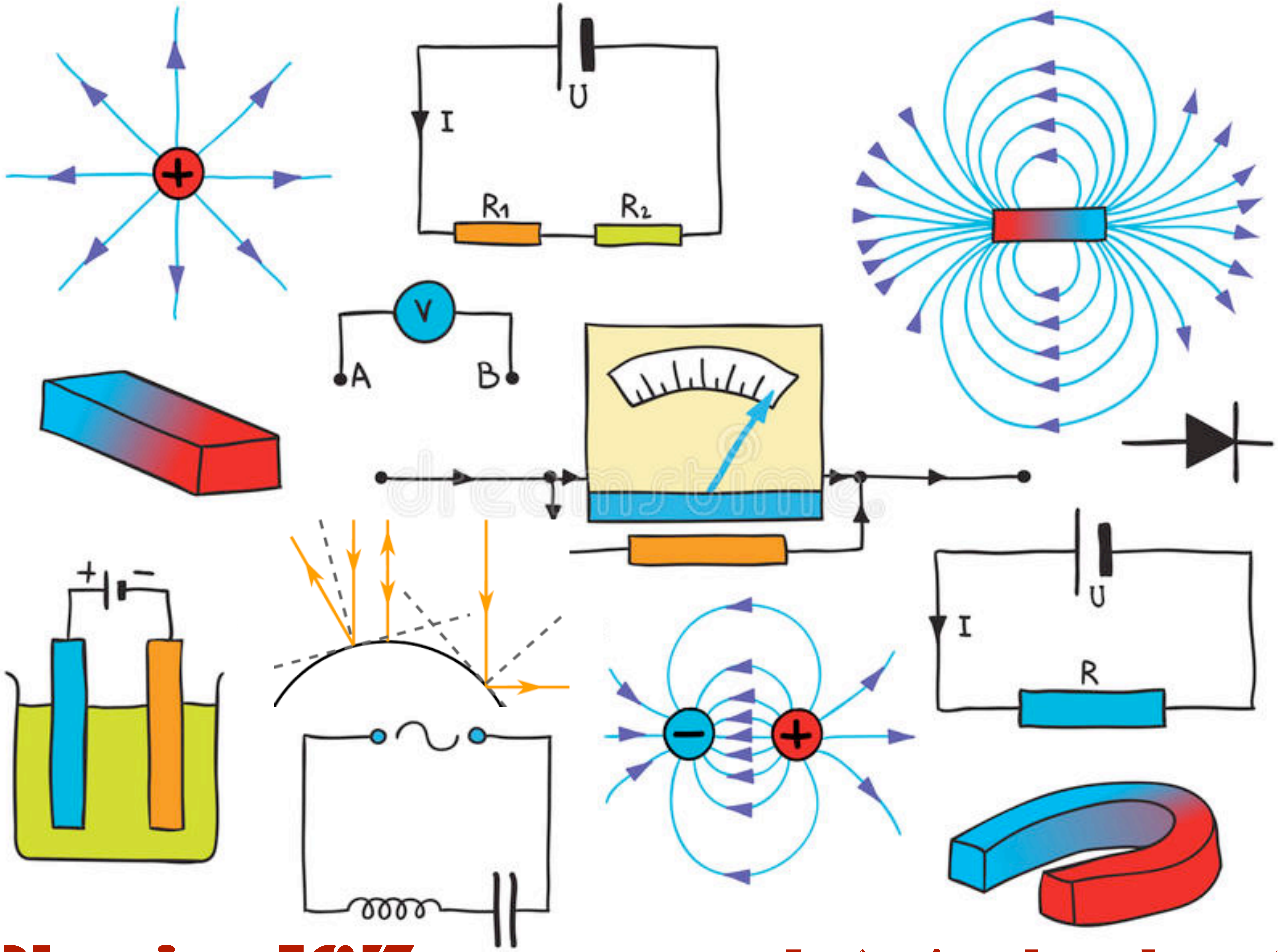


Physics 167

Luis Anchordoqui

Interference and Wave Nature Of Light

- We have been studying **geometrical optics**, where wavelength of light is much smaller than size of our mirrors and lenses and distances between them

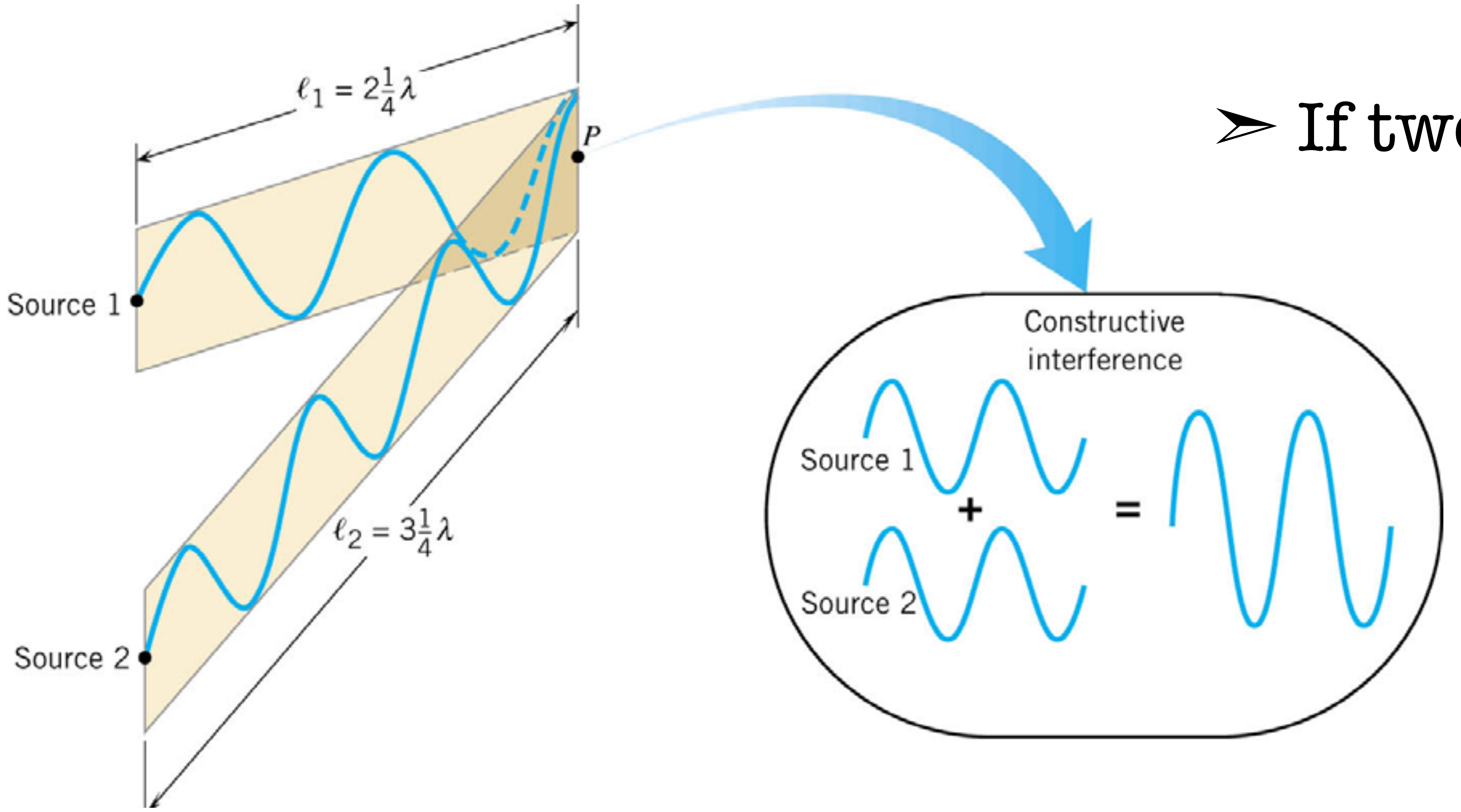
- Propagation of light is well described by linear rays except when reflected or refracted at surface of materials

- Now we will study **wave optics**, where wavelength of light is comparable to size of an obsatcle or aperture in its path

- This leads to wave phenomena of light called **interference** and **diffraction**

Principle of Linear Superposition

➢ Take two waves of equal amplitude and wavelength and have them meet at a common point



➢ If two waves are **in-phase** ☛ they meet crest -to-crest and trough-to-trough

➢ Their two amplitudes add to each other

➢ In this case, resulting wave would have an amplitude that doubled

➢ This is called **Constructive Interference (CI)**

- **Define Optical Path Difference (OPD)**

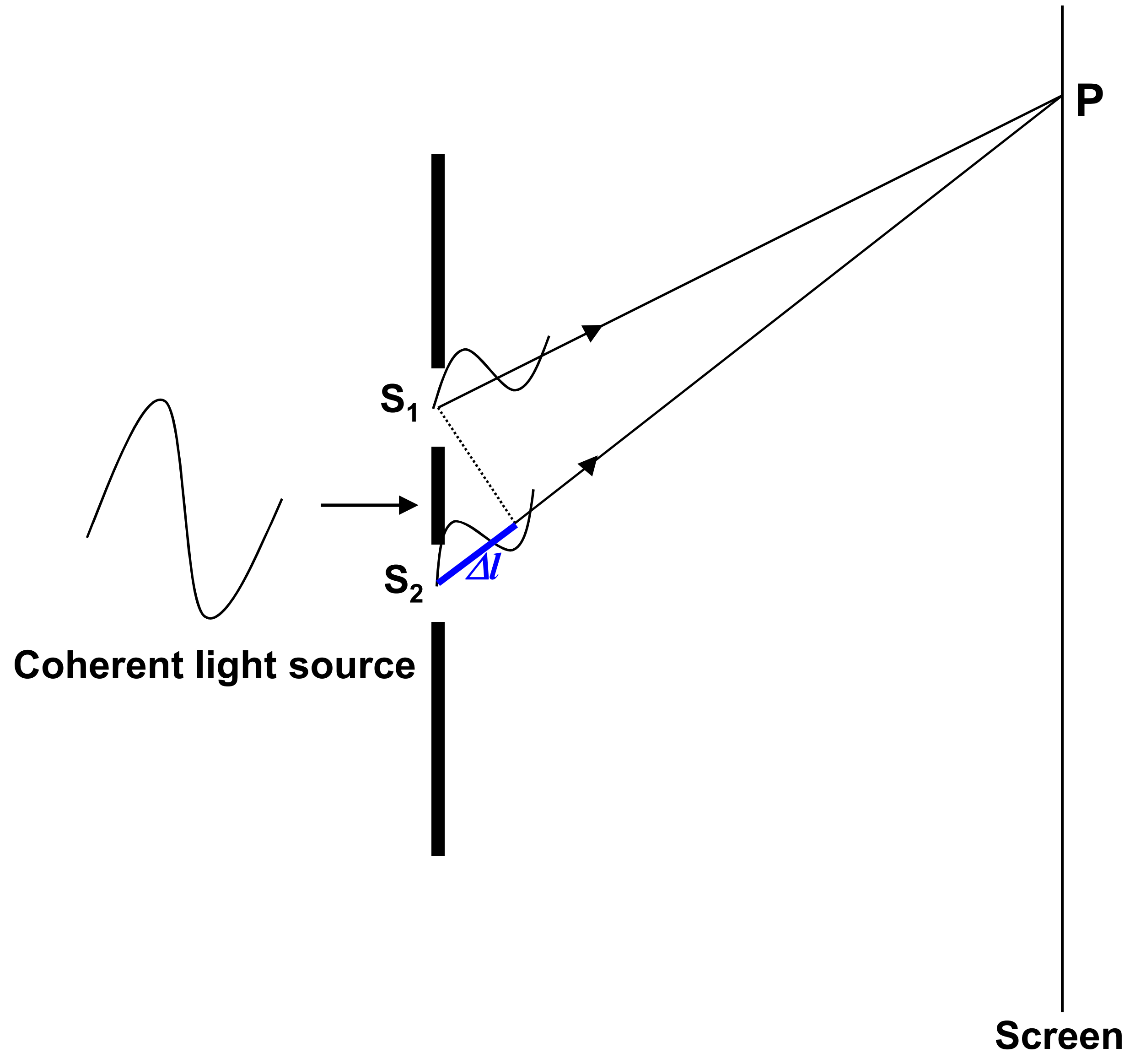
OPD = Difference in distance that two waves travel

➢ For CI to occur, we need waves to meet crest-to-crest, thus waves must differ by an integer multiple of wavelength λ

$$\mathrm{OPD} = m\lambda,\ m = 0, 1, 2, \ldots$$ **Constructive Interference**

Young's Double Slit Experiment

- Plane wave is diffracted by each of slits so that light passing through each slit covers a much larger area on screen than geometric shadow of slit
- This causes light from two slits to overlap on the screen ☛ producing interference
- Each slit acts like a coherent light source



