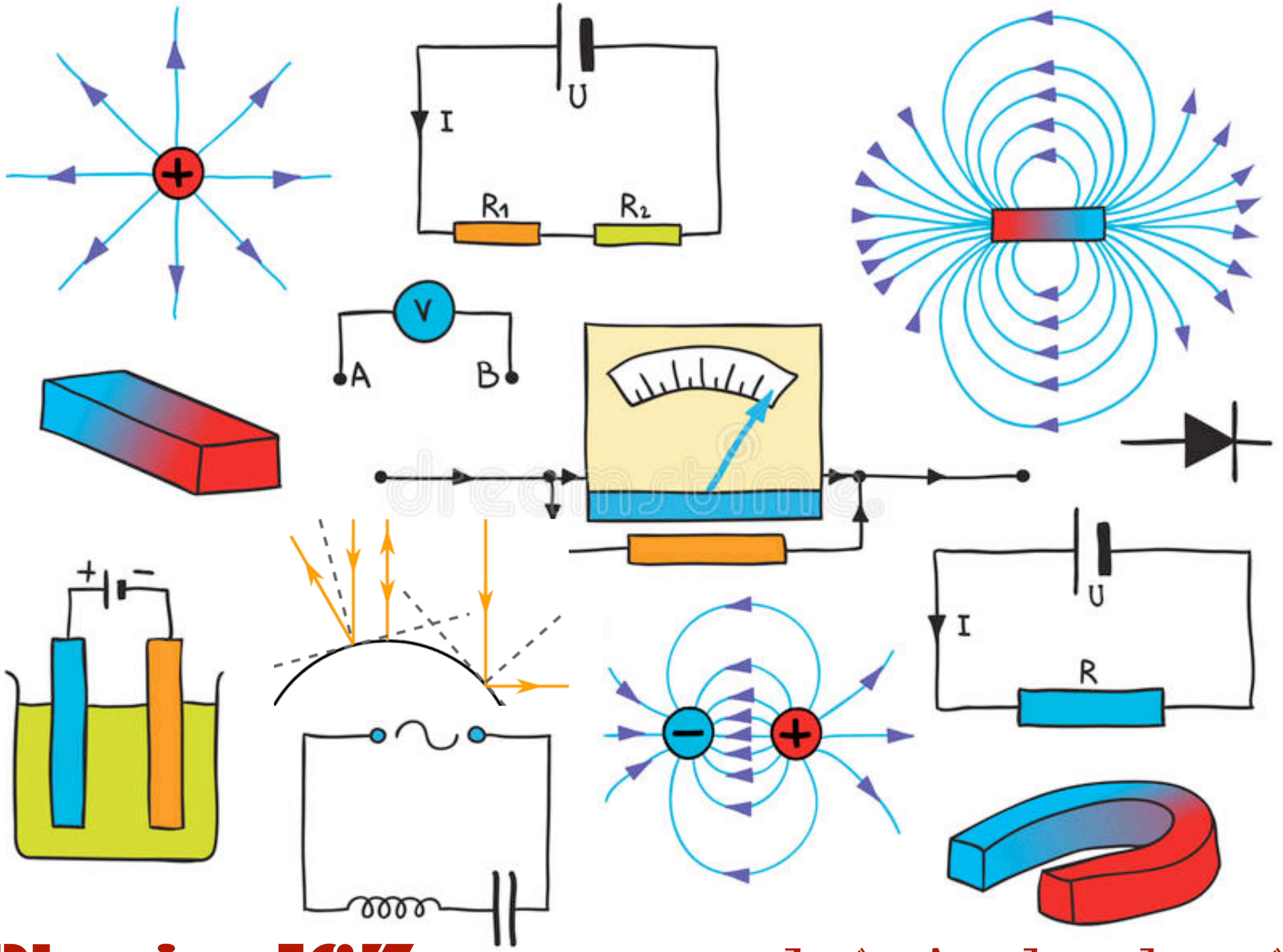
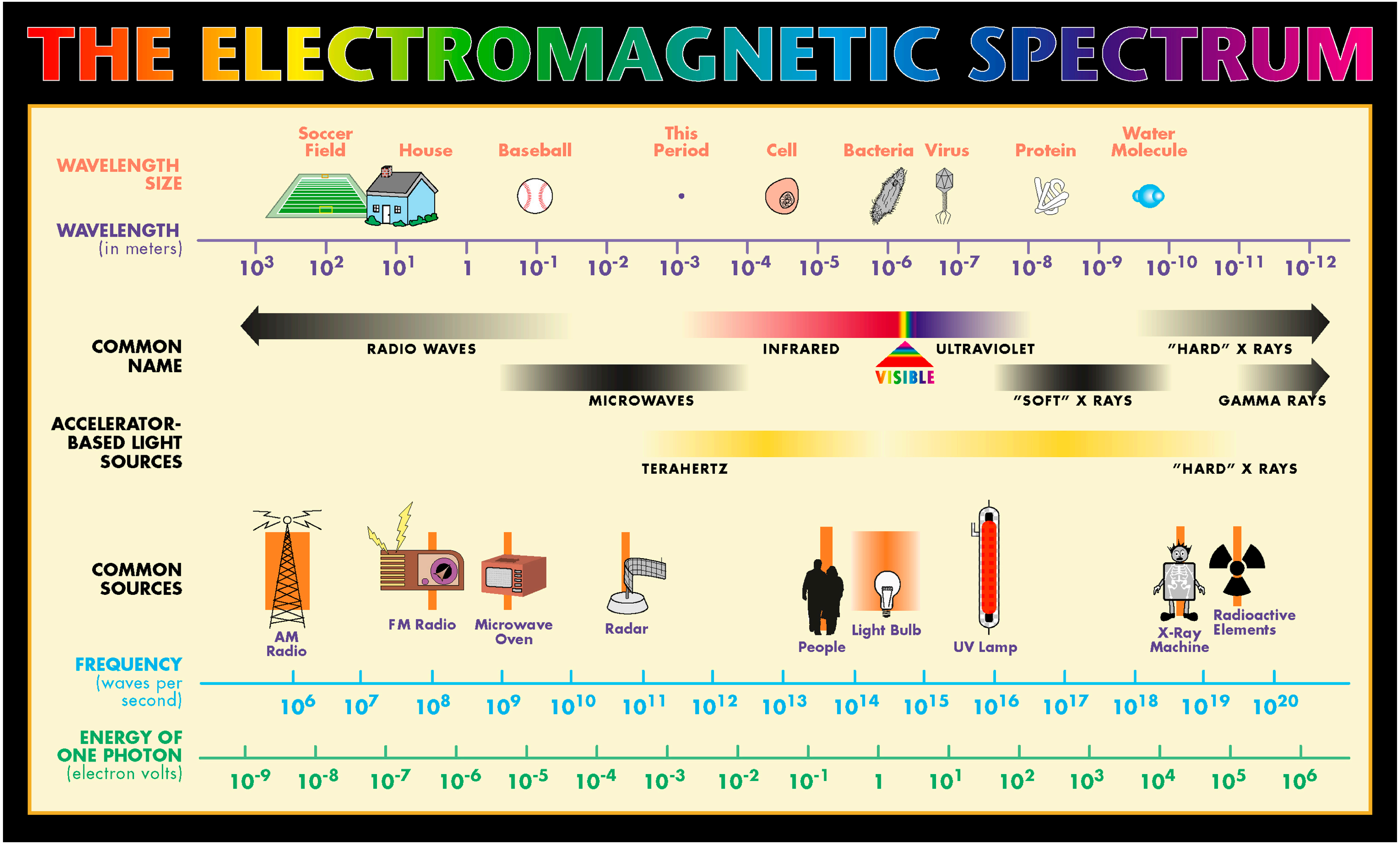


Physics 167

Luis Anchordoqui

Ray Approximation in Geometric Optics

- ➢ We just Learned that light is a wave
- ➢ Unlike particles ☛ waves behave in funny ways e.g. ☛ they bend around corners
- ➢ However ☛ smaller wavelength λ is ⇒ weaker funny effects are
- ➢ λ of light is about 100 times smaller than diameter of human hair!
- ➢ For a long time ☛ no one noticed "wave nature" of light at all
- ➢ This means that for most physics phenomena of everyday life we can safely ignore wave nature of light
- ➢ Light waves travel through and around obstacles whose transverse dimensions are much greater than wavelength and wave nature of light is not readily discerned
- ➢ Under this circunstances ☛behavior of light is described by rays obeying set of geometrical rules
- ➢ This model of light is called **ray optics**
- ➢ Ray optics is limit of wave optics when wavelength is infinitesimally small


THE ELECTROMAGNETIC SPECTRUM
WAVELENGTH SIZE
Soccer Field
House
Baseball
This Period
Cell
Bacteria
Virus
Protein
Water Molecule
WAVELENGTH (in meters)
10^{3} 10^{2} 10^{1} 1 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10} 10^{-11} 10^{-12}
COMMON NAME
RADIO WAVES
INFRARED
ULTRAVIOLET
"HARD" X RAYS
VISIBLE
MICROWAVES
"SOFT" X RAYS
GAMMA RAYS
ACCELERATOR-BASED LIGHT SOURCES
TERAHERTZ
"HARD" X RAYS
COMMON SOURCES
AM Radio
FM Radio
Microwave Oven
Radar
People
Light Bulb
UV Lamp
X-Ray Machine
Radioactive Elements
FREQUENCY (waves per second)
10^{6} 10^{7} 10^{8} 10^{9} 10^{10} 10^{11} 10^{12} 10^{13} 10^{14} 10^{15} 10^{16} 10^{17} 10^{18} 10^{19} 10^{20}
ENERGY OF ONE PHOTON (electron volts)
10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6}

➢ To study more classical aspects of how light travels

- We will ignore time variations ☛ (10^{14} Hz too fast to notice)
- We will assume light travels through a transparente medium in straight line
- Light can change directos in 3 main ways

1. Boucing off objects (reflection)
2. Entering objects (e.g. glass) and bending (refraction)
3. Getting caught and heating object (absorption)

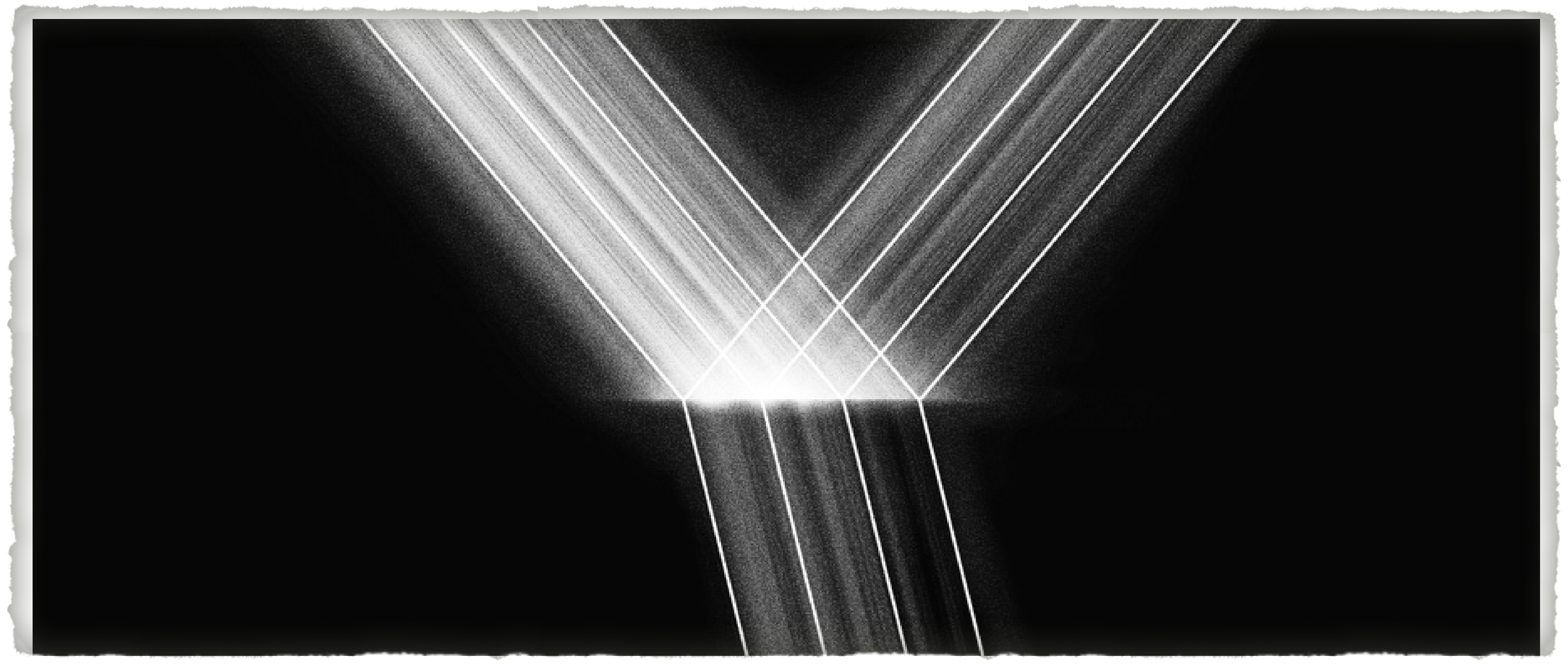
➢ In other words

- We consider that light travels in form of rays
- Rays are emitted by lights sources and can be observed when they reach optical detector
- We further assume that optical rays propagate in optical media
- To keep things simple ☛ we will assume that media are transparent

