1,x-y}\frac{1}{\sqrt{2}} = F_1\frac{\sqrt{2}}{\sqrt{3}}\frac{1}{\sqrt{2}} = F_1\frac{1}{\sqrt{3}}$$

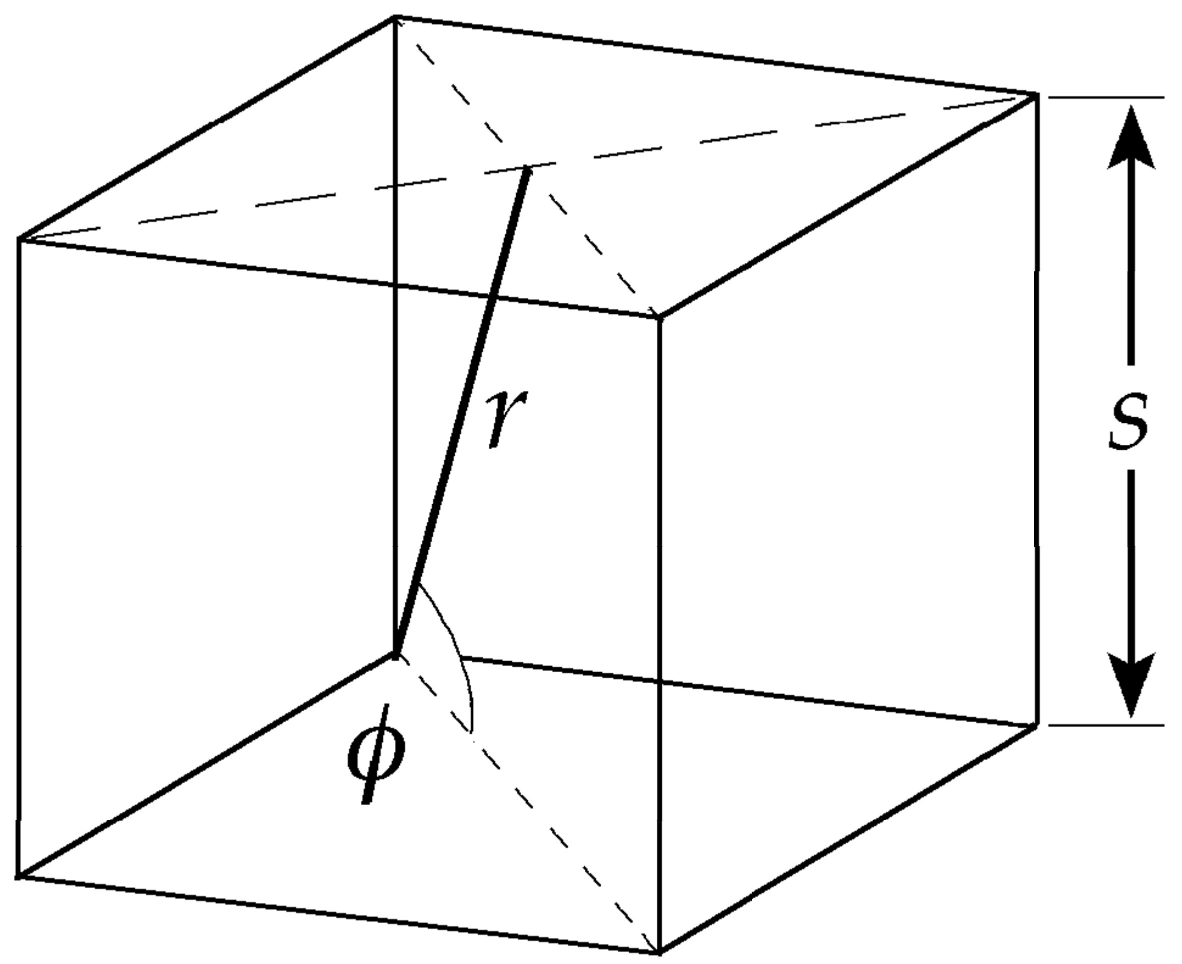
Coulomb's Law and Electric Field

- Since we know charge 1 is a distance $s\sqrt{3}$ away, we can calculate the full force F_1 easily, and complete the expression for $F_{1,y}$ that is $F_{1,y} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(s\sqrt{3})^2}\frac{1}{\sqrt{3}} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}\frac{1}{3\sqrt{3}}$
- Now we have the y component for the force from every charge; the net force in the y direction is just the sum of all those:

$$F_{y,\text{net}} = F_{1,y} + F_{2,y} + F_{5,y} + F_{6,y} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}\left[1 + \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}}\right] = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}\left[1 + \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right]$$

- Since the problem is symmetric in the x, y, and z directions, all three components must be equivalent
- The force is then $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}\left[1 + \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right](\hat{\imath} + \hat{\jmath} + \hat{k}) = 1.90\frac{1}{4\pi\epsilon_0}\frac{q^2}{s^2}(\hat{\imath} + \hat{\jmath} + \hat{k})$

Coulomb's Law and Electric Field



Solution (ii)

- $F = \sqrt{F_x^2 + F_y^2 + F_z^2} = 3.29 \frac{1}{4\pi\epsilon_0} \frac{q^2}{s^2}$ away from the origin

Solution (iii)

- There is zero contribution from the same face due to symmetry.
- The opposite face contributes $\frac{q \quad \sin\phi}{\pi\epsilon_0 r^2}$ where $r = \sqrt{\frac{(\sqrt{2}s)^2}{4} + s^2} = \sqrt{1.5}\, s = 1.22s$ and $\sin\phi = s/r$
- All in all $F = \frac{q \quad s}{\pi\epsilon_0 r^3} = 2.18 \frac{1}{4\pi\epsilon_0} \frac{q}{s^2}$

Solution (iv)

- The direction is $\hat{k}$


aNy