- Two waves meet at point P on a screen
- Δl is optical path difference of two light waves coming from S_1 and S_2
- Two waves interfere with each other, and if

  $\Delta l = m\lambda$ Constructive interference, and we see a bright spot

  $\Delta l = (m + \frac{1}{2})\lambda$ Destructive interference and we see a dark spot

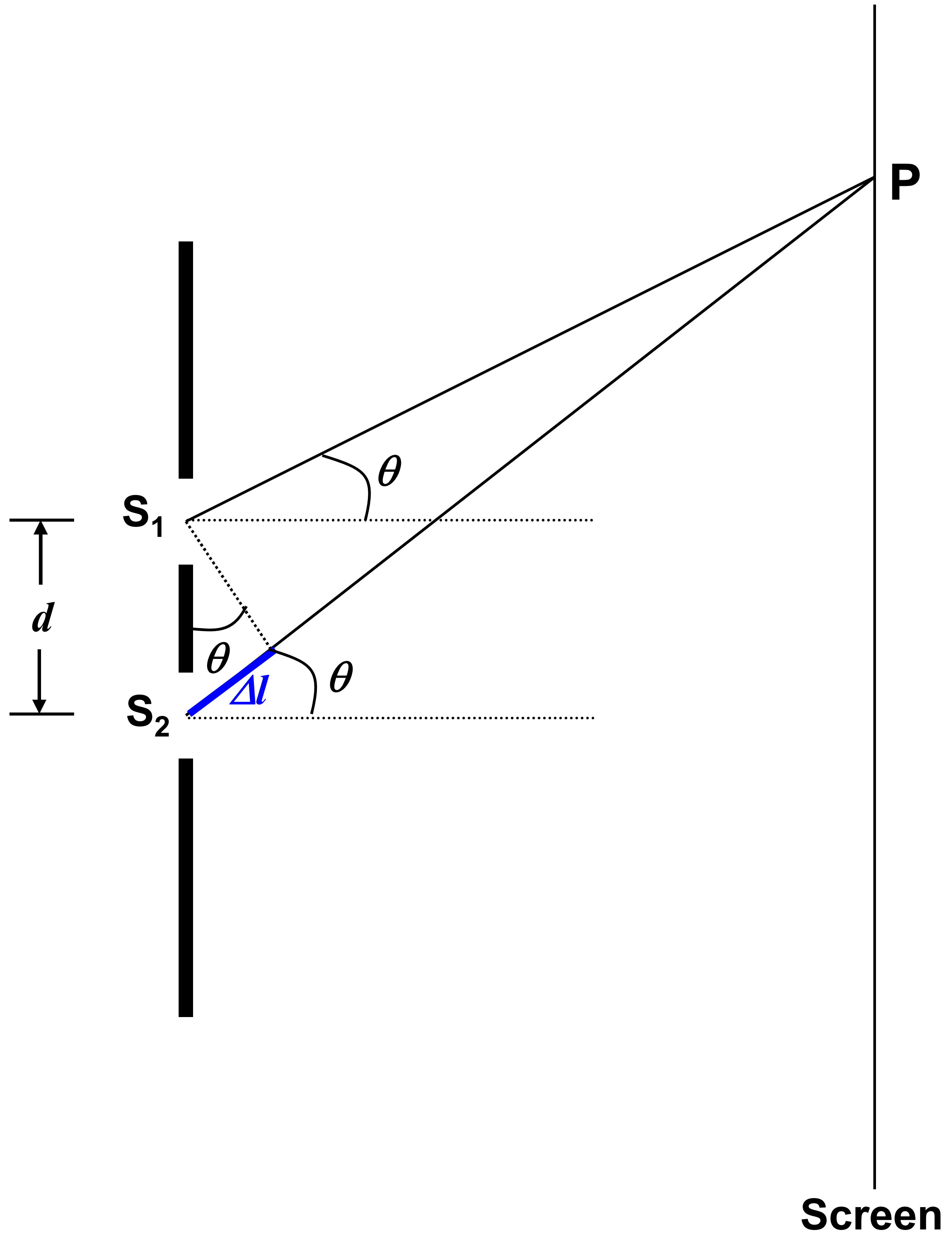
- Thus, we should see alternating bright and dark regions (called **fringes**) as we move along screen and above two conditions are satisfied

- Can we find a relationship between fringes and wavelength of light?

- Answer is ☛ YES

- Assume screen is far away from slits which are small
- This is called Fraunhofer approximation
- Since slits are very close together ☛ θ is same for each ray
- From figure we see that

$$\sin\theta = \frac{\Delta l}{d} \quad \Rightarrow \Delta l = d\sin\theta$$



- We know that for constructive interference ☛ $\Delta l = m\lambda$

- $d\sin\theta = m\lambda$ for **constructive interference**

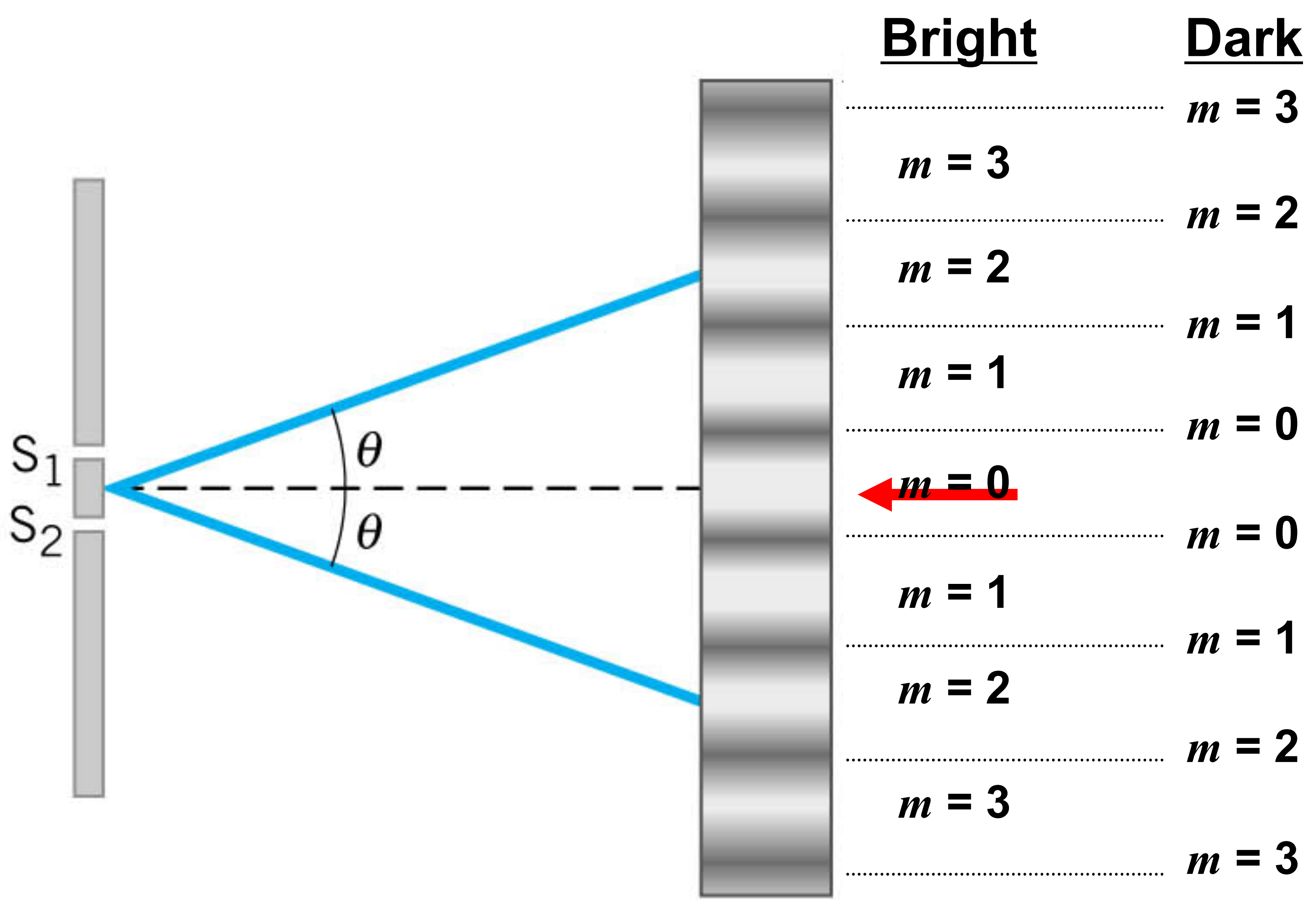
and ...

m is **order** of finge

$d\sin\theta = (m+\frac{1}{2})\lambda$ for **destructive interference**

These are interference conditions for double slit

➢ This is what a typical double slit interference pattern would look like



➢ Notice there are alternating light and dark fringes

➢ Also note that central fringe at $\theta = 0$ is a bright fringe

➢ It is also brightest of bright fringes

➢ Order of bright fringes starts at central bright fringe

➢ Order of dark fringes starts right above and below central bright fringe

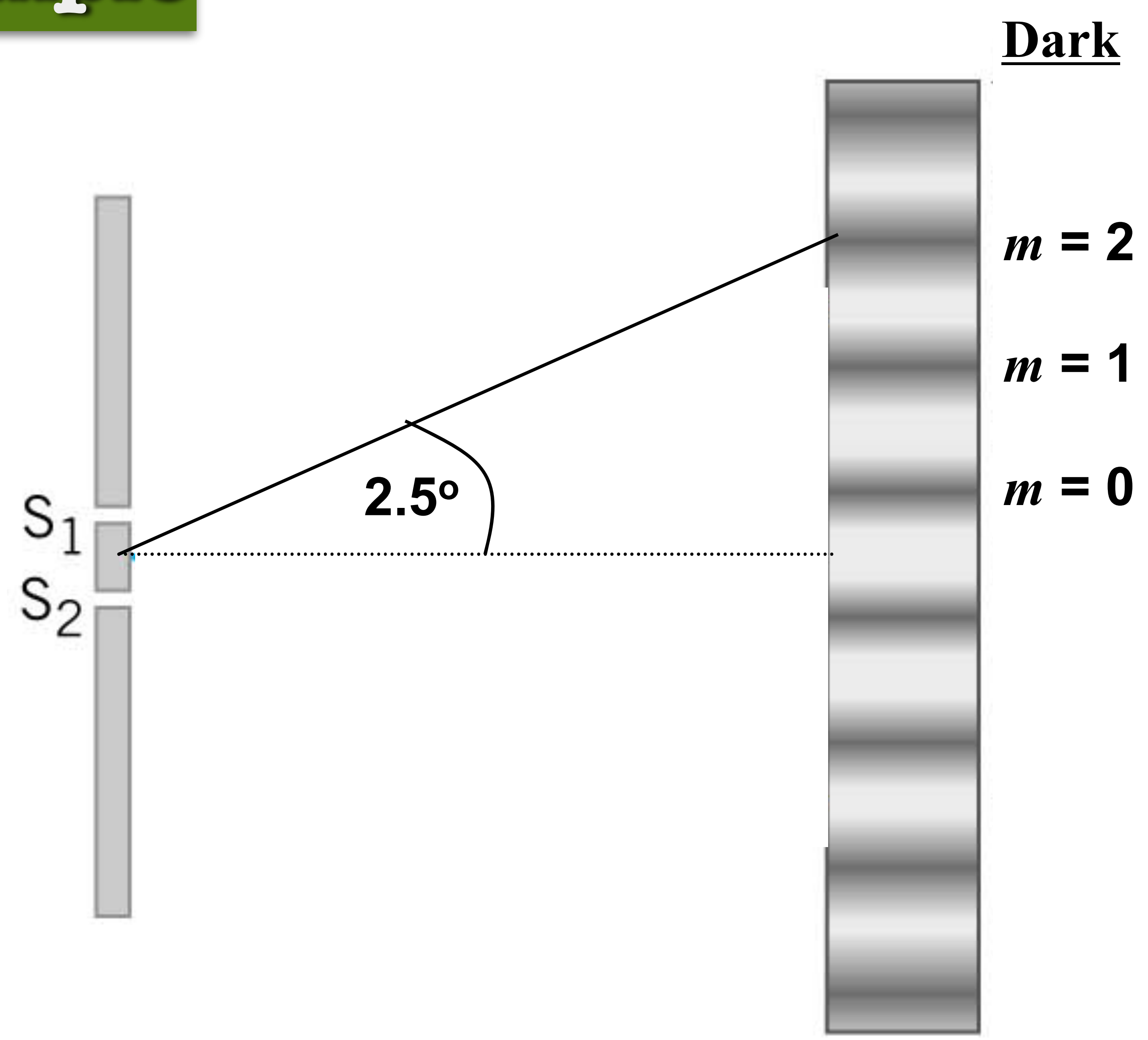
➢ Second dark fringe on either side of central bright fringe is 1st order dark fringe, or $m = 1$

Remember ☛ order means *m*

✱ Young's experiment provided strong evidence for wave nature of light

✱ If it was completely particle like☛ we could only get two fringes on screen not an interference pattern!