Fermat's Principle

- ➢ When light ray travels between any two points its path is one that requires smallest time interval

- ➢ Obvious consequence of this principle ☛ paths of light rays traveling in homogeneous medium are straight lines because straight line is shortest distance between two points

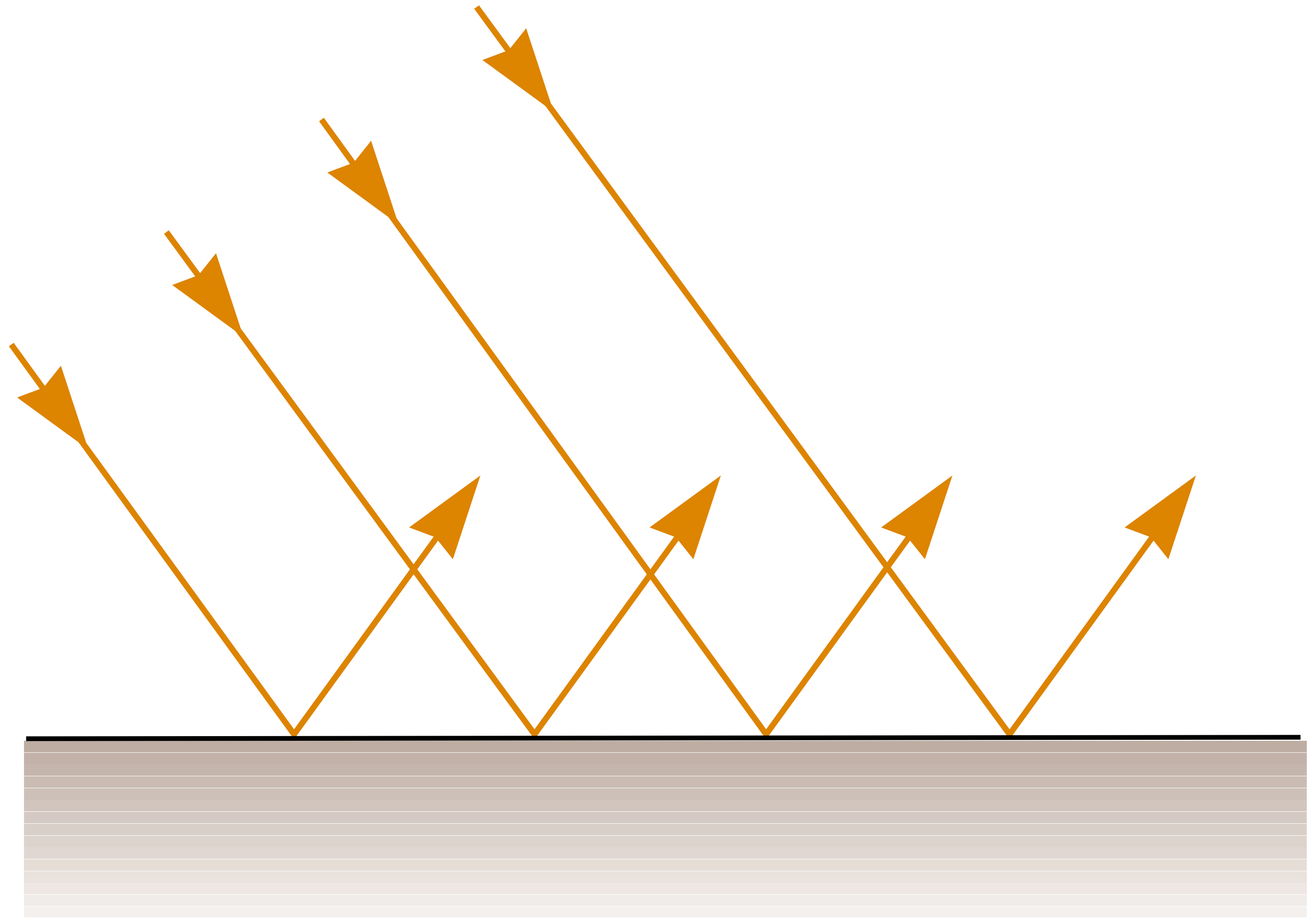


Reflection

➢ When light ray traveling in medium encounters with another medium part of incident light is reflected

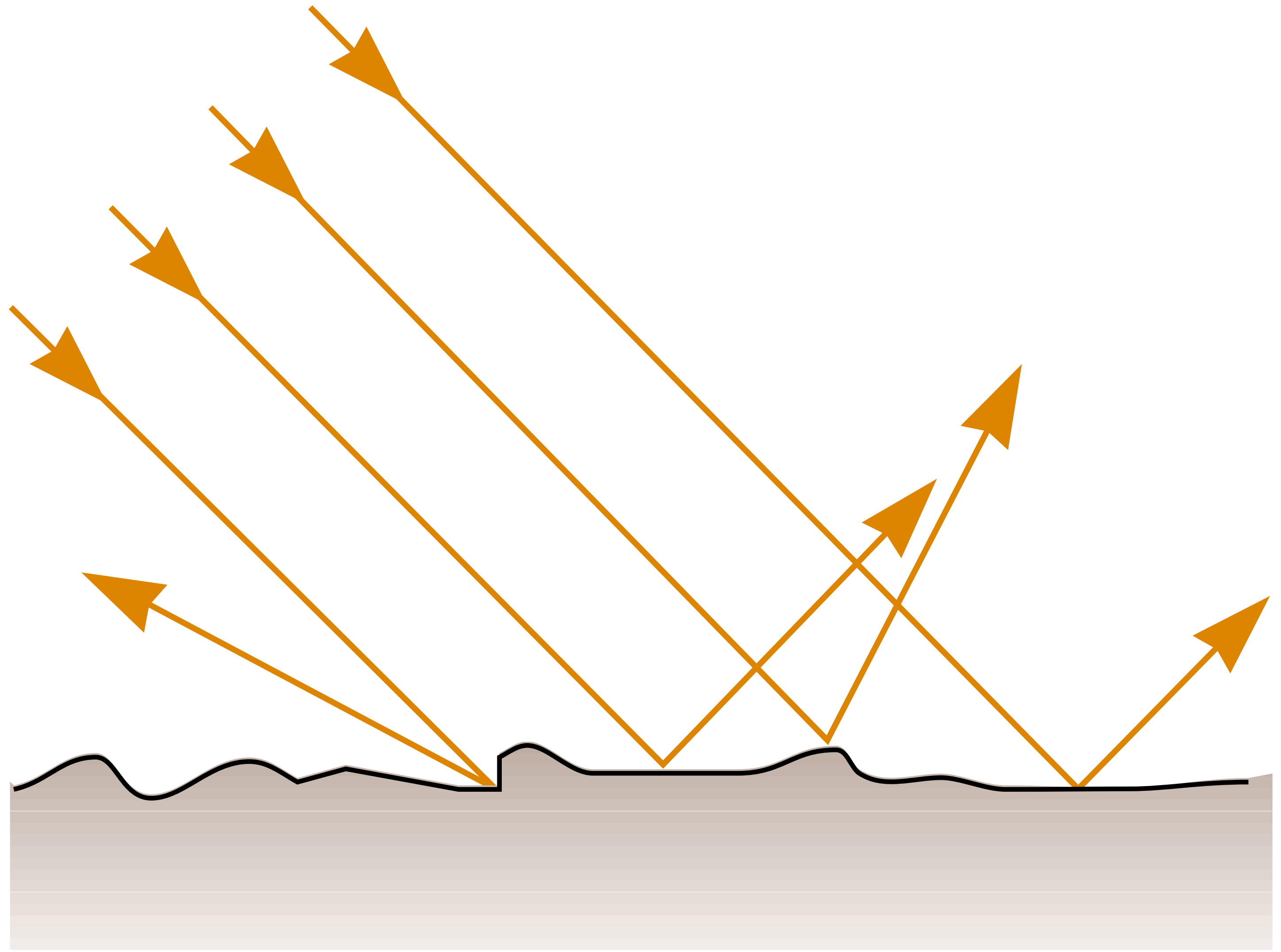
➢ Reflection of light from smooth surface is called **specular reflection**

➢ Reflected rays are parallel to each other as indicated in ☛



➢ Reflection from rough surface is known as **diffuse reflection**

➢ If reflecting surface is rough surface reflects rays not as a parallel set but in various directions as shown in ☛



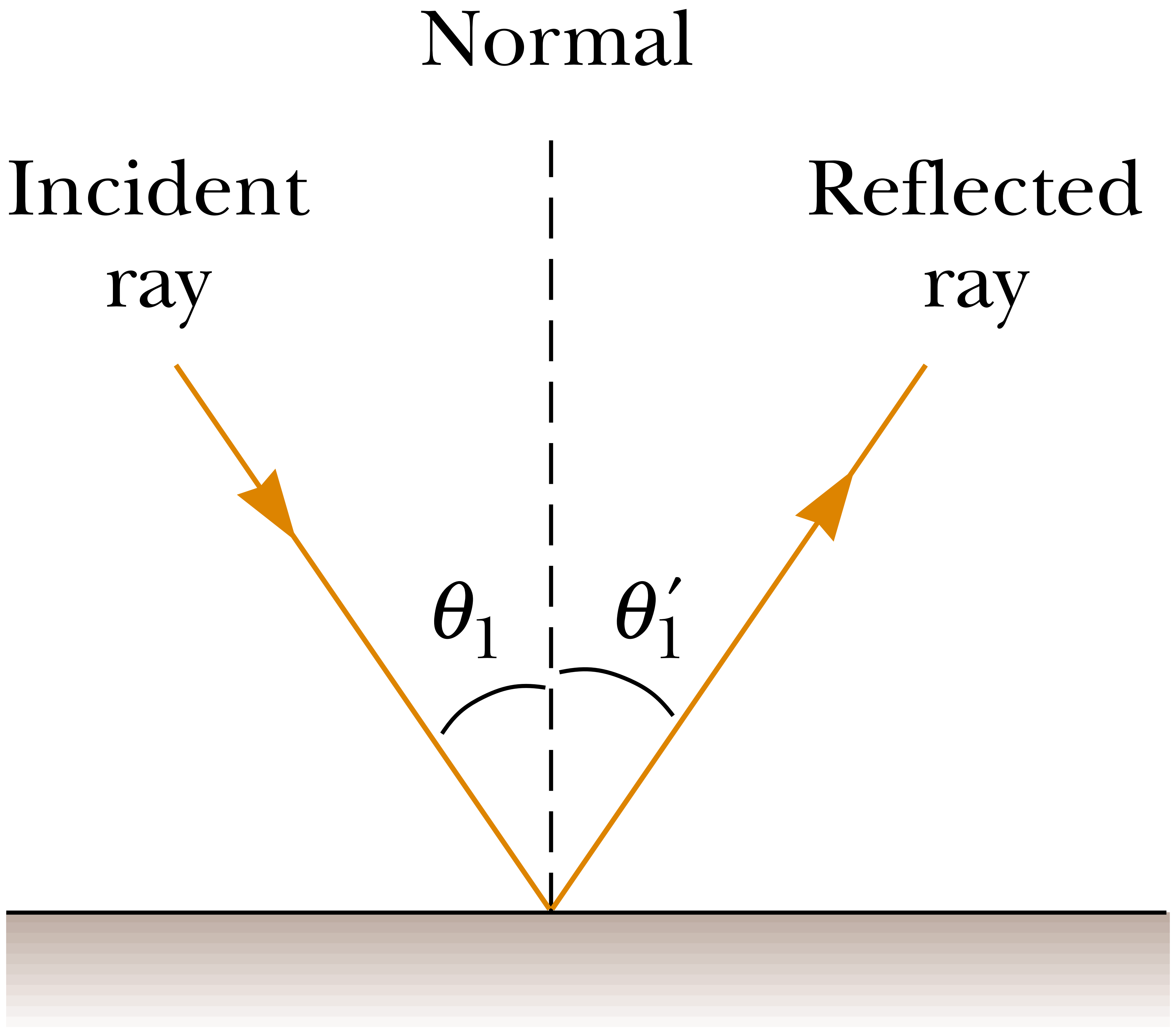
➢ Surface behaves as smooth surface if surface variations are much smaller than wavelength of incident light