Example



- In a Young's double-slit experiment, angle that locates 3rd dark fringe oneither side of central bright maximum is 2.5°
- Slits have a separation distance $d = 3.8 \times 10^{-5}\, m$
- What is wavelength of light?
- What is the order?
- It is 2nd order dark fringe, or m = 2

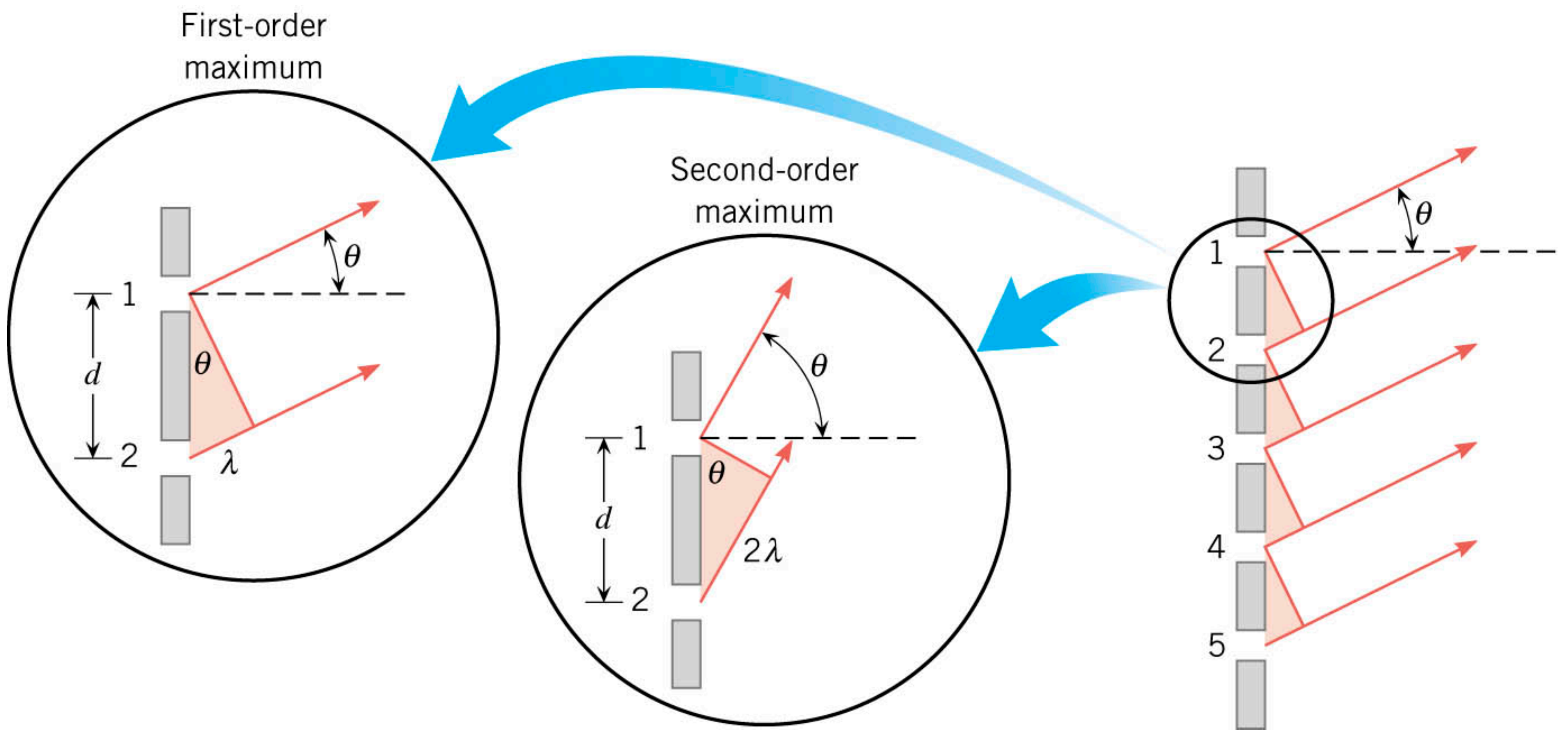
- Since it's a dark fringe ☛ we know it must be destructive interference

$$d \sin\theta = (m + \tfrac{1}{2})\lambda$$

$$\lambda = \frac{d \sin\theta}{m + \frac{1}{2}} \Rightarrow \lambda = \frac{(3.8 \times 10^{-5})(\sin 2.5^o)}{2 + \frac{1}{2}} \quad \Rightarrow \lambda = 6.63 \times 10^{-7}\ \text{m} = \boxed{663\ \text{nm}}$$

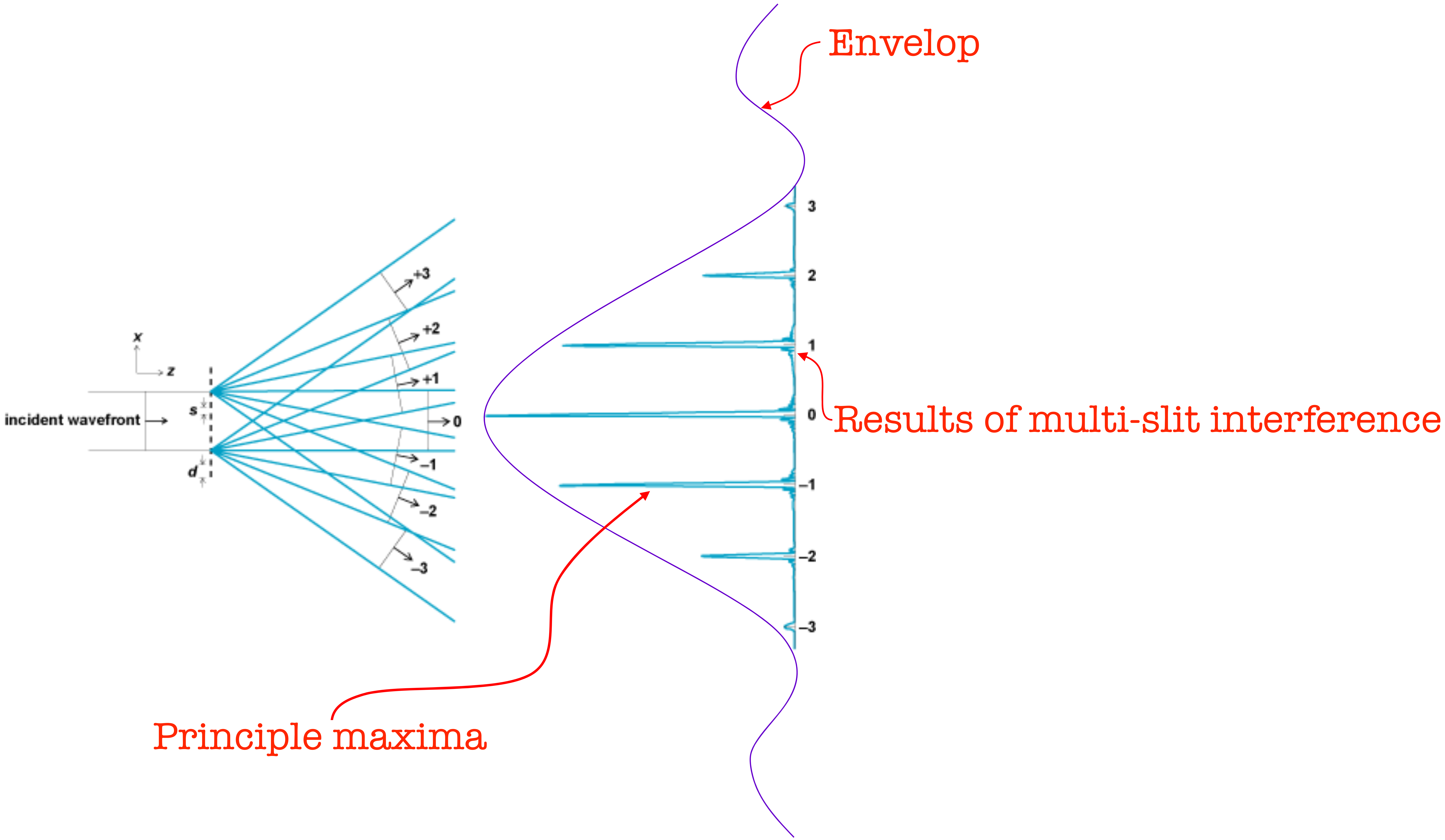
Diffraction Grating

- ➢ What if we shined light on many close-spaced slits?
- ➢ What would we expect to see?
- ➢ Such an instrument is called a **Diffracting Grating**
- ➢ Some of them can have tens of thousands of slits per cm
- ➢ Again we see alternating bright and dark fringes
- ➢ Each slit acts as a source of wavelets



- ➢ (m =1 and 2) maxima (bright fringes) develop

Fringe Formation of Multiple Diffraction



Origins of Quantum Mechanics

- ➢ Quantum mechanics was born in early 20th century due to collapse of deterministic classical mechanics

- ➢ Collapse resulted from Discovery of various phenomena which are inexplicable with classical physics

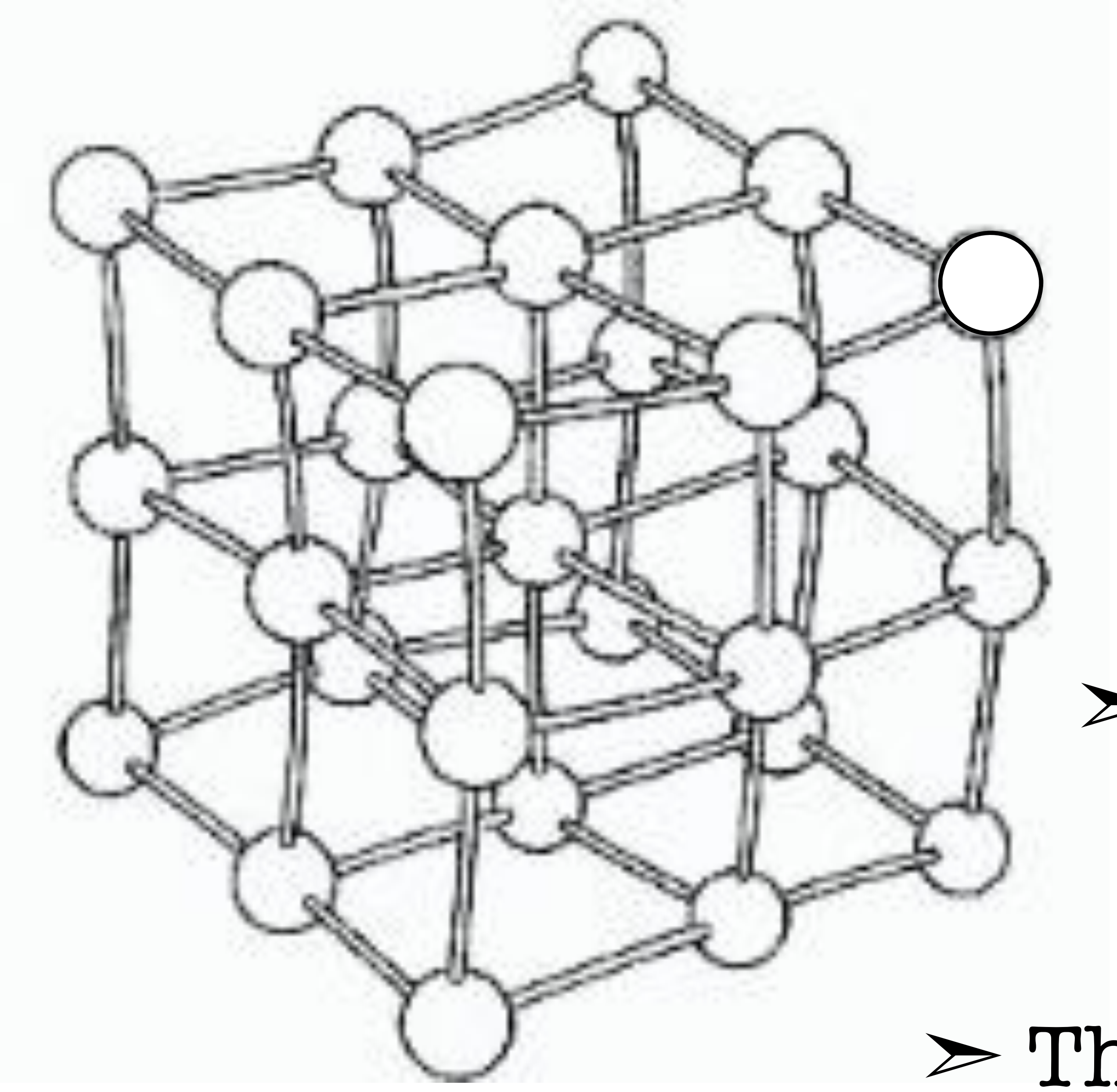
- ➢ Pathway to quantum mechanics invariably begins with Planck and his analysis of blackbody spectral data

Stefan-Boltzman Law

- ➢ Rate at which objects radiate energy ☛ $L \propto AT^4$
- ➢ At normal temperatures ☛ $\approx 300\ K$ not aware of this radiation because of its low intensity
- ➢ At higher temperatures ☛ sufficient IR radiation to feel heat
- ➢ At still higher temperatures ☛ $\mathcal{O}$ (1000 K) objects actually glow such as a red-hoy electric stove burner
- ➢ At temperatures above 2000 K objects glow with a yellow or whitish color ☛ filament of lightbulb

Blackbody Radiation

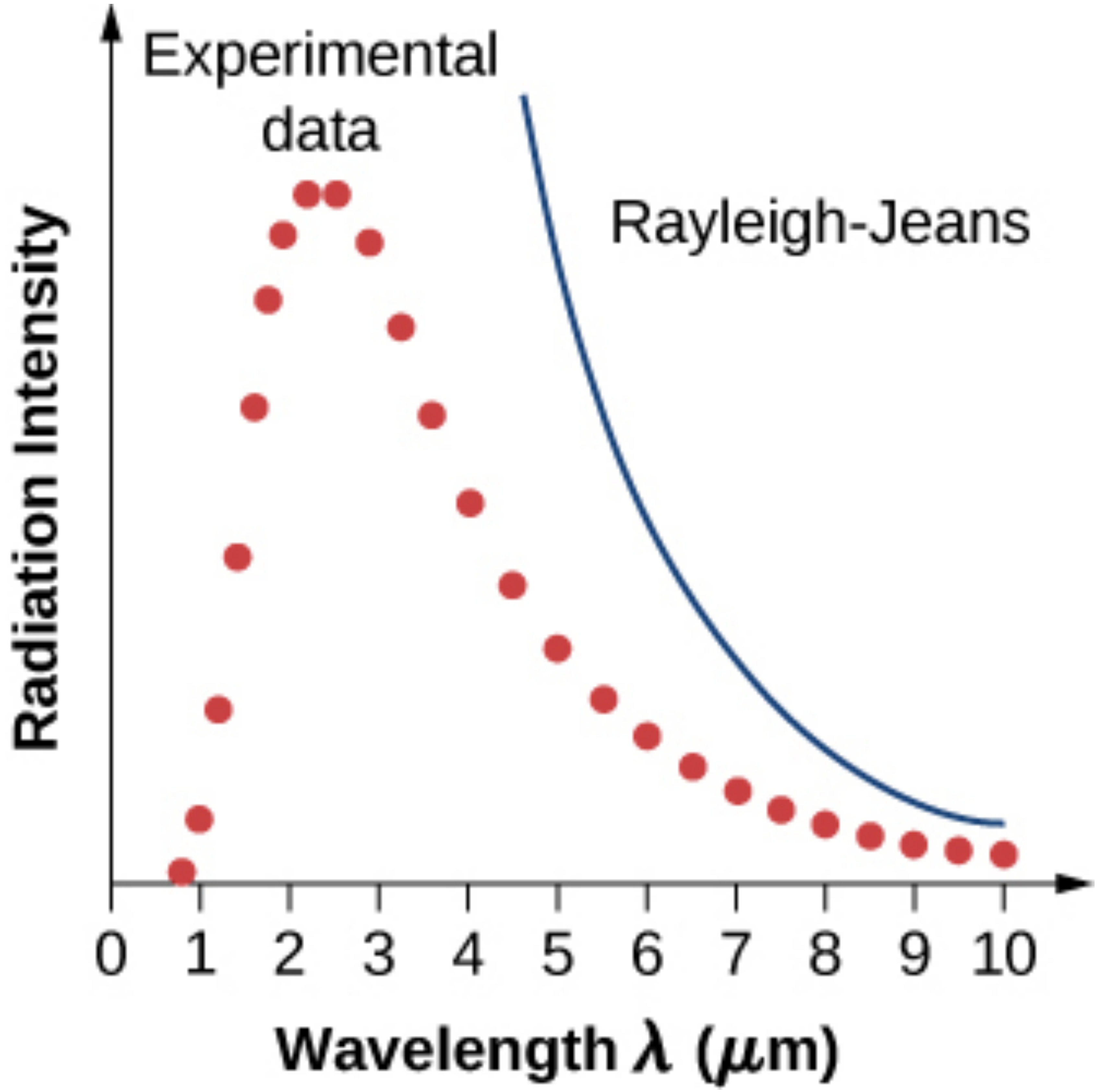
- ➢ A body that absorbs and emits all of radiation incident on it is called an **ideal blackbody**
- ➢ A blackbody is a piece of matter, and like all matter, it is componed of atoms
- ➢ We can treat atoms in solid as being connected by invisible springs



- ➢ Each atom will vibrate, or oscilase, in 3-dimensions
- ➢ This is called **simple harmonic approximation**
- ➢It is strictly **classical physics**
- ➢ Vibrating atoms absorb and emit radiation, and classical physics tells us that intensity of radiation emitted by oscillatlrs is $I(\lambda) = \frac{4kT}{\lambda^2}$
- ➢ This is called **Rayleigh-Jeans Law**
- ➢ It is a classical result
- ➢ k in above equation is Boltzmann cosntant $k = 1.38 \times 10^{-23}$ J/K

Ultraviolet Catastrophe

➢ If we make a plot of radiation intensity emitted by atomic oscillators versus wavelength, we would get



$$I(\lambda) = \frac{4kT}{\lambda^2}$$

➢ Classical theory only gets it right at large wavelengths, but fails at low wavelengths

➢ This is known as **Ultraviolet Catastrophe**

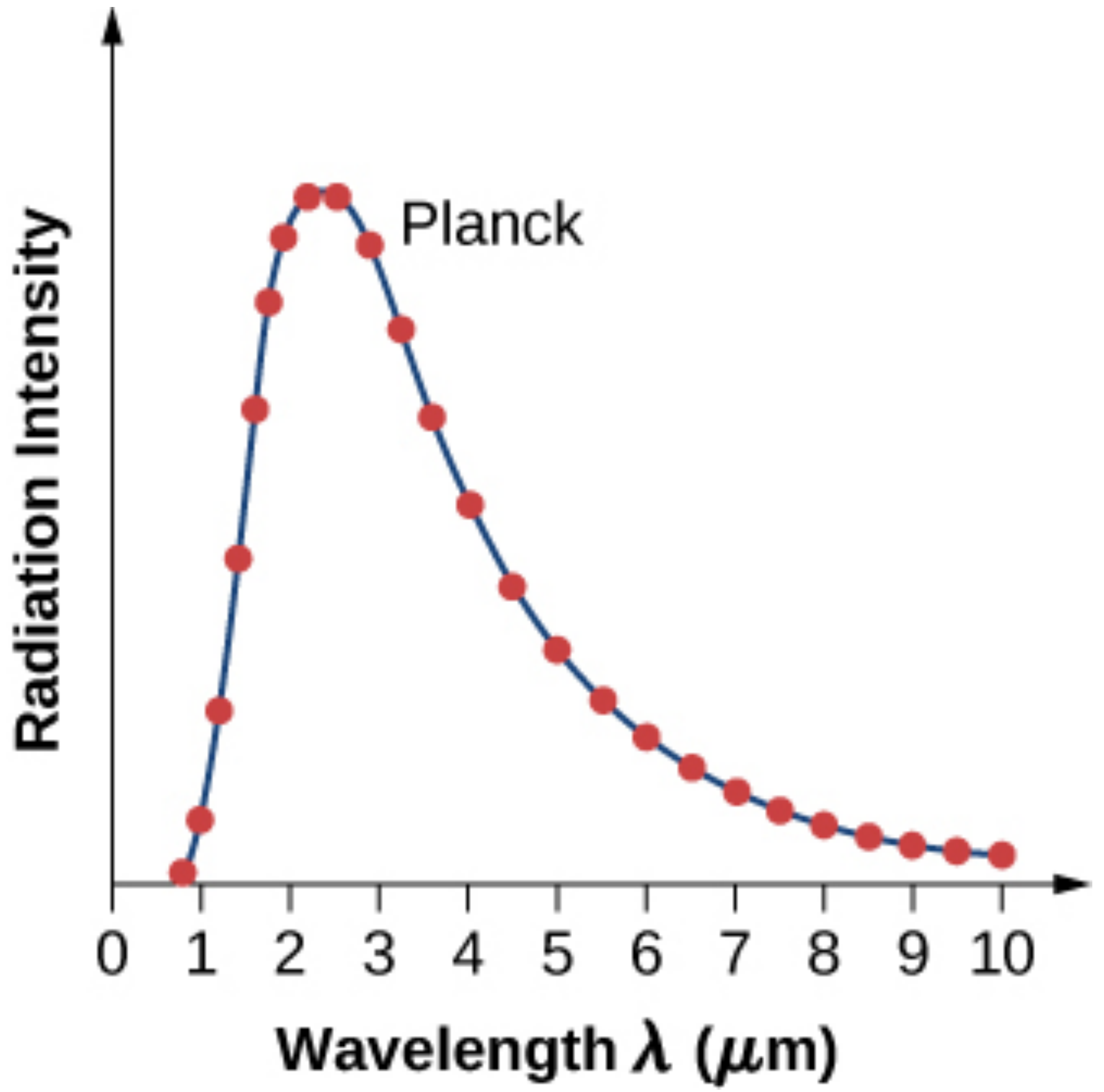
Planck's Hypothesis of Energy Quanta

- ➢ Planck showed that he could get good agreement between theory and data if he assumed that **energy of atomic oscillators was a discreet variable**
- ➢ In other words ☛ energy could only have certain discreet values

$$E = 0, \varepsilon, 2\varepsilon, 3\varepsilon, \ldots n\varepsilon$$

- ➢ Energy has to be proportional to frequency ☛ $\varepsilon \propto f$
- ➢ Make this an equality ☛ $\varepsilon = hf$
- ➢ Proportionality constant is Planck's constant h

$$h = 6.626 \times 10^{-34}\ \text{J} \cdot \text{s}$$



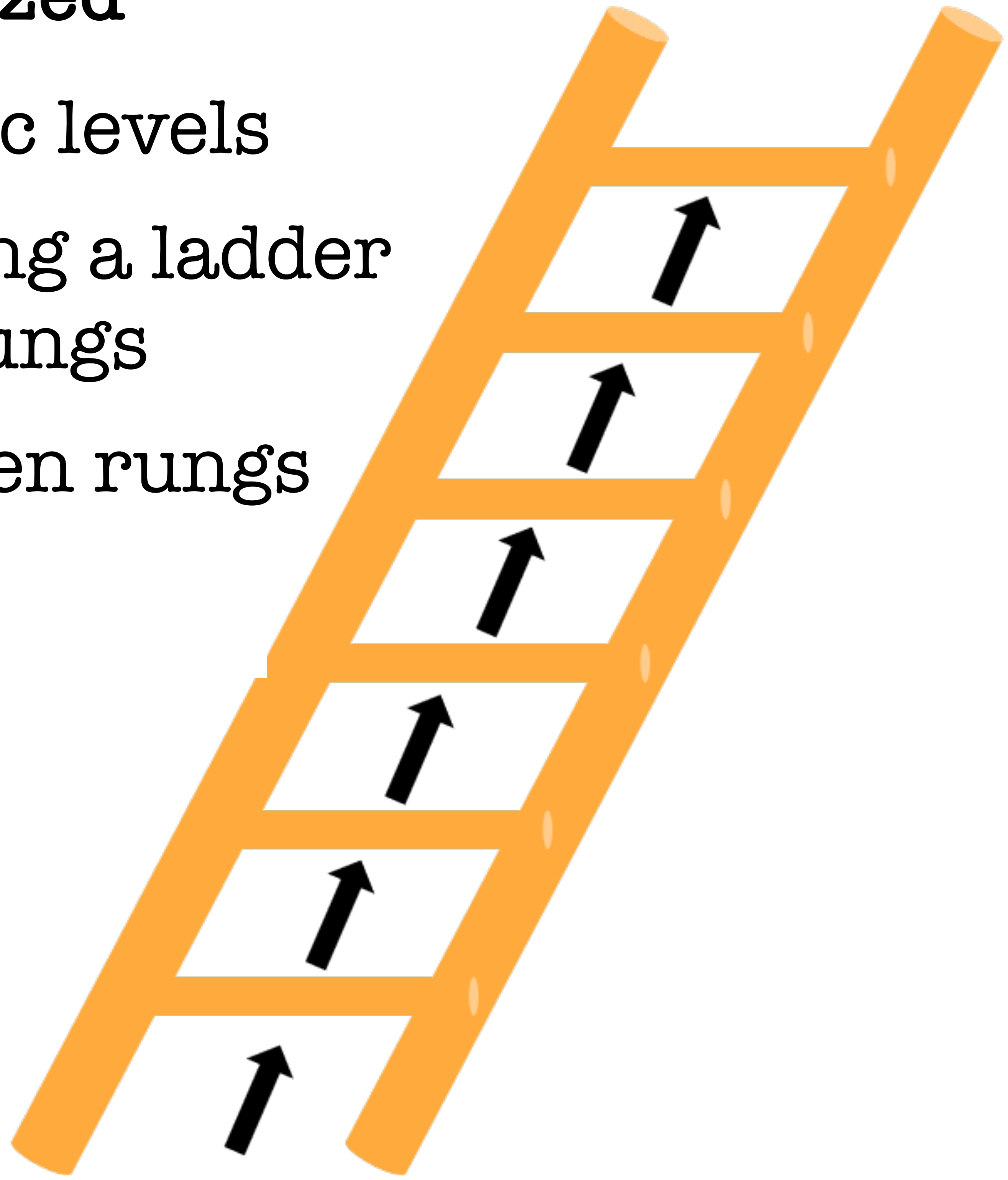
Energy is allowed to only have certain values ☛ it is quantized