- ➢ Difference between these two kinds of reflection explains why it is more difficult to see while driving on a rainy night
- ➢ If road is wet ☛ smooth water surface specularly reflects most of your headlight beams away from your car (and perhaps into eyes of oncoming drivers)
- ➢ When road is dry ☛ its rough surface diffusely reflects part of headlight beam back towards you allowing to see highway more clearly

- ➢ We'll concern ourselves only with specular reflection and use term reflection to mean specular reflection

Law of reflection

- Consider light ray traveling in air and incident at angle on flat smooth surface
- Incident and reflected rays make angles θ_1 and θ_1' with respect to normal
- Experiments and theory show that



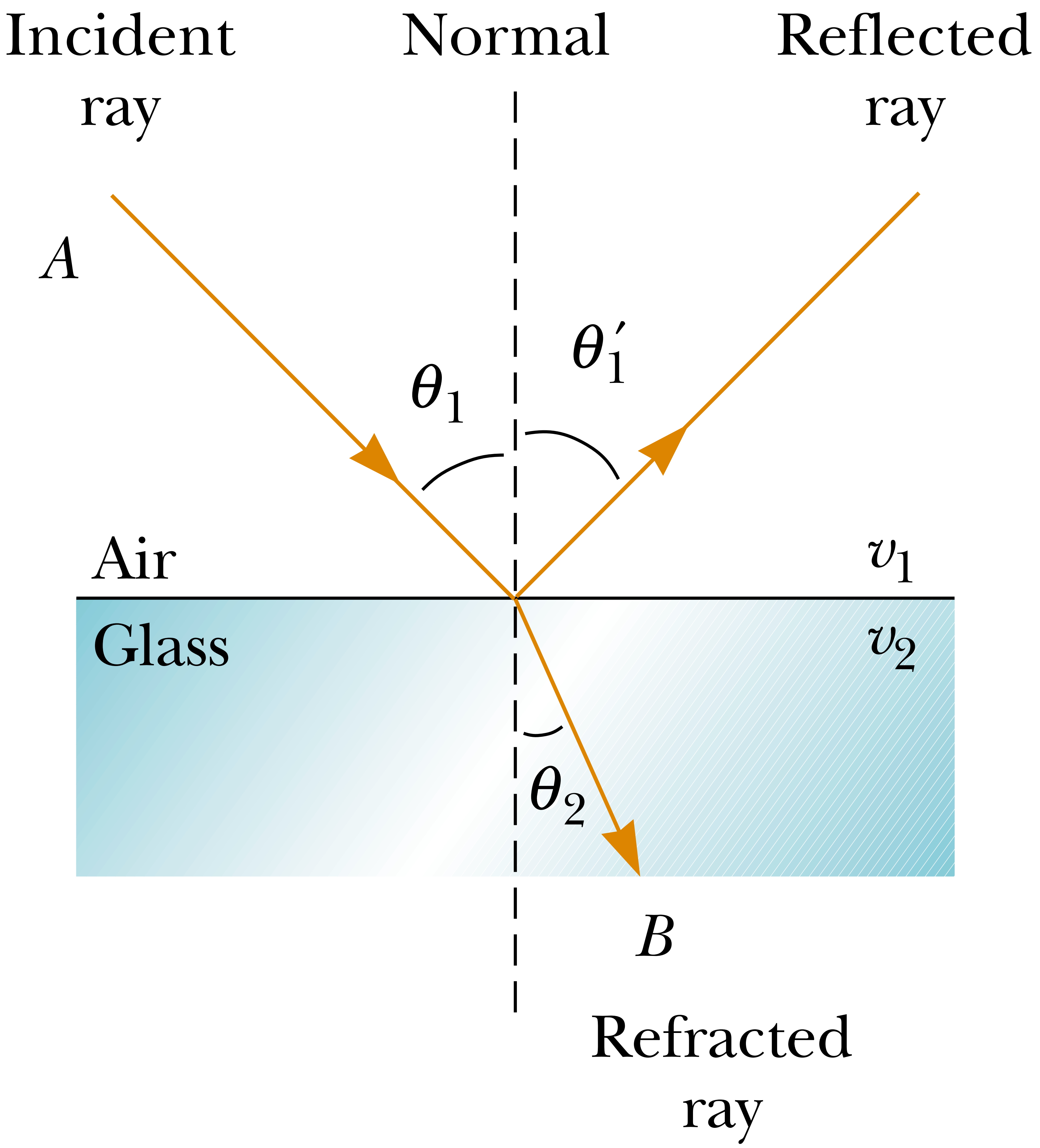
angle of reflection equals angle of incidence

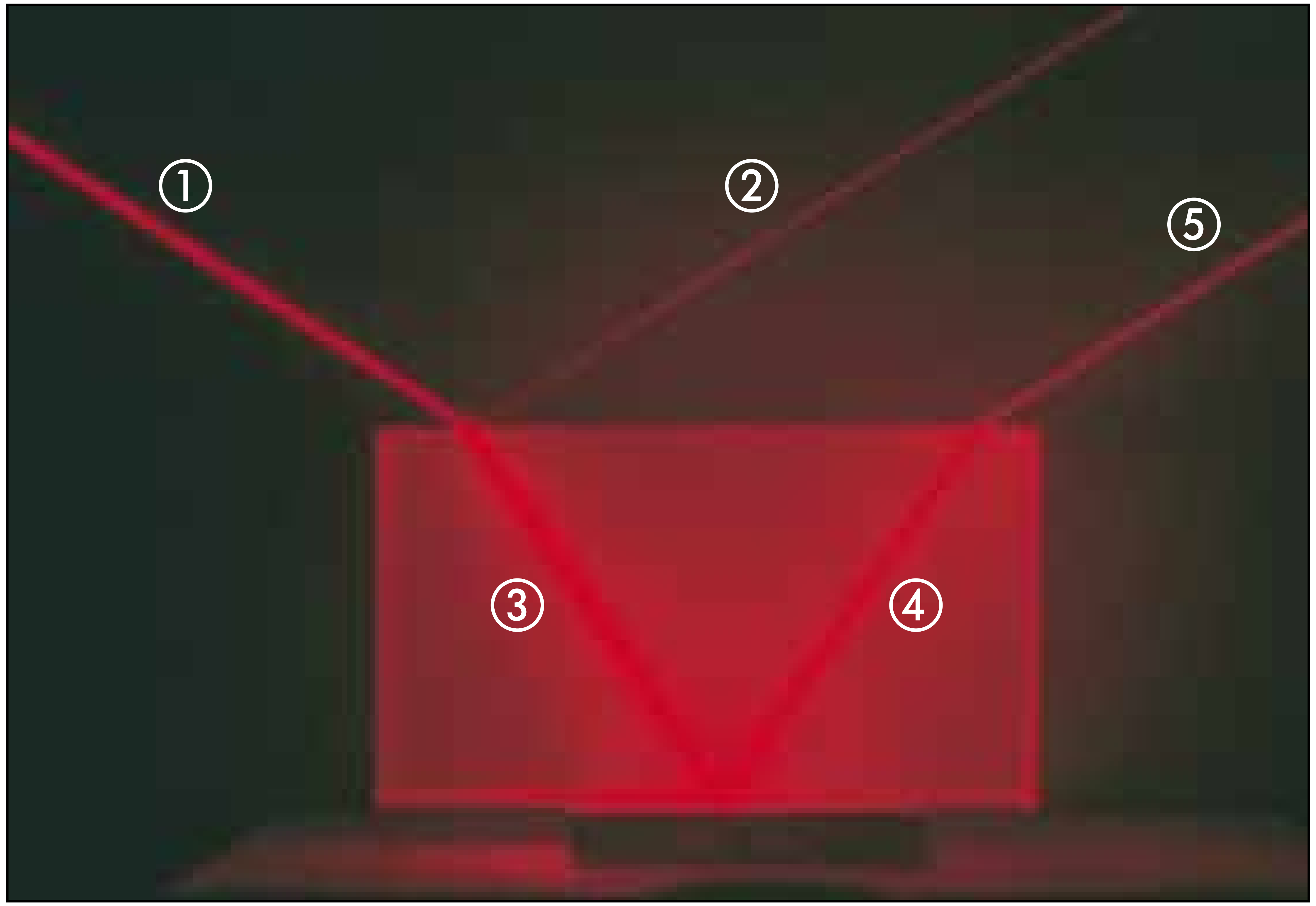
$$\theta_1' = \theta_1$$

- Normal is a line drawn perpendicular to surface at point where incident ray strikes surface

Refraction

➢ When light ray traveling in medium encounters with another medium part of energy is reflected and part enters second medium





➢ Ray that enters second medium is bent at boundary and is said to be **refracted**

➢ Incident ray, reflected ray, and refracted ray all lie in same plane

- Light only travels at $c \simeq 3 \times 10^8$ m/s in vacuum
- In materials ☛ it is always slowed down
- Index of refraction ☛ how fast light travels through material

$$\text{index of refraction} = n = \frac{\text{speed of light (in vacuum)}}{\text{speed of light (in medium)}}$$

- Bigger n ☛ slower light travels

Material	Index of Refraction (n)
Vacuum	1.000
Air	1.000277
Water	1.333333
Ice	1.31
Glass	About 1.5
Diamond	2.417