Energy is quantized

Or restricted to specific levels

This is like how climbing a ladder must be done using rungs

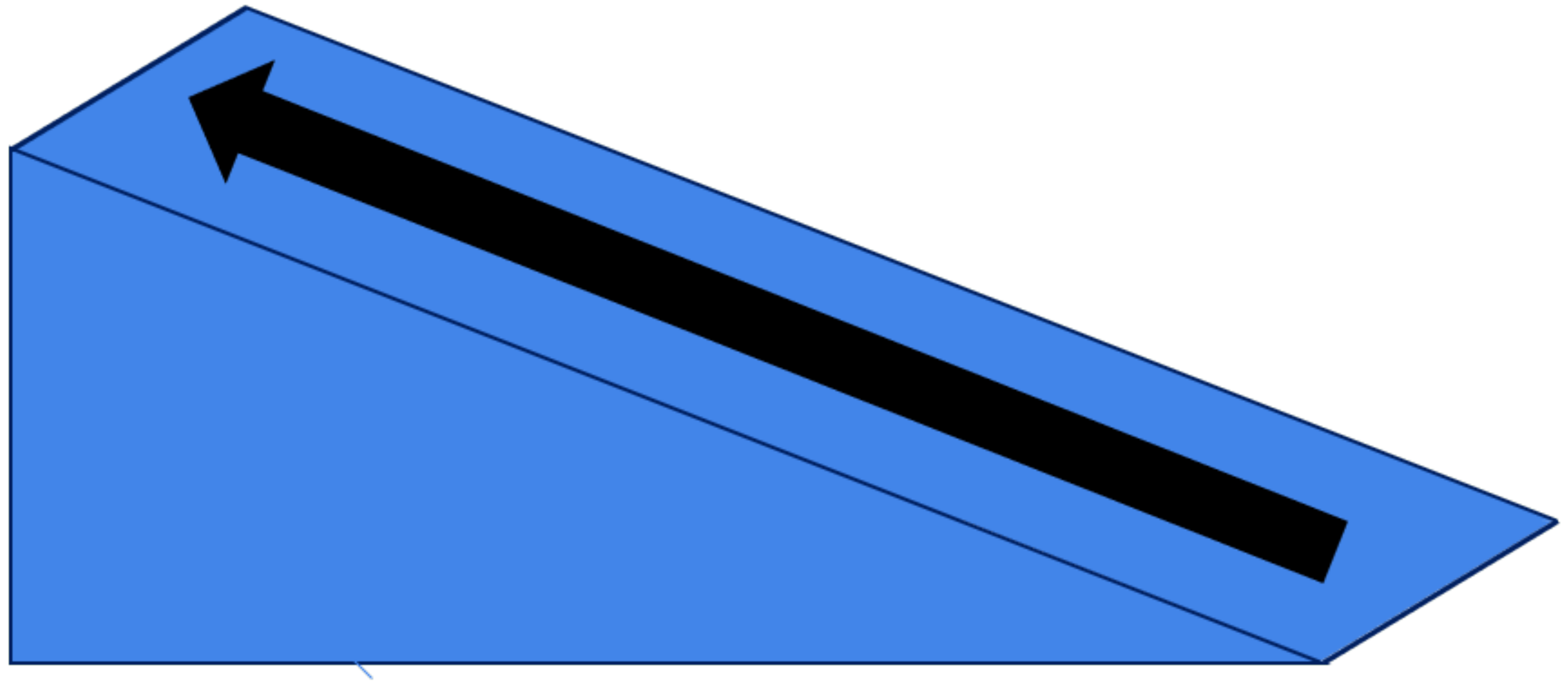
You cannot step between rungs to climb



Energy is not continuous

Climbing this ramp, you can stop at any point

A ramp does not have discreet or specific values, like energy



$$E = nhf, n = 0,1,2,3,...$$

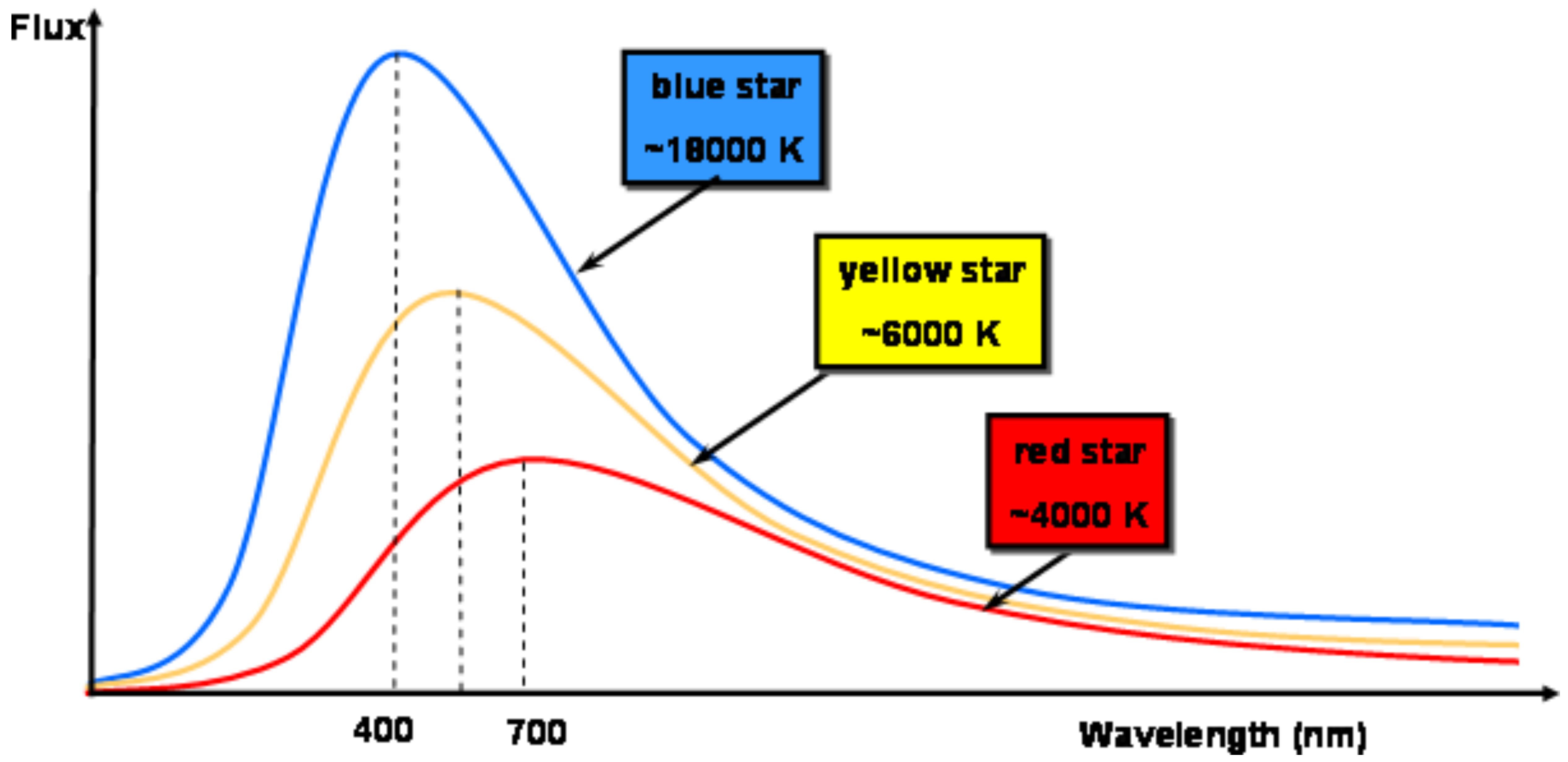
➢ Energy comes in discreet packets of (hf) called **quanta** or **quantum of energy**

Wien's displacement law

- ➢ Wavelenghth $\lambda_{\max}$ at which spectral emittance reaches maximum decreases as T is increased in inverse proportion to T

$$\lambda_{max} T = 2.90 \times 10^{-3} \mathrm{mK}$$

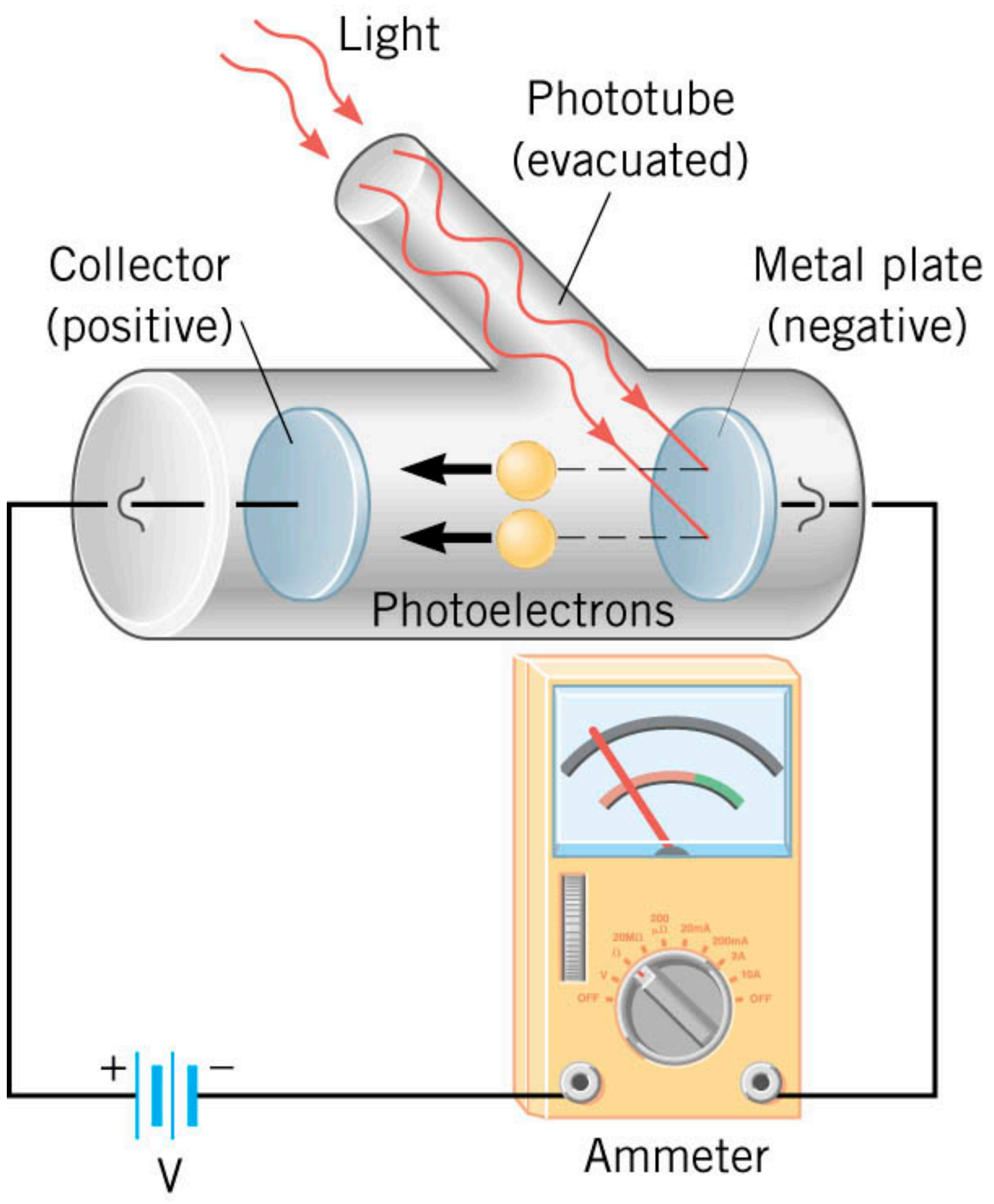
- ➢ Qualitatively consistent with observation that heated objects first begin to glow with red color and at higher temperatures color becomes more yellow