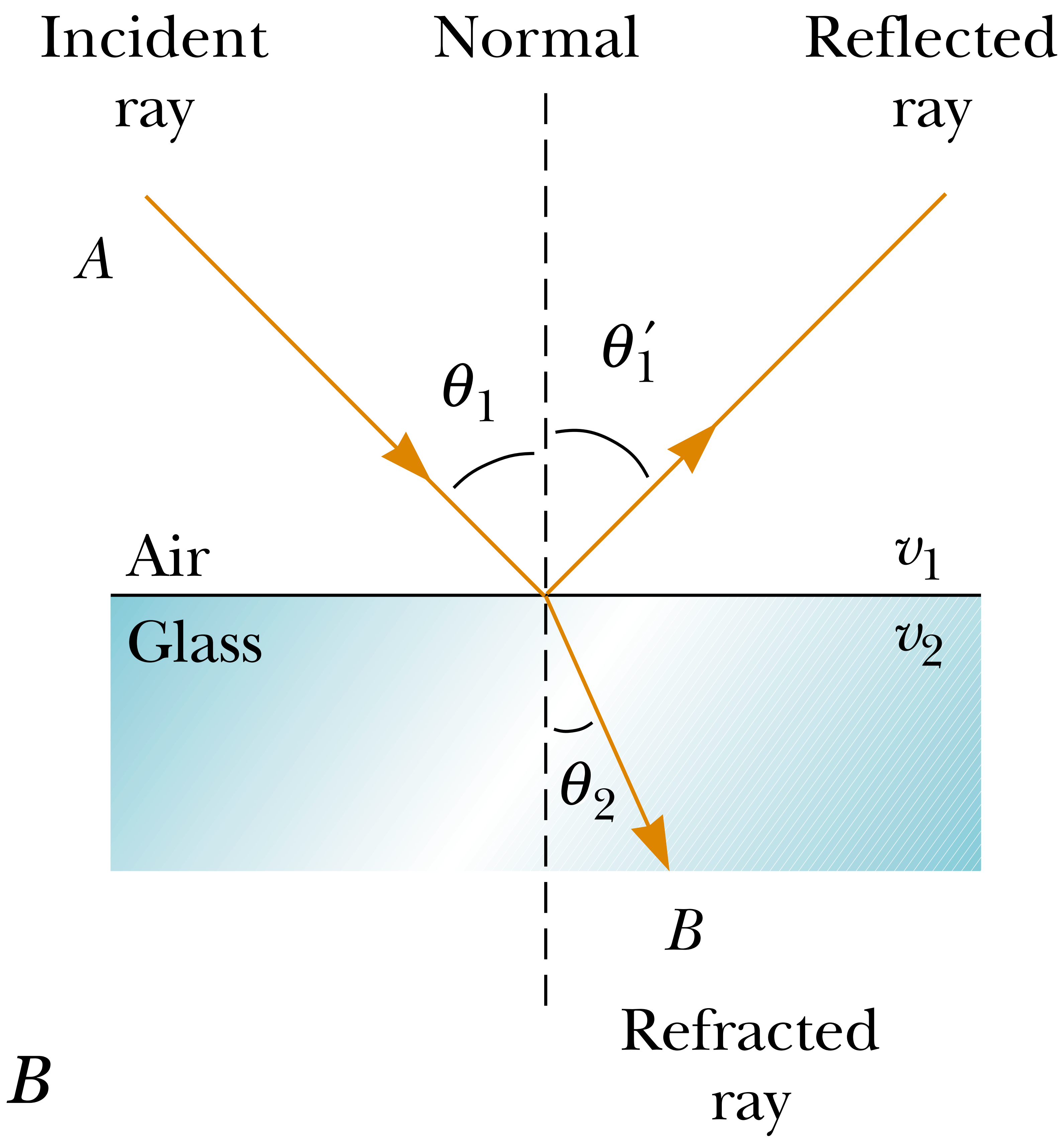
Angle of refraction θ_2

➢ Depends on properties of two media and on angle of incidence

speed of in second medium

$$\frac{\sin\,\theta_2}{\sin\,\theta_1} = \frac{v_2}{v_1} = \text{constant}$$

speed of light in first medium



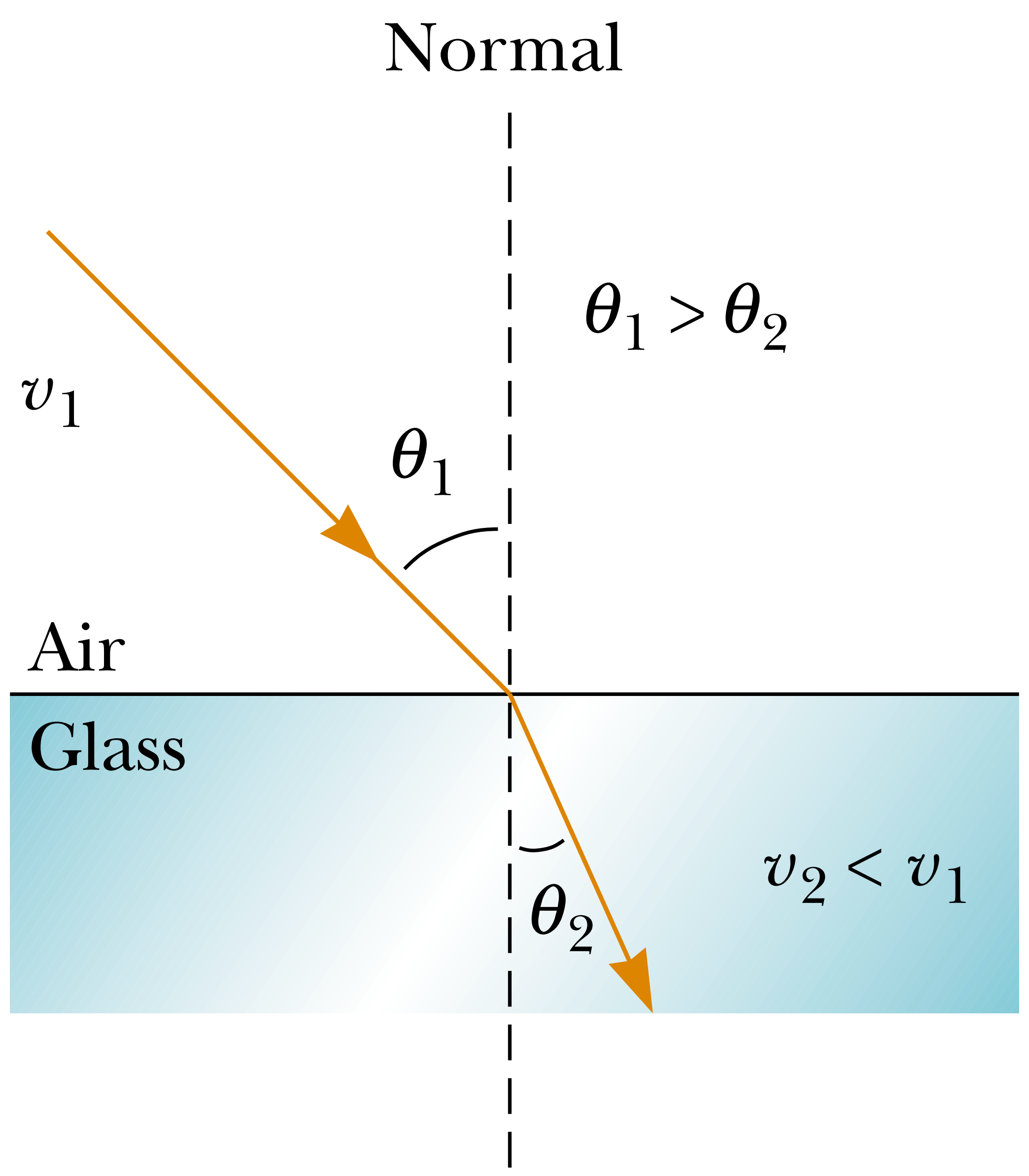
➢ Path of a light ray through a refracting surface is reversible

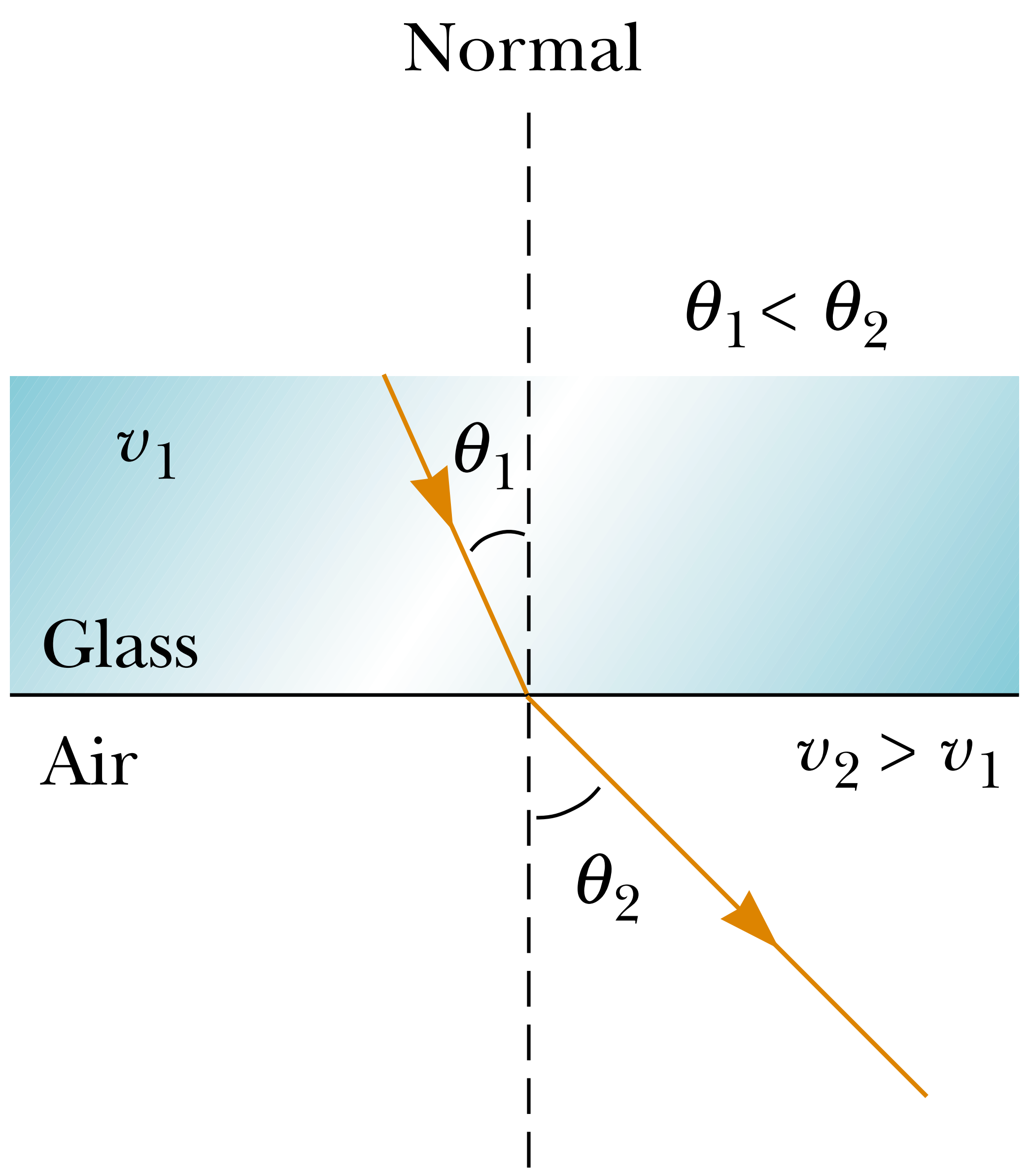
➢ For example ☛ ray shown in figure travels from point A to point B

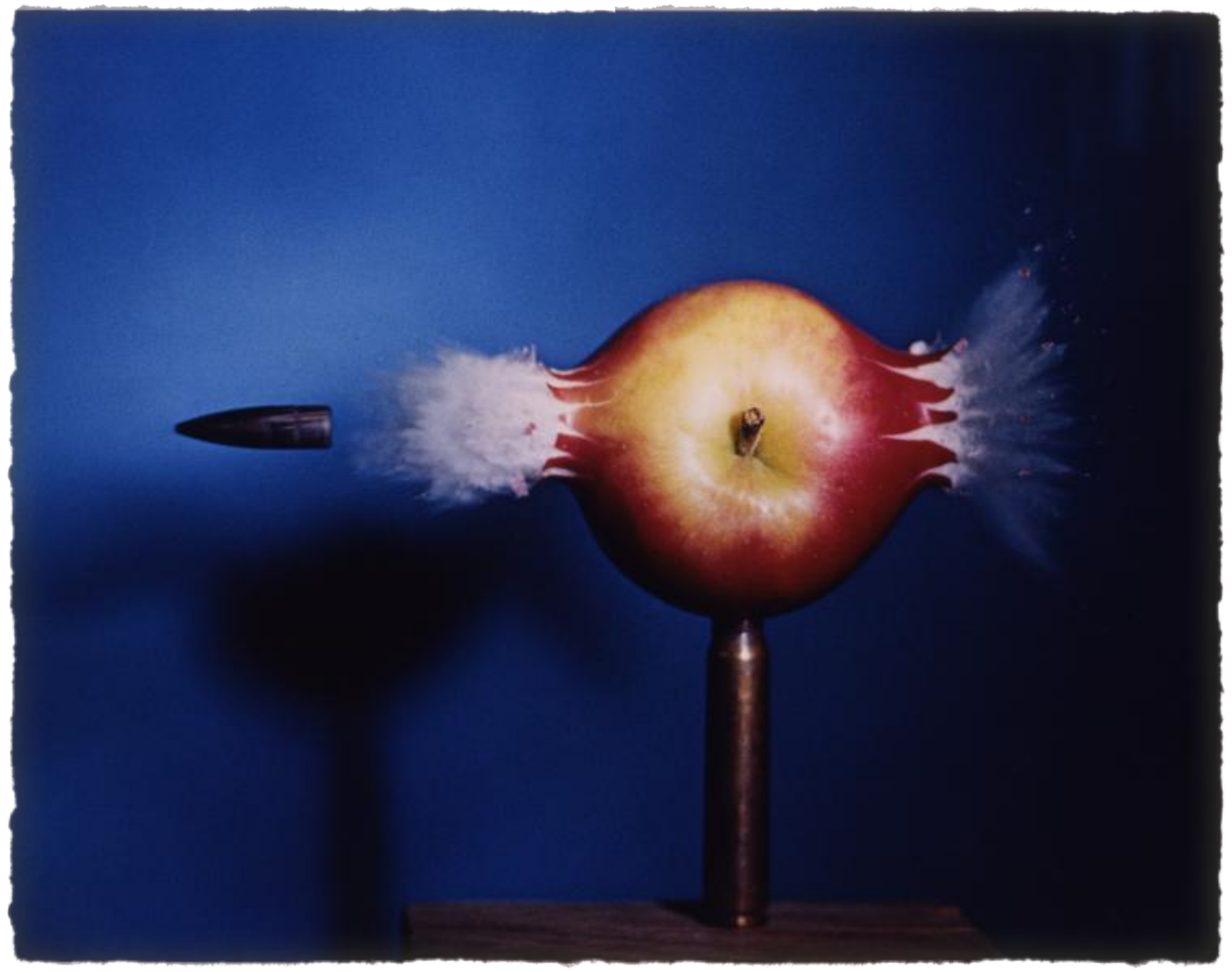
➢ If ray originated at B ☛ it would travel to left along line BA to reach point A and reflected part would point downward and to left in glass

Behavior of light as it passes from air into another substance and re-emerges into air is often source of confusion

- When light travels in air its speed is 3.00 x 10^8 m/s but this speed is reduced to ≈ 2 x 10^8 m/s when light enters block of glass
- When light re-emerges into air its speed instantaneously increases to its original value of 3.00 x 10^8 m/s
- This is far different from what happens when bullet is fired through apple
- In this case speed of bullet is reduced as it moves through apple because some of its original energy is used to tear apart apple fiber
- When bullet enters air once again it emerges at speed it had just before leaving apple

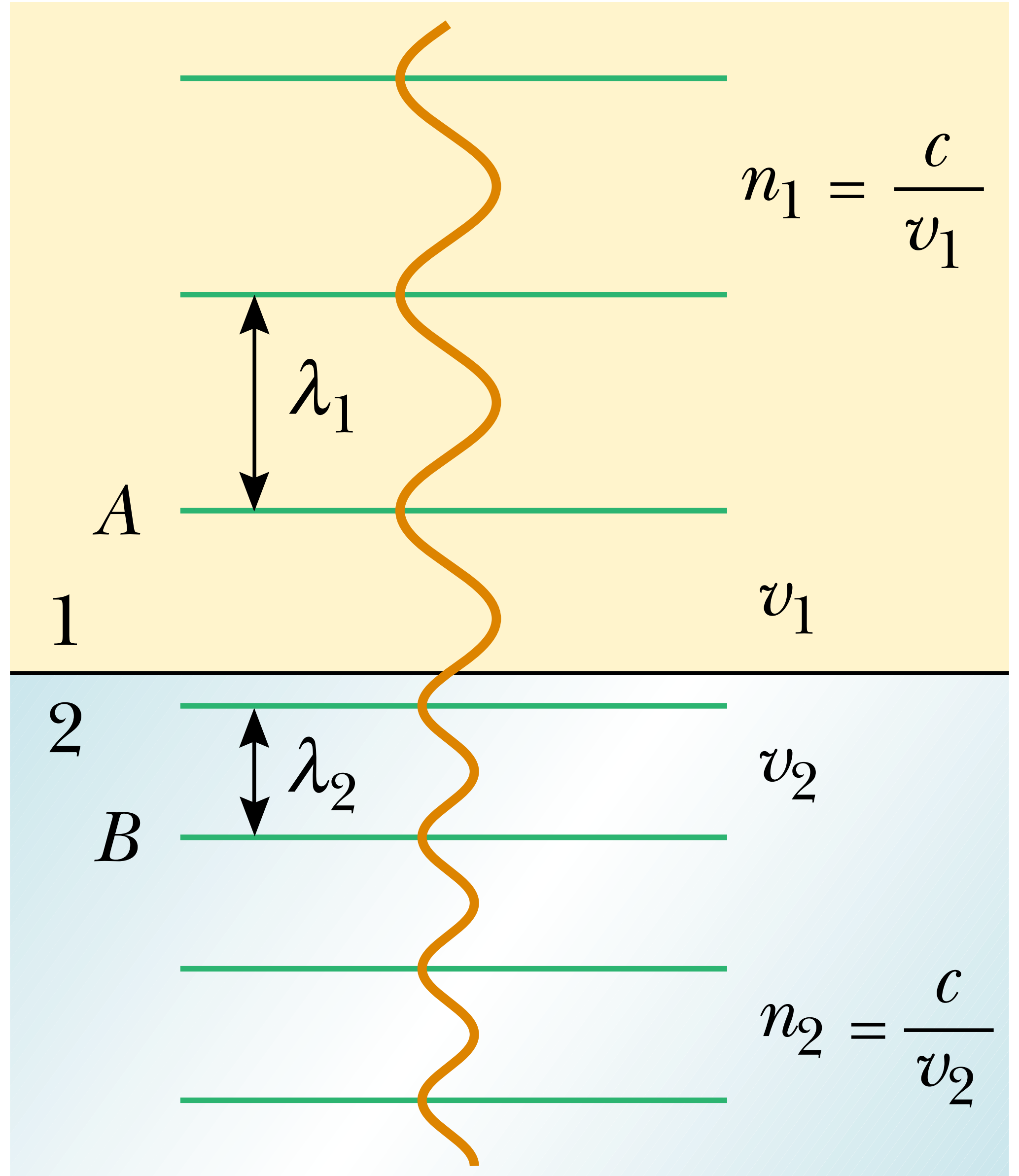






As light travels from one medium to another its frequency does not change but its wavelength does

➢ Waves pass observer at point A in medium 1 with certain frequency and are incident on boundary between medium 1 and medium 2



➢ Frequency with which waves pass observer at point B in medium 2 must equal frequency at which they pass point A

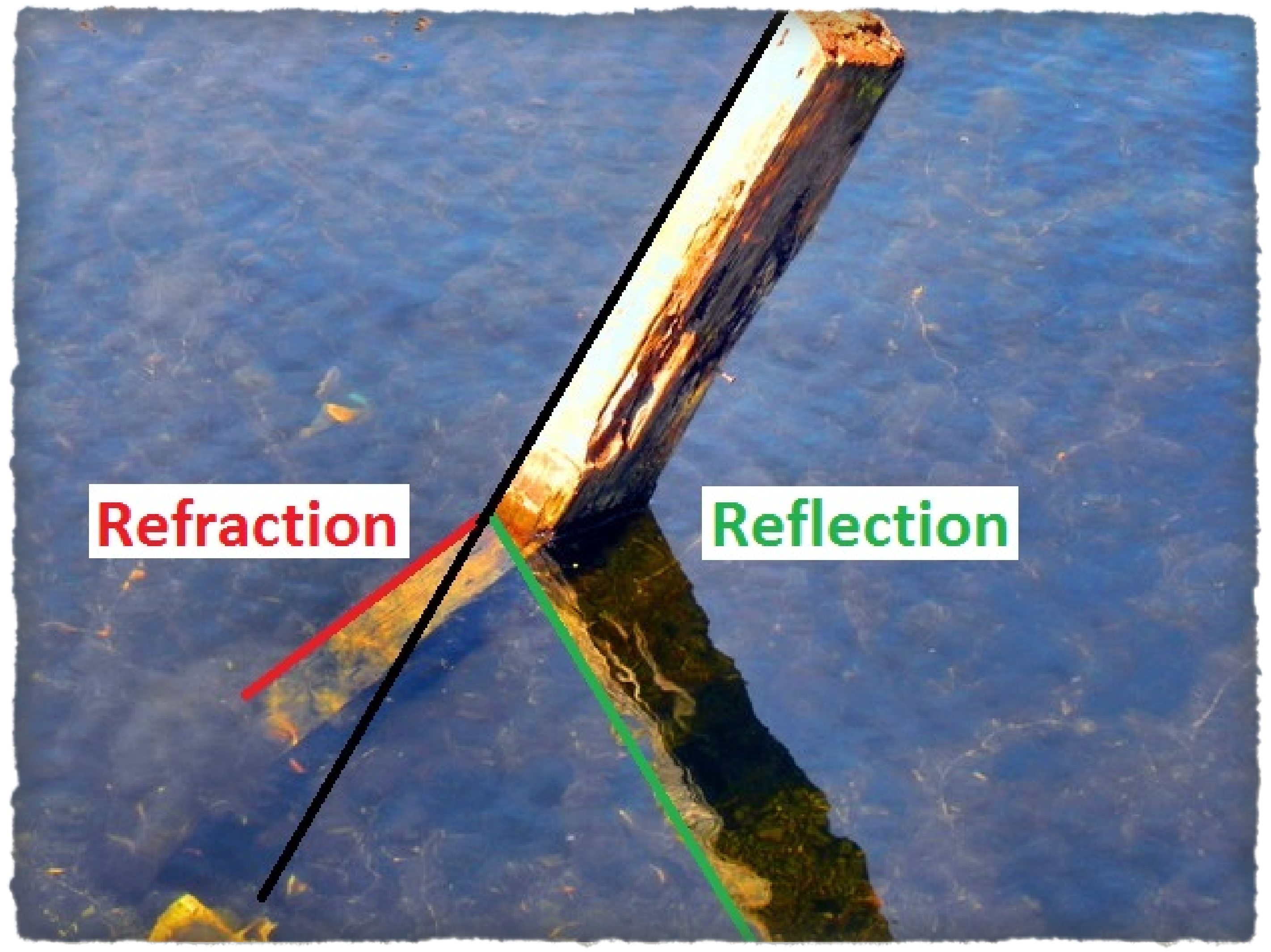
➢ If this were not the case energy would be piling up at boundary ☛ ($E = hf$)

➢ Because relationship $v = f\lambda$ must be valid in both media

$$f_1 = f_2 = f \quad ☛ \quad v_1 = f\lambda_1 \text{ and } v_2 = f\lambda_2$$

➢ Because $v_1 \neq v_2$ it follows that $\lambda_1 \neq \lambda_2$

- ➢ Relationship between index of refraction and wavelength $\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$
- ➢ This gives ☛ $\lambda_1 n_1 = \lambda_2 n_2$
- ➢ If medium 1 is vacuum (or for all practical purposes air) then ☛ $n_1 = 1$
- ➢ Index of refraction of any medium ☛ $n = \frac{\lambda_{\text{vacuum}}}{\lambda_n}$
- ➢ Because ☛ $n > 1, \lambda_n < \lambda$