Photoelectric Effect

➢ In 1887 Heinrich Hertz produced and detected electromagnetic waves, thus proving Maxwell's theory

➢ He also discovered something called **Photoelectric Effect**

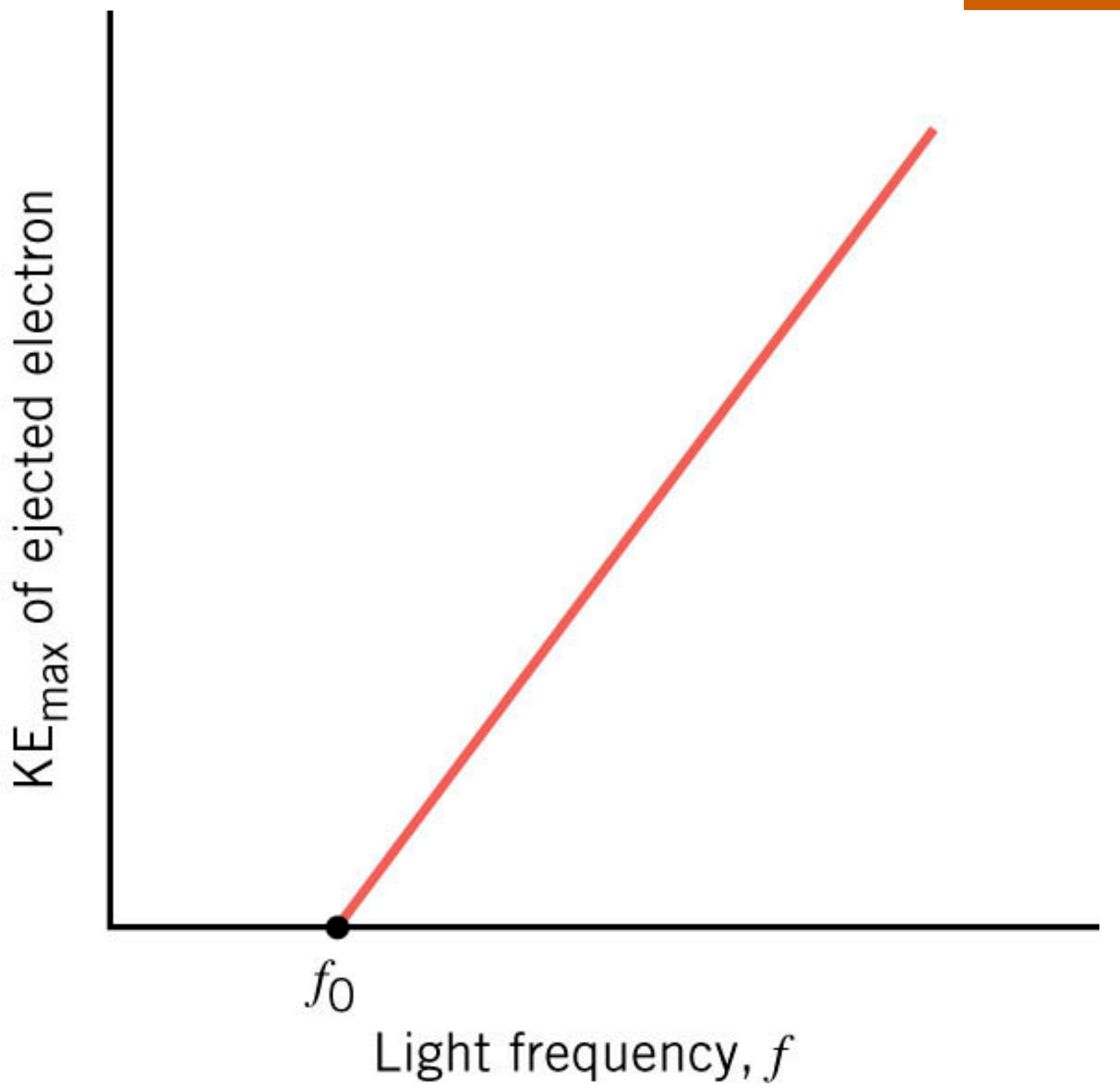


➢ This results in a current flow in circuit as shown

➢ Important Characteristics of photoelectric effect

1- Only light with a frequency above some minimum value, f_0 will result in electrons being ejected -regardless of light intensity

➢ Let's look at a plot of KE of ejected electrons vs. frequency of light shining on metal:



Notice ☛ No electrons are ejected from metal for frequencies below some f_0

$f < f_0$ no ejected electrons

$f \geq f_0$ electrons are ejected from metal's surface

f_0 is called **Threshold Frequency**

➢ Now choose some constant value for frequency $f \geq f_0$, so that electrons are being ejected from metal

2- Maximum KE of ejected electrons remains constant, even if intensity of light is increased

➢ Classically, we would expect higher intensity light to eject electrons with greate KE

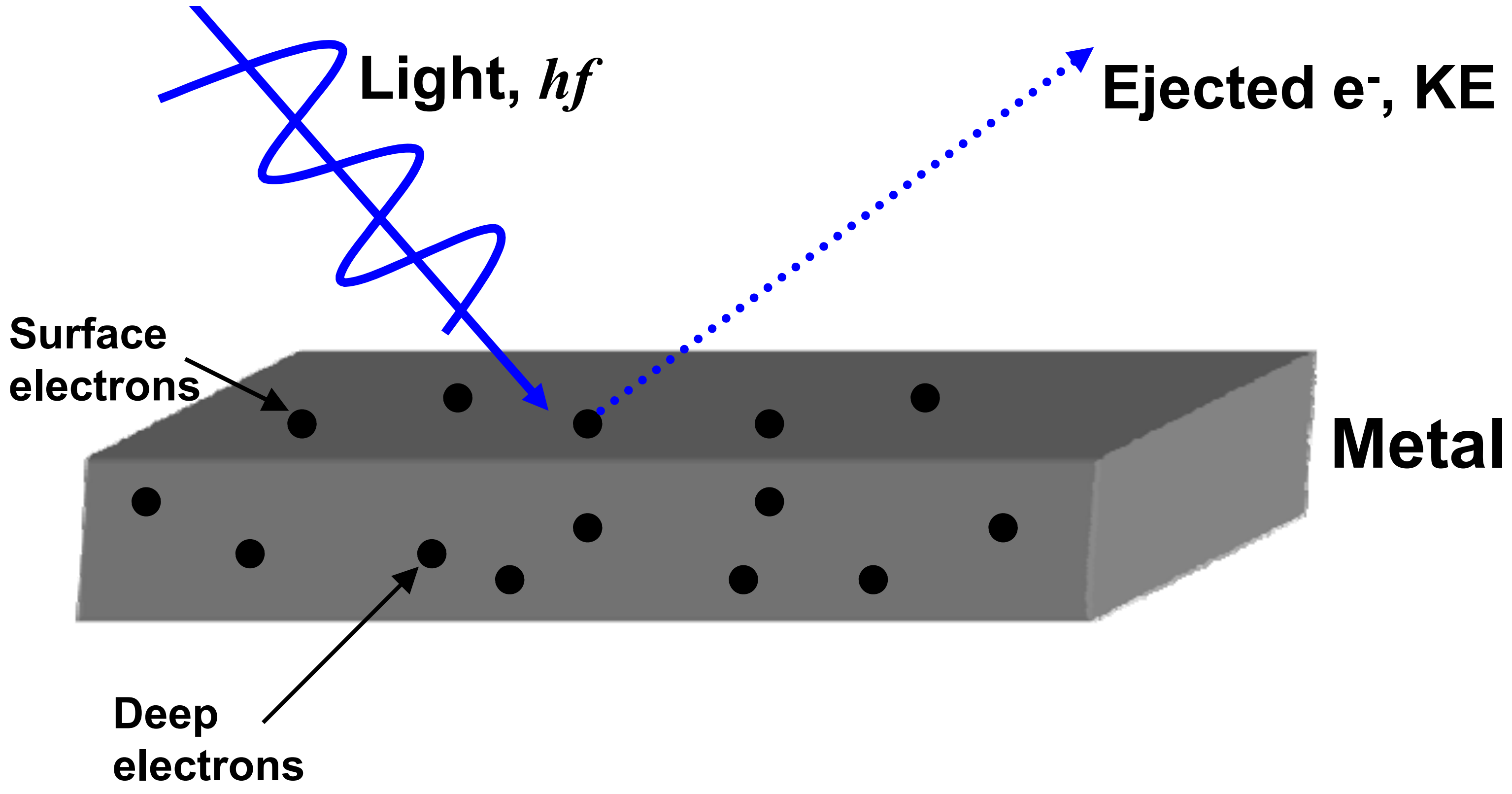
➢ It doesn't happen

- ➢ We would also expect that if we used very low intensity light, that it would take a long time for electrons to build up enough energy to be ejected from metal's surface
- ➢ That doesn't happen either!
- ➢ Even if light intensity is very low, electrons are still ejected from metal's surface, almost instantaneously, as long as $f \geq f_0$
- ➢ Einstein assumed that light was composed of discrete packets (particles) of energy called **photons**
- ➢ Photon energy is given by ☛

$$E = hf = \frac{hc}{\lambda}$$

- ➢ More intense light is more photons it carries ☛ but each photon still has a energy ☛ $E = hf$
- ➢ Now let's examine photoelectric effect in a little more detail

- Free electrons occupy entire volume of metal
- However, electrons close to metal's surface (surface electrons) are more weakly bound to metal than deep electrons
- But even though surface electrons are more weakly bound, there is still a minimum binding energy I must overcome to get them out of metal



- This is called **Work Function** (W_0) of metal
- It is an energy, and it is typically on order of a few eV
- During effect, a photon of light ($f > f_0$) with energy hf strikes metal and electrons are ejected with energy KE

- By conservation of energy ☛ following relationship must be true

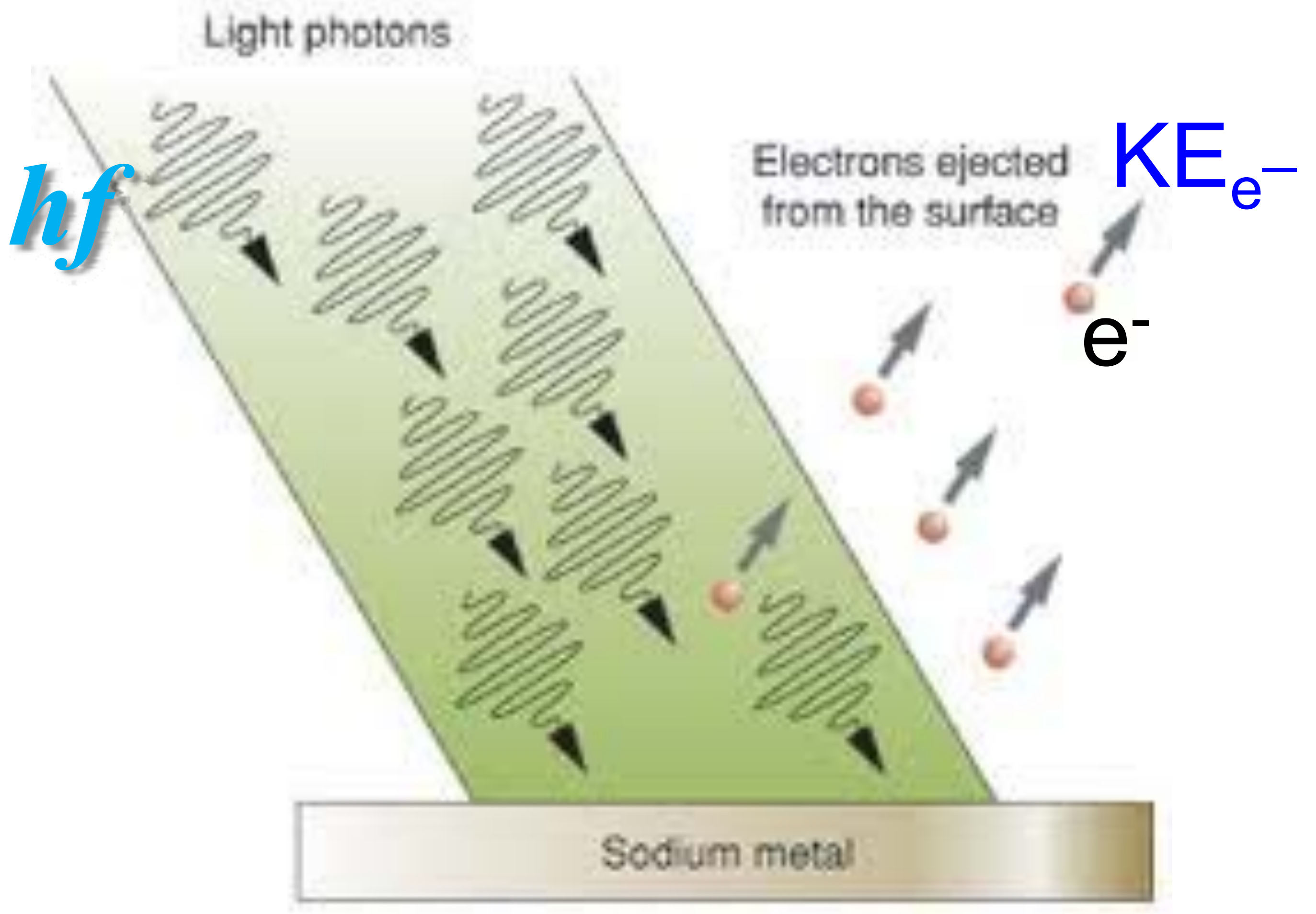
Photon energy - Binding energy = KE of ejected electrons