- ➢ If we replace v_2/v_1 in refraction angle relation with n_1/n_2

Snell's law of refraction ☛ $n_1 \sin\theta_1 = n_2 \sin\theta_2$

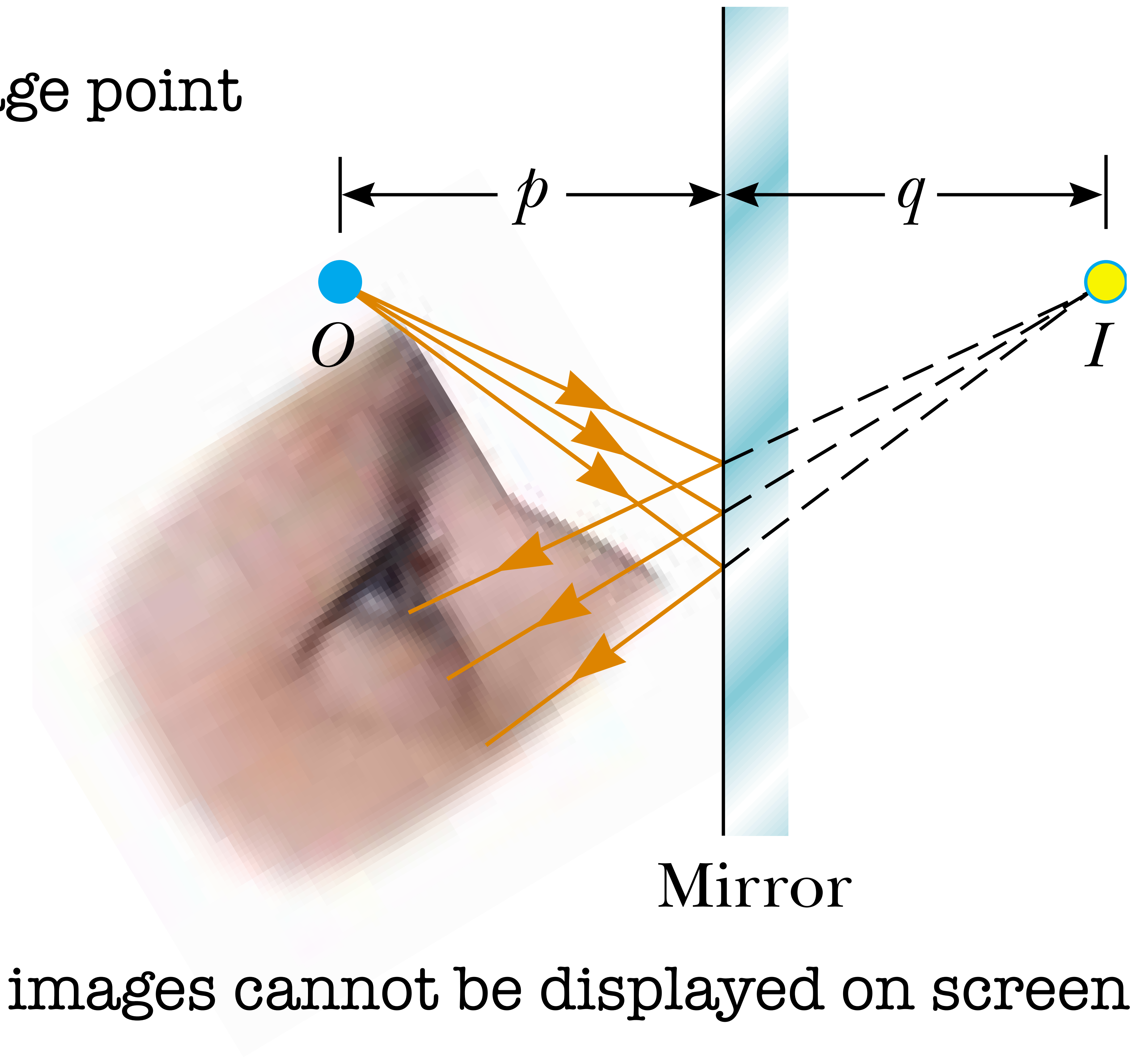
Images Formed by Flat Mirrors

➢ Images are classified as **real** or **virtual**

➢ Real image ☛ formed when light rays pass through and diverge from image point

➢ Virtual image ☛ formed when light rays don't pass through image point

but only appear to diverge from that point



➢ Image of object seen in flat mirror is always virtual

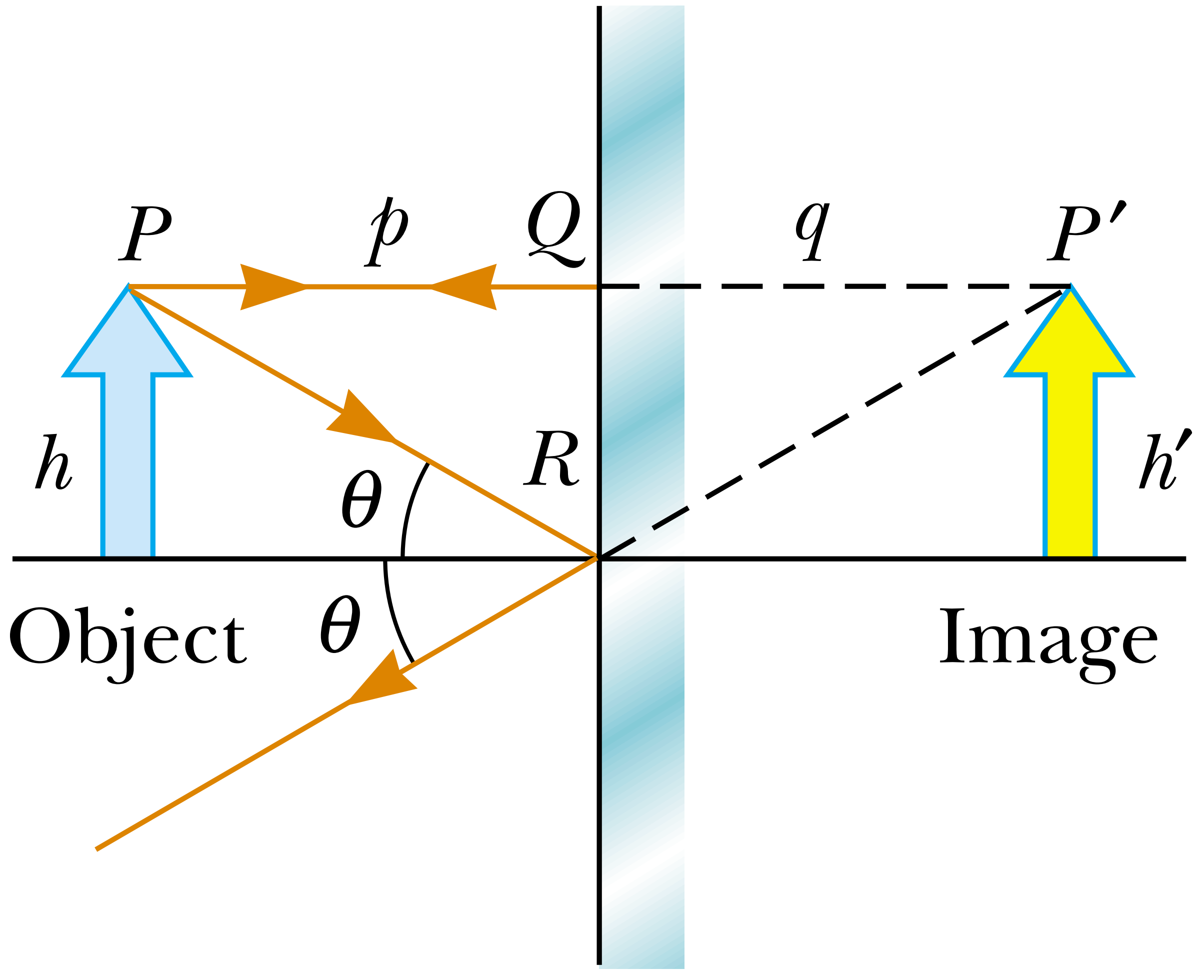
➢ Real images can be displayed on screen (e.g. movie) but virtual images cannot be displayed on screen

Properties of images of extended objects formed by flat mirrors

➢ There are infinite number of choices of direction in which light rays could leave each point on object we need only two rays to determine where image is formed

➢ One ray starts at *P* follows horizontal path to mirror and reflects back on itself

➢ Second ray follows oblique path *PR* and reflects according to law of reflection



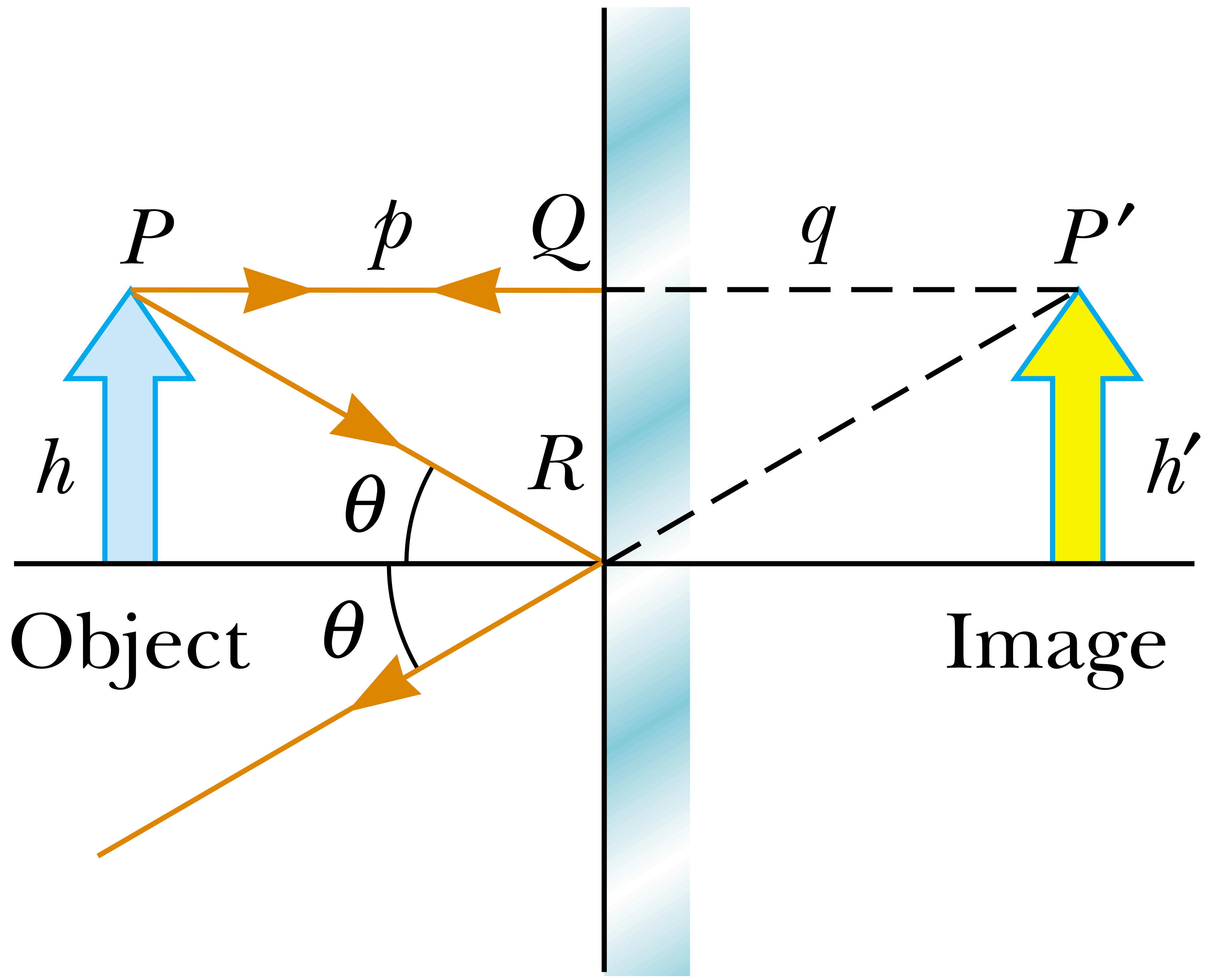
➢ An observer in front of mirror would trace two reflected rays back to point at which they appear to have originated ☛ which is point *P'* behind mirror

➢ Because triangles *PQR* and *P'QR* are congruent ☛ *PQ* = *P'Q*

➢ **Image formed by object placed in front of flat mirror is as far behind mirror as object is in front**

➢ Geometry reveals that object height h equals image height h'

➢ Define **lateral magnification** M of image as follows



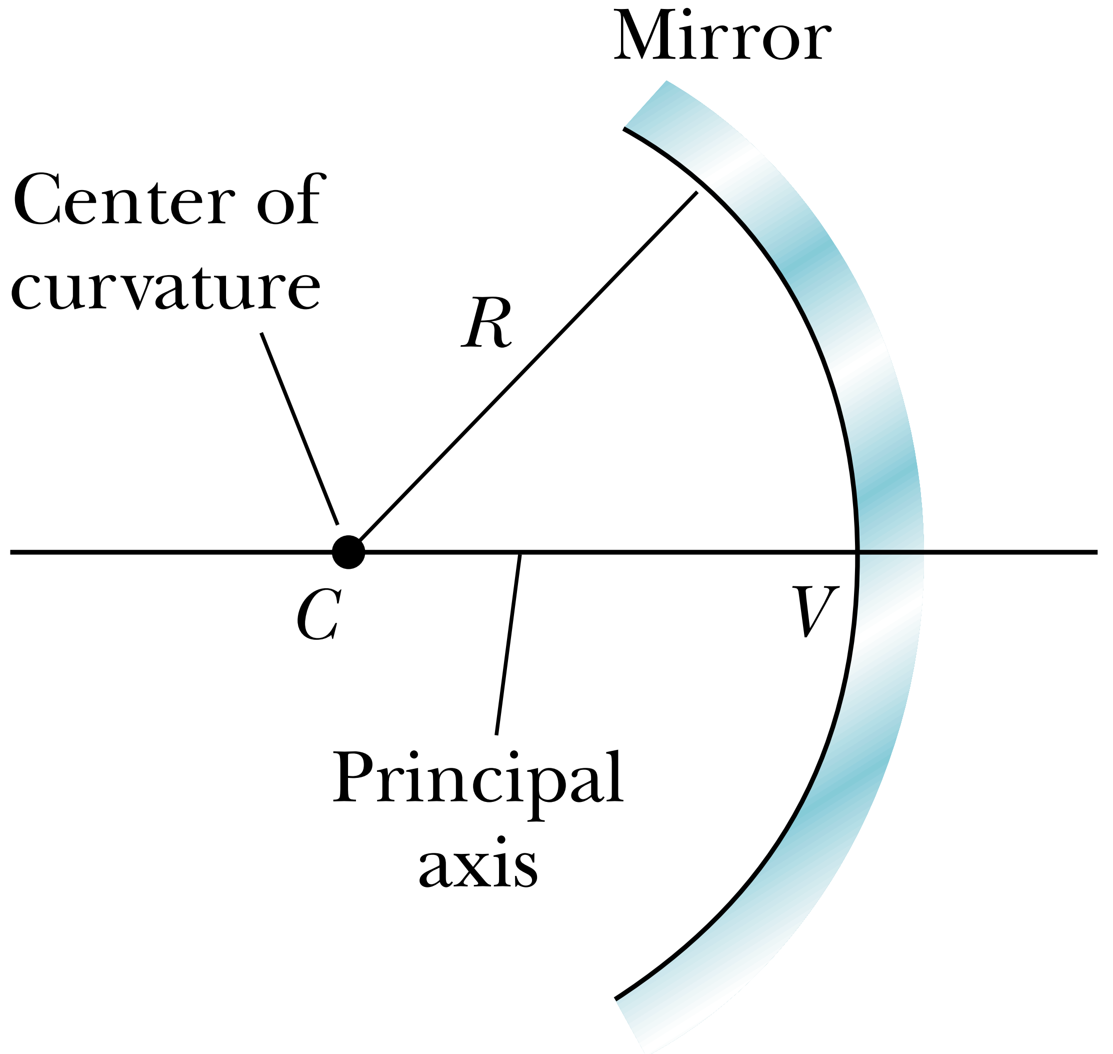
$$M = \frac{\text{Image height}}{\text{Object height}} = \frac{h'}{h}$$

➢ This is general definition of lateral magnification for image from any type of mirror

➢ For flat mirror $M = 1$ for any image because $h' = h$

Images Formed by Spherical Mirrors

➢ Spherical mirror has shape of section of sphere

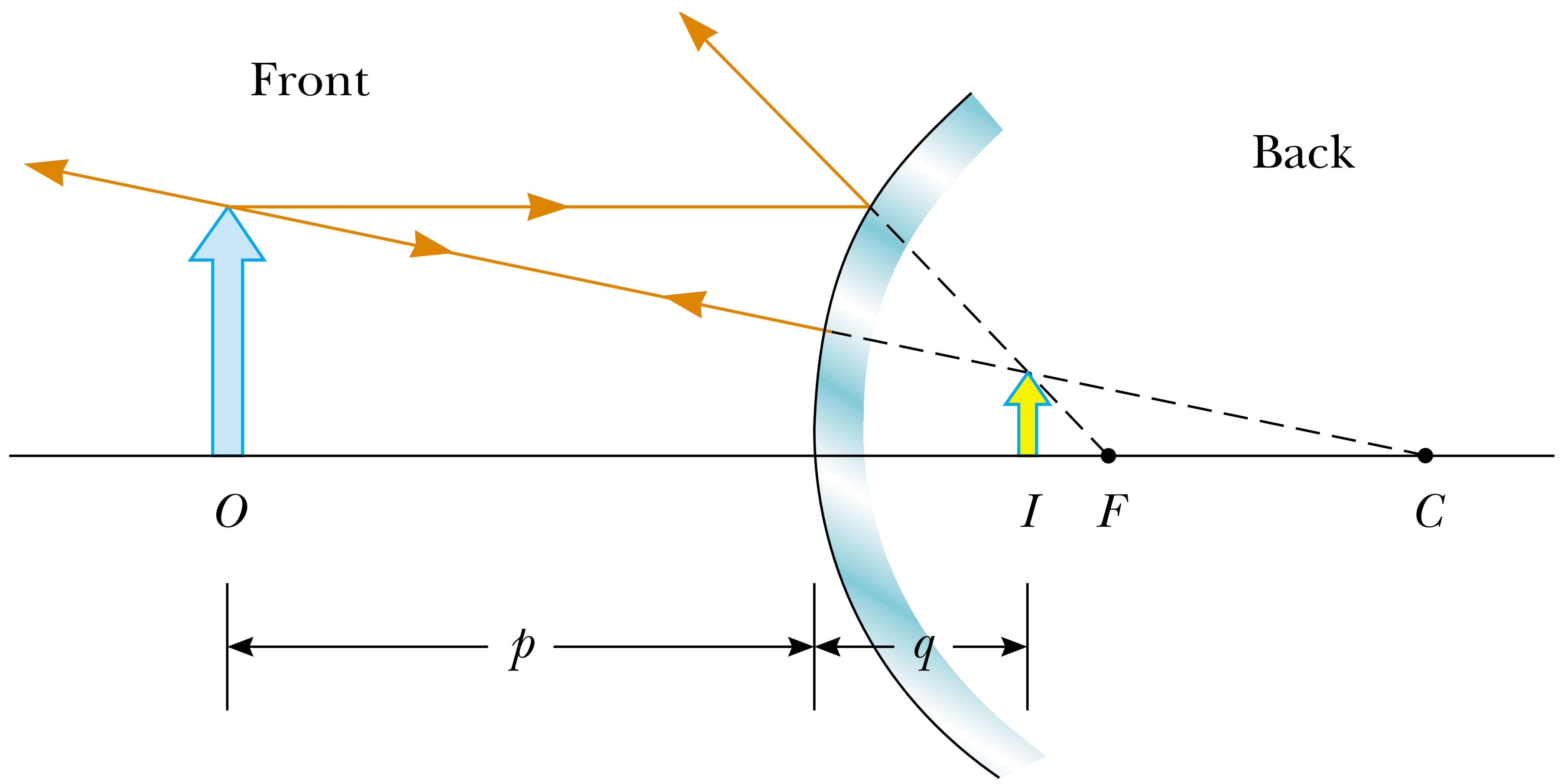


Concave Mirror

➢ Mirror has a radius of curvature *R* and its center of curvature is point *C*

principal axis of mirror ☛ line through *V* and *C*

Convex Mirror



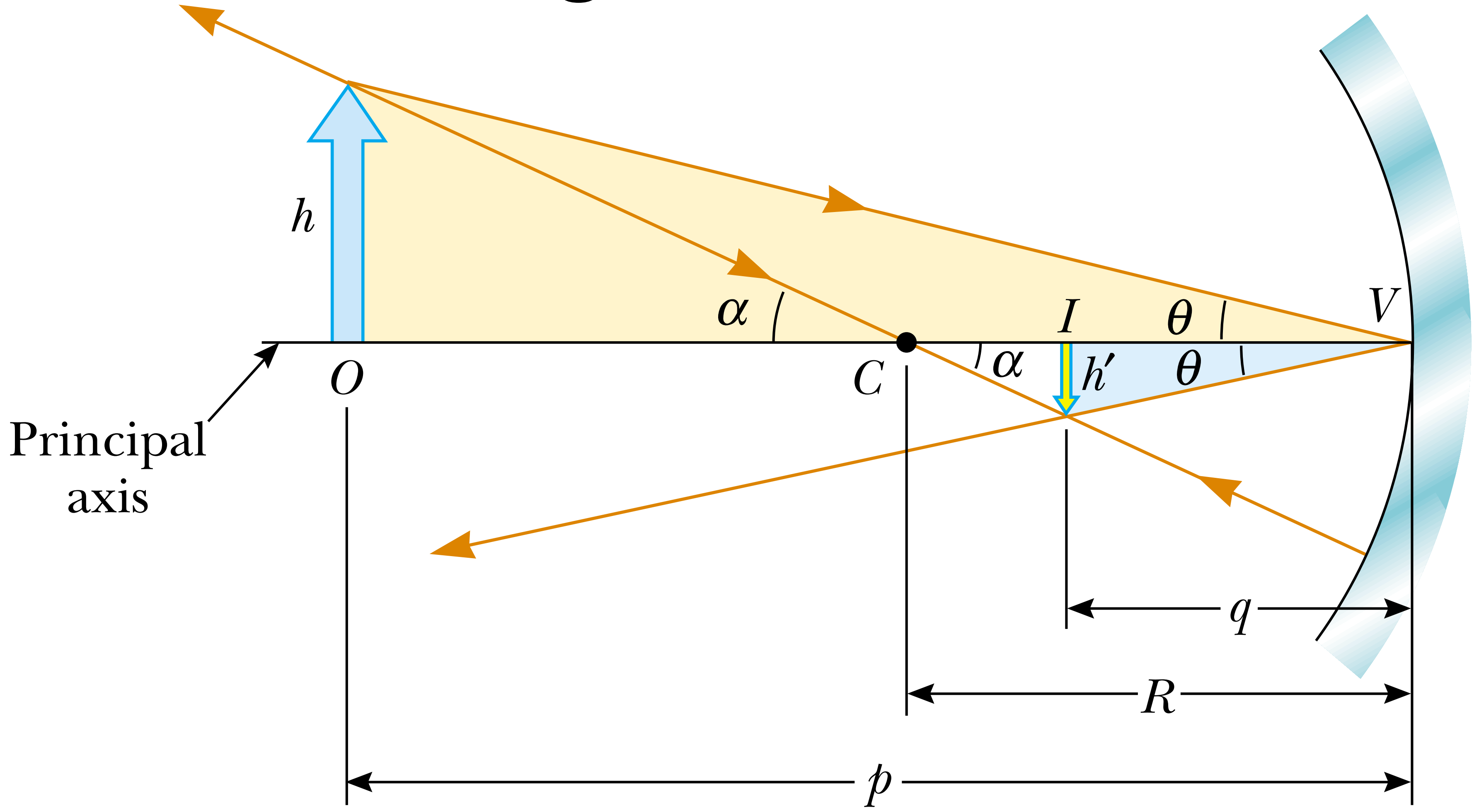
➢ Image in **convex mirror** is virtual because reflected rays only appear to originate at image point