$$hf - W_0 = KE_{e^-}$$

This is Einstein equation for Photoelectric Effect

Einstein Theory

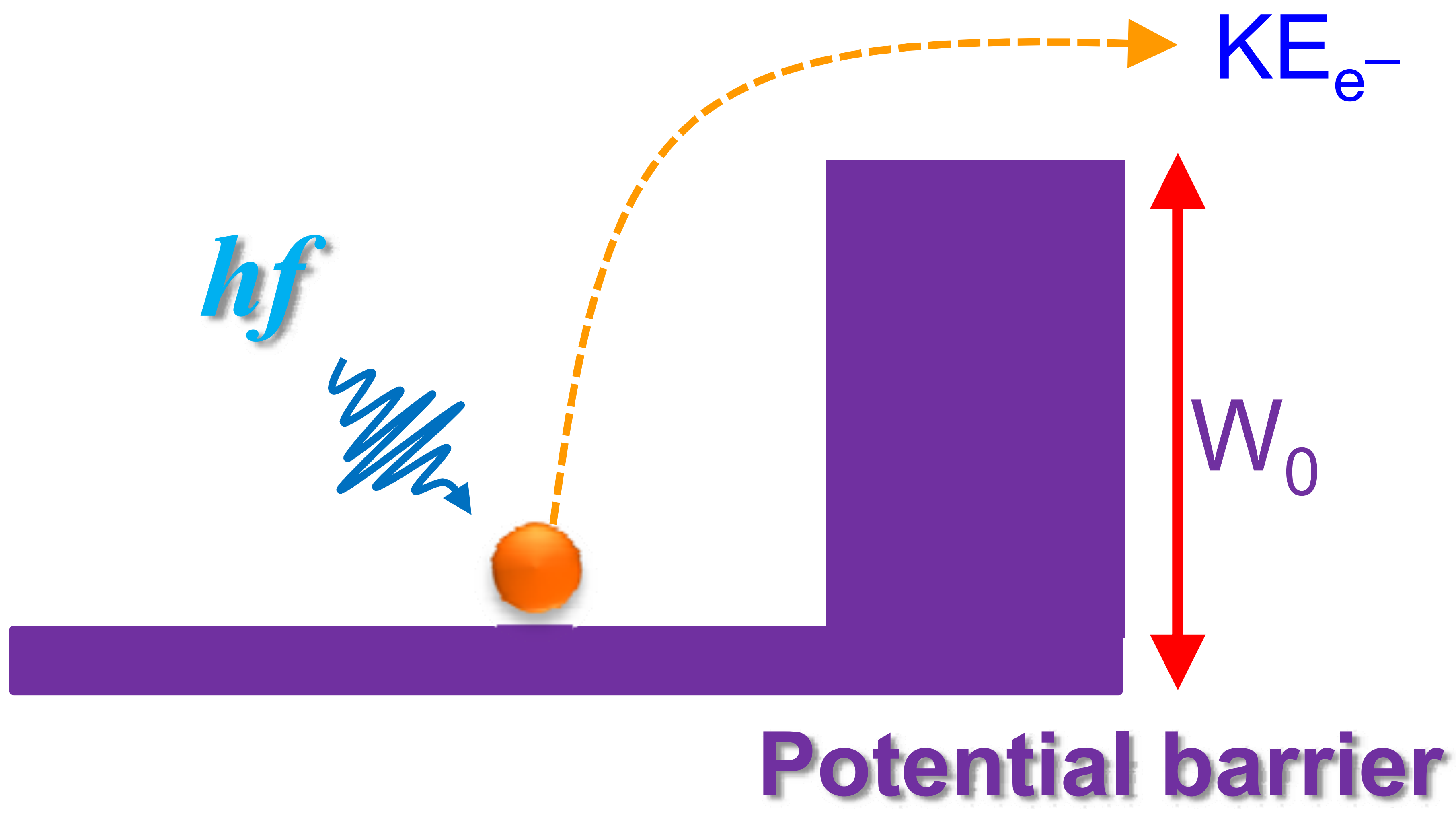
Simple picture view



- Light consits of photons (hf)
- One electron can discretely absorb one photon
- Electron use photon energy to overcome potential barrier

Energy conservation

$$hf - W_0 = KE_{e-}$$



Whether or not electrons can get out depends on frequency of light not intensity of light !!!

de Broglie Wavelength

➢ In view of particle properties for light waves **photons** de Broglie ventured to consider reverse phenomenon

➢ Assign wave properties to matter ☛ to every particle with mass m and momentum $\vec{p}$ ☛ associate

$$\lambda = h/|\vec{p}|$$

➢ Assignment of energy and momentum to matter in (reversed) analogy to photons

$$E = hf \quad \text{and} \quad |\vec{p}| = h/\lambda$$

- All objects have a de Broglie wavelength - baseballs, cars, even you and me!!
- But remember, in order for wave effects to be seen, such as interference and diffraction, wavelength must be comparable to size of opening or obstacle
- For fun ☛ let's calculate human body de Broglie wavelength

$$\lambda_{\text{Human}} = \frac{h}{p} = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(90.7\ \text{kg})(6.7\ \text{m/s})} = \boxed{1\times10^{-36}\ \text{m}}$$

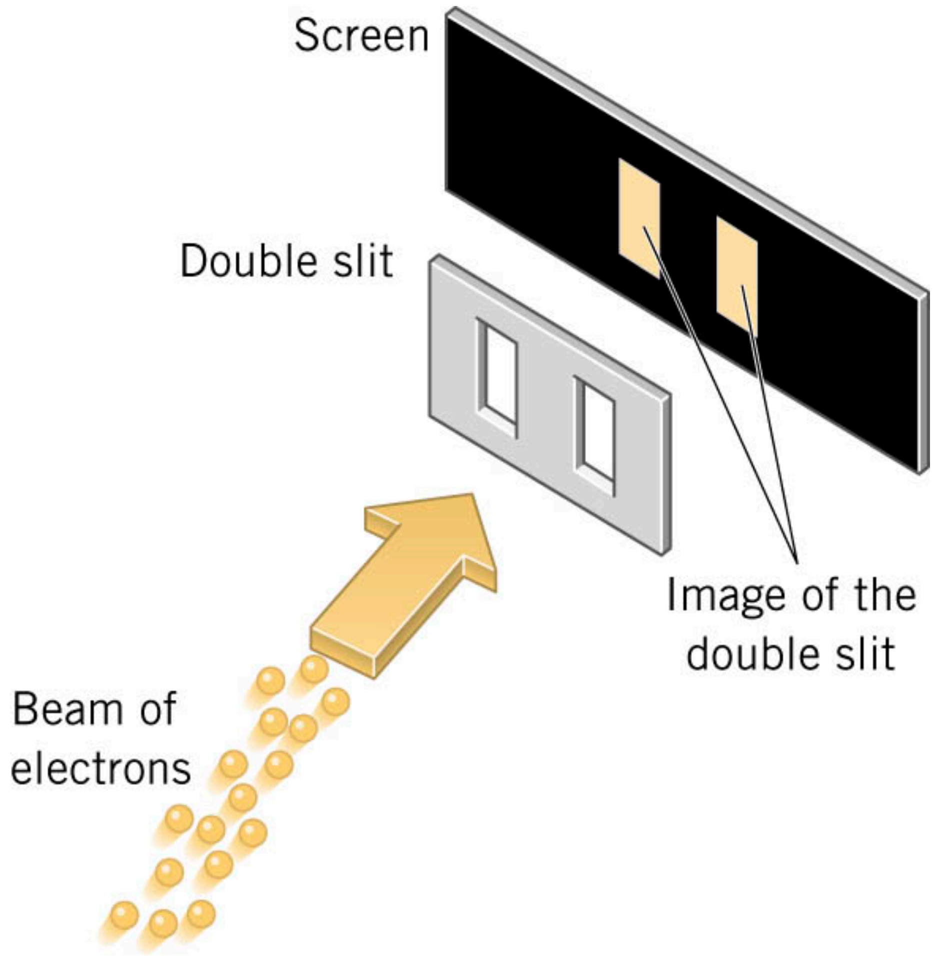
So what does this number mean??

- Size of an atom is roughly 1 x 10 $^{-10}$ m
- So my de Broglie wavelength is some 26 orders of magnitude smaller than size of an atom!!!
- Which means ….we don't observe wave-like properties with everyday objects, baseballs, humans, etc
- We need sub-atomic particles to observe wave-like properties!!

➢ Now let's repeat Young's double-slip experiment, but this time let's shoot electrons (particles) at slits instead of light

What would we expect to see??

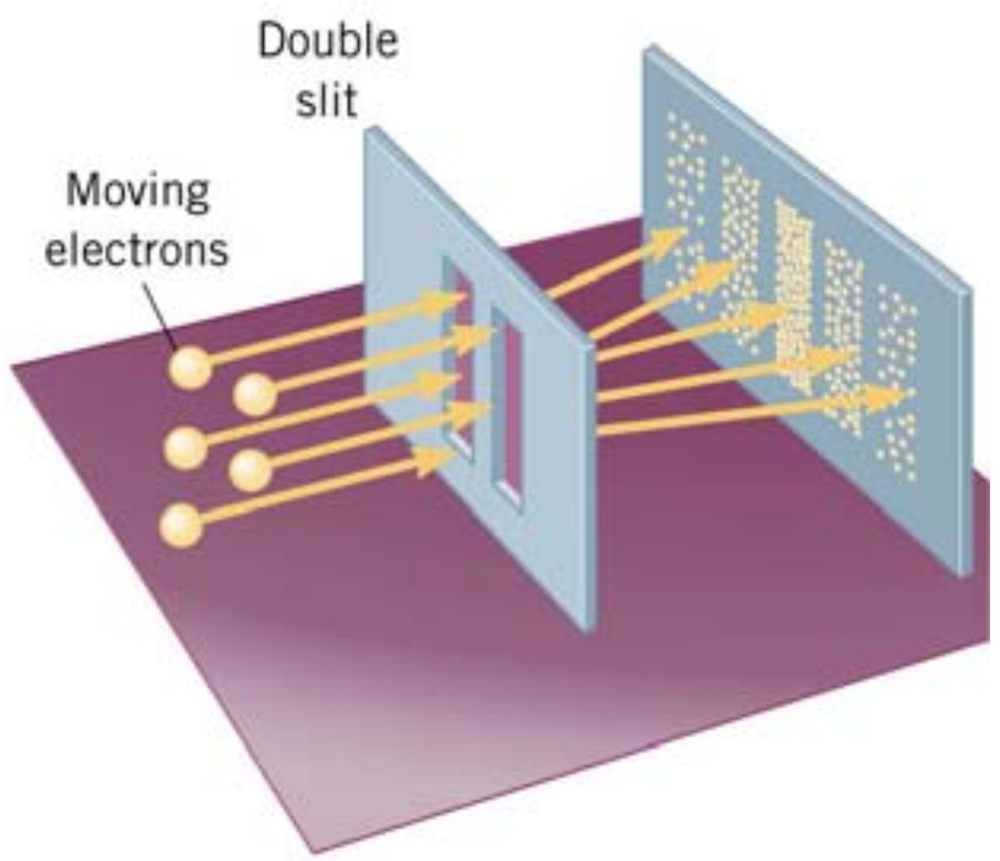
➢ We might expect screen to appear with two bright fringes, one directly behind each slit



➢ What we actually see is shown in figure at botton

alternating dark and bright fringes

➢ **In other words ☛ electrons have acted like waves and interfered with each other to produce classic interference pattern!**



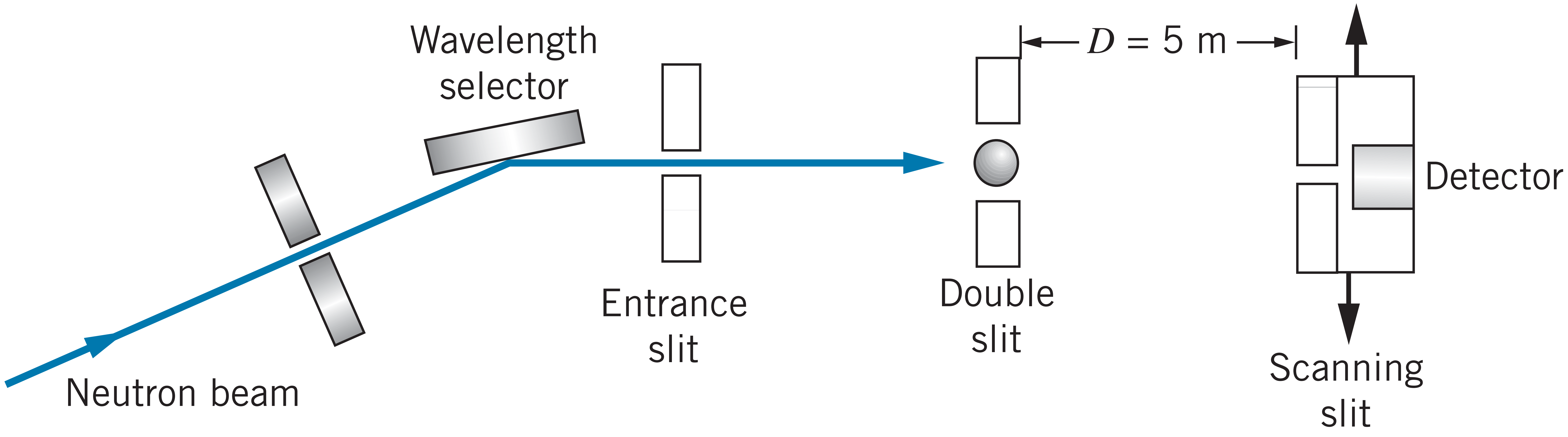
➢ Our notion of electron as being a tiny discrete particle of matter does not account for fact that electron can behave as a wave in some circunstances

➢ It exhibits a dual nature -bahaving sometimes like a particle, and sometimes like a wave

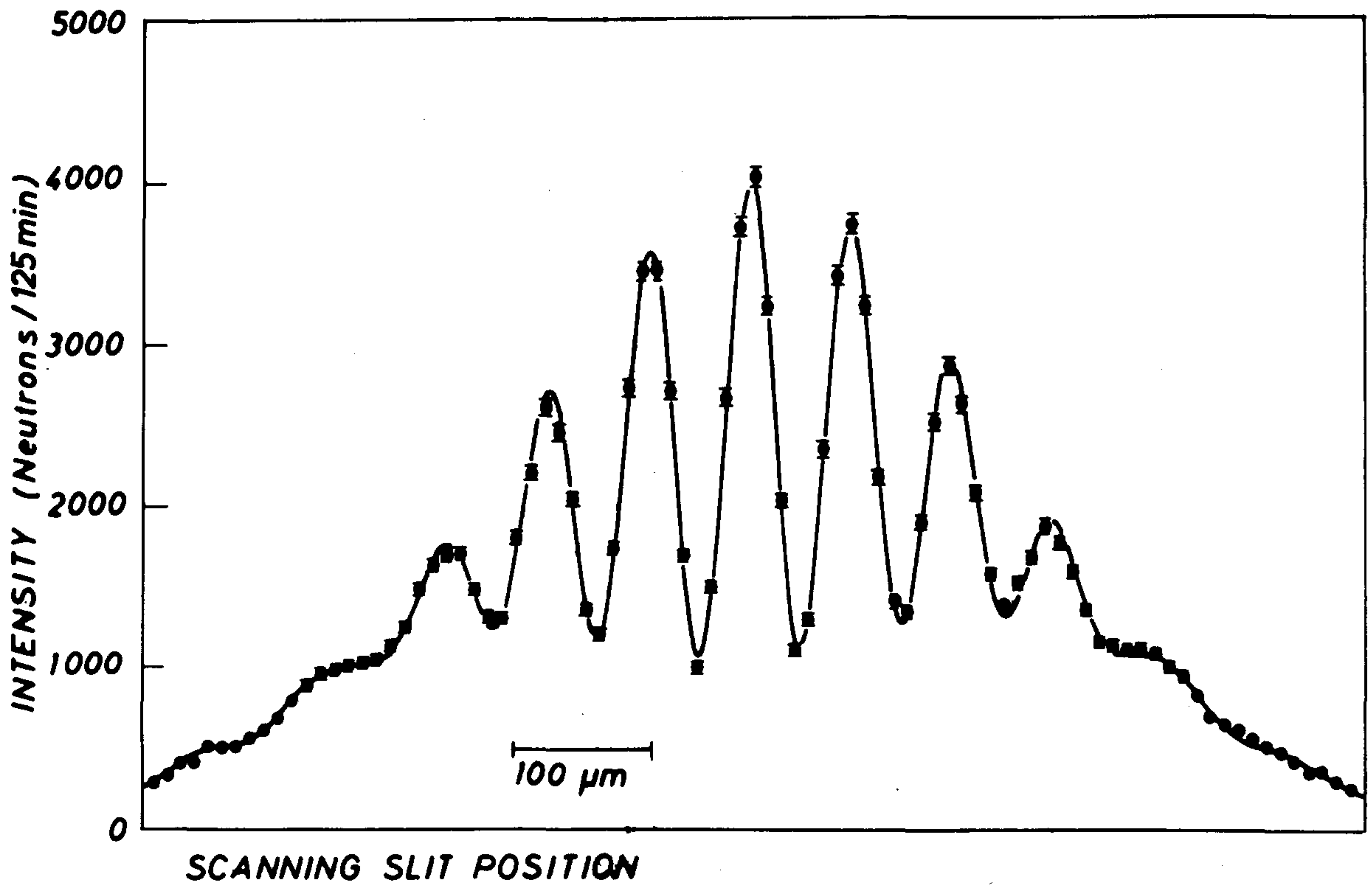
➢ Things are even weirder than this!!....

Neutron Double-Slit Experiment

- Parallel beam of neutrons falls on double-slit
- Neutron detector capable of detecting individual neutrons
- Detector register discrete particles localized in space and time
- This can be achieved if neutron source is weak enough

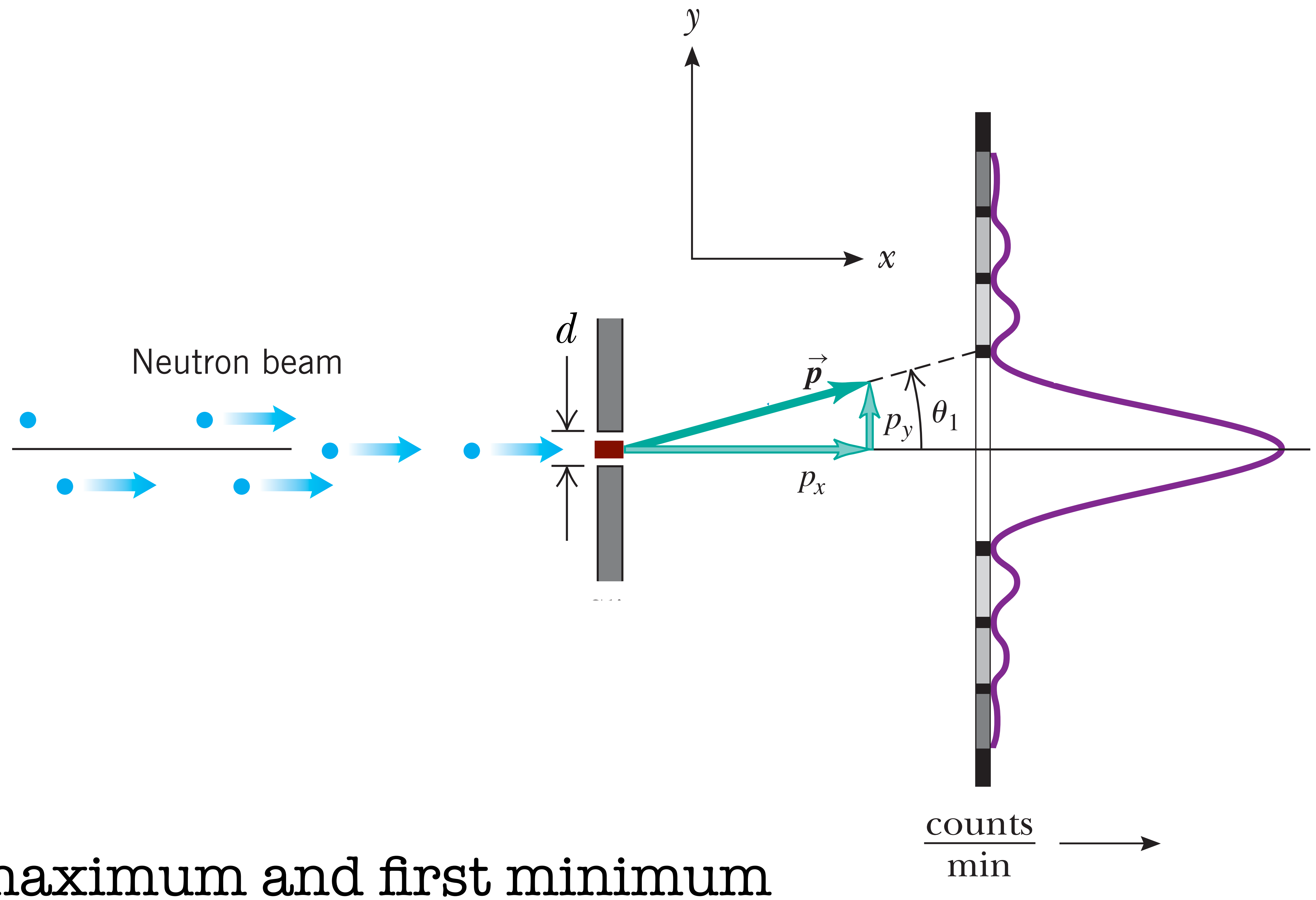


- Neutron kinetic energy ☛ 2.4×10^{-4} eV
- de Broglie wavelength ☛ 1.85 nm
- Center- to-center distance between two slits ☛ $d = 126\ \mu\text{m}$



Estimating spacing ☛ $(y_{n+1} - y_n) \approx 75\ \mu\text{m}$

$$\lambda = \frac{d\,(y_{n+1} - y_n)}{D} = 1.89\ \text{nm}$$



- θ_1 ☛ angle between central maximum and first minimum
- For $m = 1$ ☛ $\sin\theta_1 = \lambda/d$
- Neutron striking screen at outer edge of central maximum must have component of momentum p_y as well as a component p_x
- From geometry ☛ components are related by $p_y/p_x = \tan\theta_1$
- Use approximation $\tan\theta_1 = \theta_1$ and $p_y = p_x\,\theta_1$

Heisenberg's Uncertainty Principle

- All in all ☛ $p_y = p_x \lambda/d$
- Neutrons striking detector within central maximum i.e. angles between $(-\lambda/d + \lambda/d)$

  have y-momentum - component spread over $(-p_x\lambda/d, +p_x\lambda/d)$
- Symmetry of interference pattern shows $\langle p_y \rangle = 0$
- There will be an uncertainty Δp_y at least as great as $p_x\lambda/d$

$$\Delta p_y \geq p_x \lambda/d$$

- Narrower separation between slits d broader is interference pattern and greater is uncertainty in p_y
- Using de Broglie relation $\lambda = h/p_x$ and simplifying

$$\Delta p_y \geq p_x \frac{h}{p_x d} = \frac{h}{d}$$

Heisenberg's Uncertainty Principle (cont')

What does this all mean??

- d ≡ Δy represents uncertainty in y-component of neutron position as it passes through double-slit gap
- Both y-position and y-momentum-component have uncertainties related by ☛ $\Delta p_y \Delta y \geq h$
- We reduce Δp_y only by reducing with of interference pattern
- To do this ☛ increase d which increases position uncertainty Δy
- Conversely we decrease position uncertainty by narrowing doubl-slit gap interference pattern broadens and corresponding momentum uncertainty increases


aNy