➢ Image is always upright and smaller than object

- Calculate image distance q from knowledge of object distance p and radius of curvature R
- By convention these distances are measured from center point V
- Consider two rays leaving tip of object
- First ray passes through center of curvature C of mirror hitting mirror perpendicular to mirror surface and reflecting back on itself
- Second ray strikes mirror at V and reflects obeying law of reflection
- Image of tip of arrow is located at point where these two rays intersect

$$\tan\,\theta = h/p \quad \text{and} \quad \tan\,\theta = -h'/q$$

- Negative sign is introduced because image is inverted so h' is taken to be negative



- Magnification of image is ☛ $M = \frac{h'}{h} = -\frac{q}{p}$
- Two triangles have α as one angle

$$\tan\alpha = \frac{h}{p - R} \quad \text{and} \quad \tan\alpha = -\frac{h'}{R - q}$$

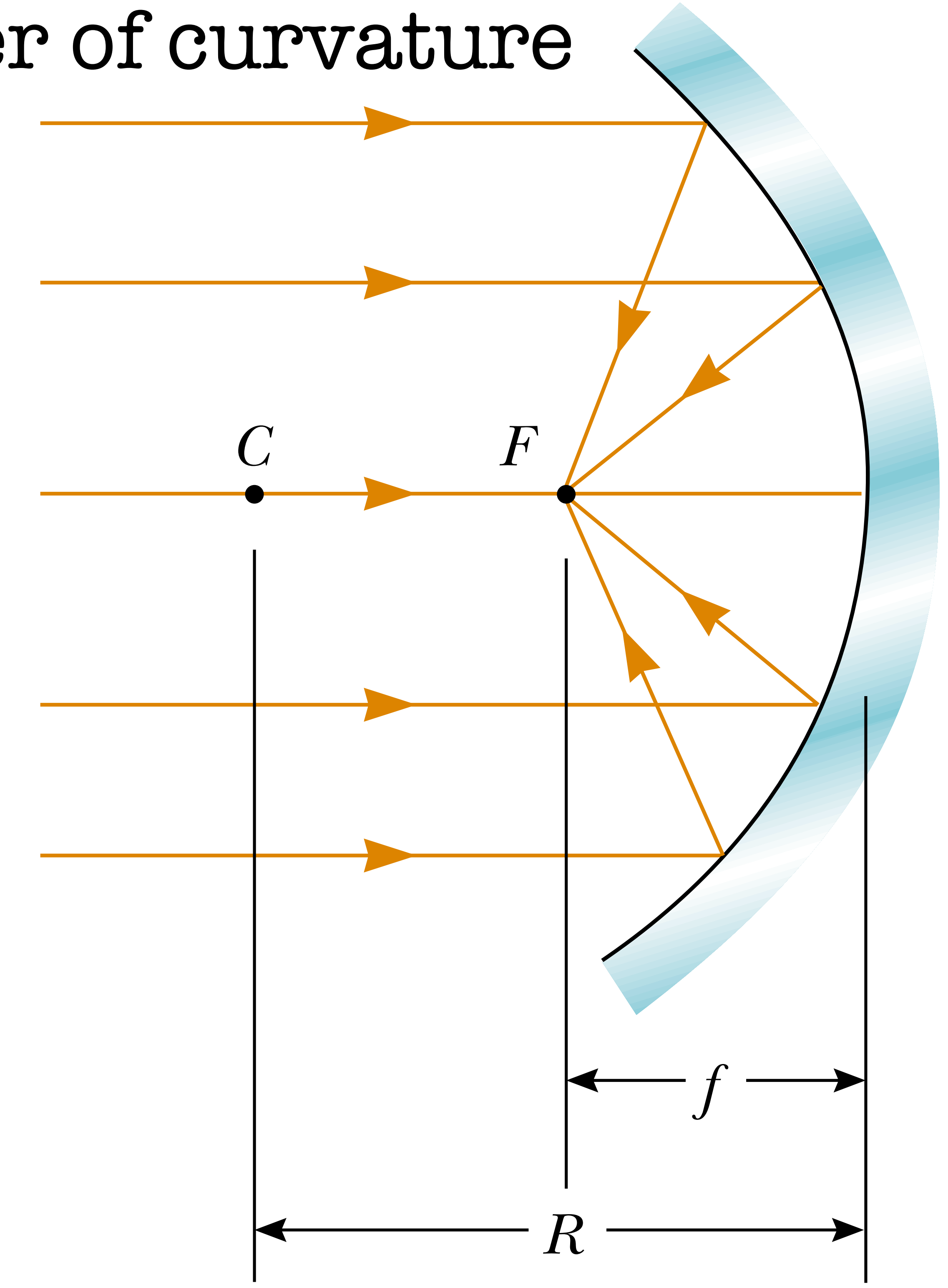
$$\frac{h'}{h} = -\frac{R - q}{p - R} \quad ☛ \quad \frac{R - q}{p - R} = \frac{q}{p}$$

- Simple algebra reduces this to **mirror equation**

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$$

- If $p \gg R \Rightarrow 1/p \approx 0 \Rightarrow p \to \infty$ and so $q \approx R/2$

➢ When object is very far from mirror image point is halfway between center of curvature and center point on mirror



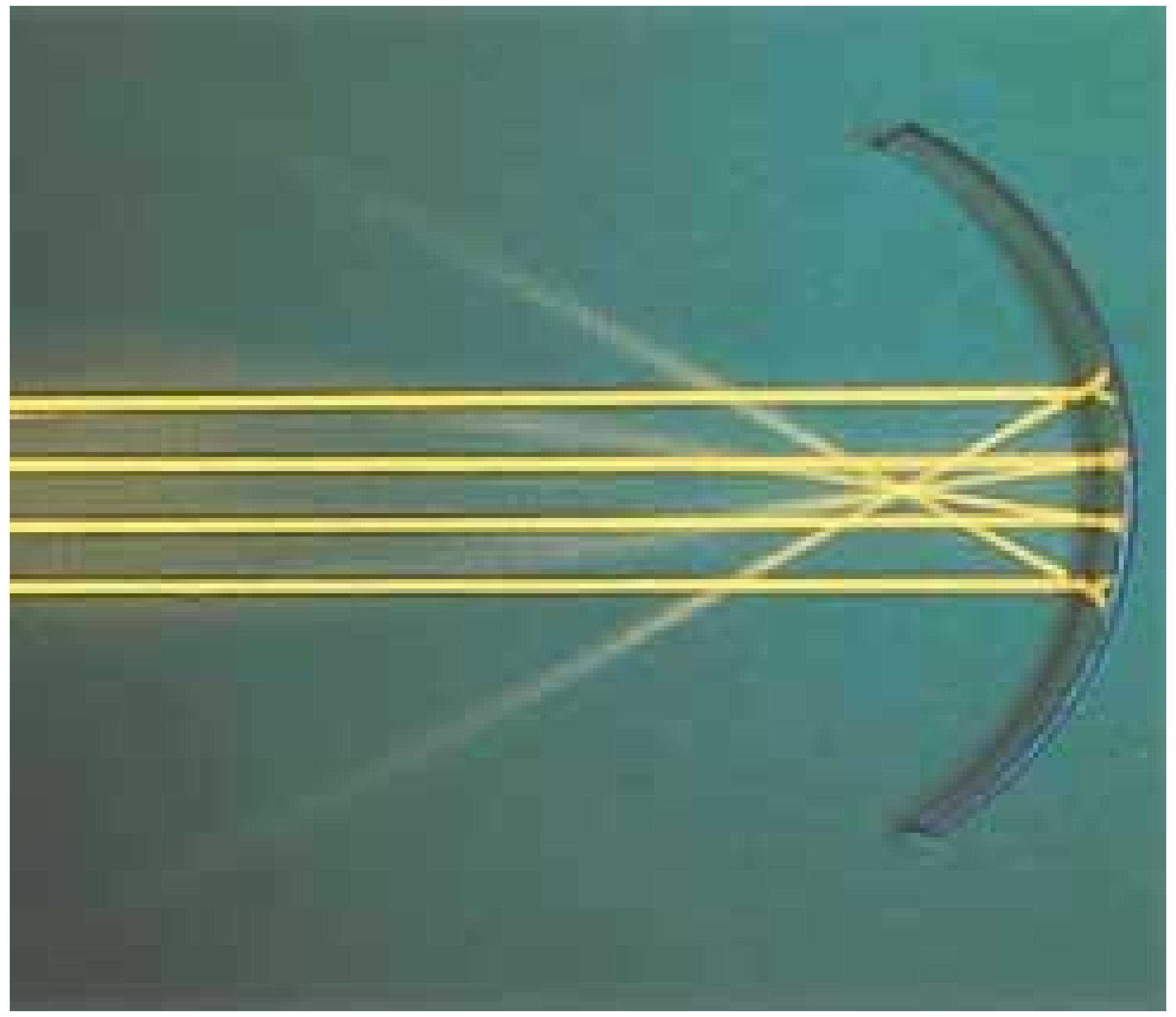
➢ Image point in this special case **is @ focal point *F***

and image distance is **focal length** f

$$f = \frac{R}{2}$$

➢ Focal length is parameter particular to given mirror

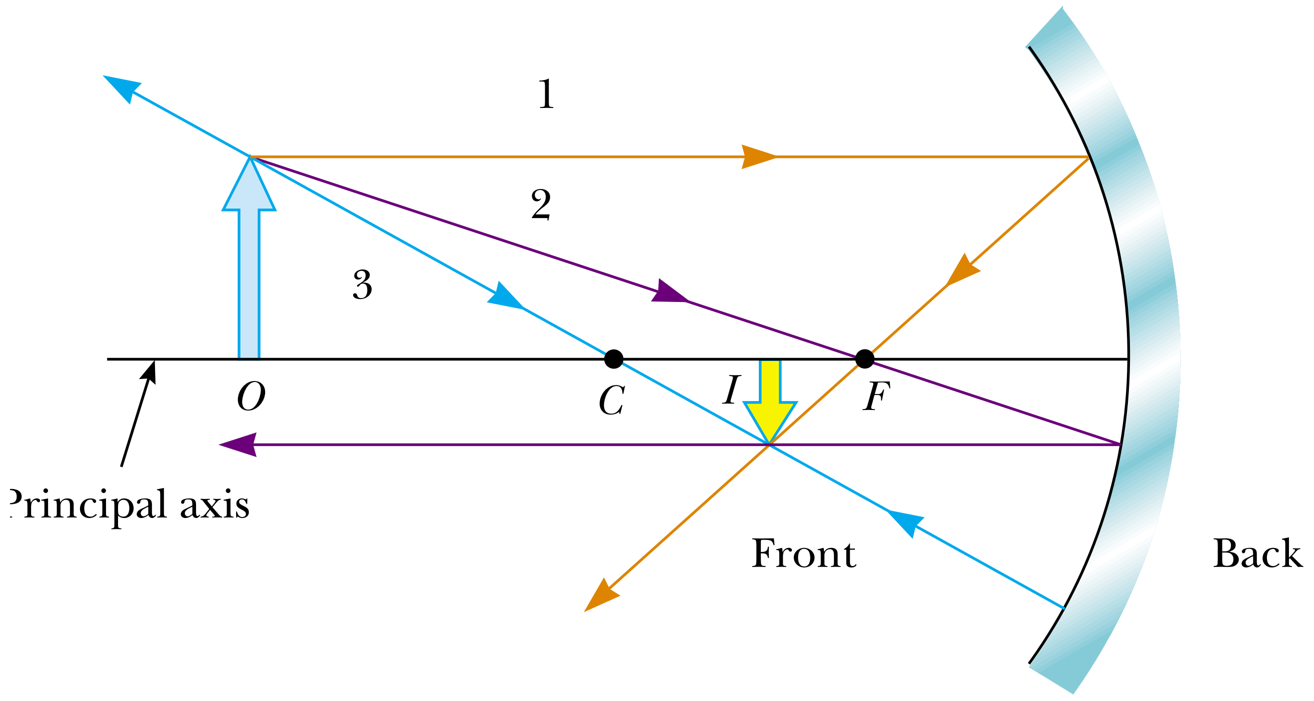
and can be used to compare one mirror to another

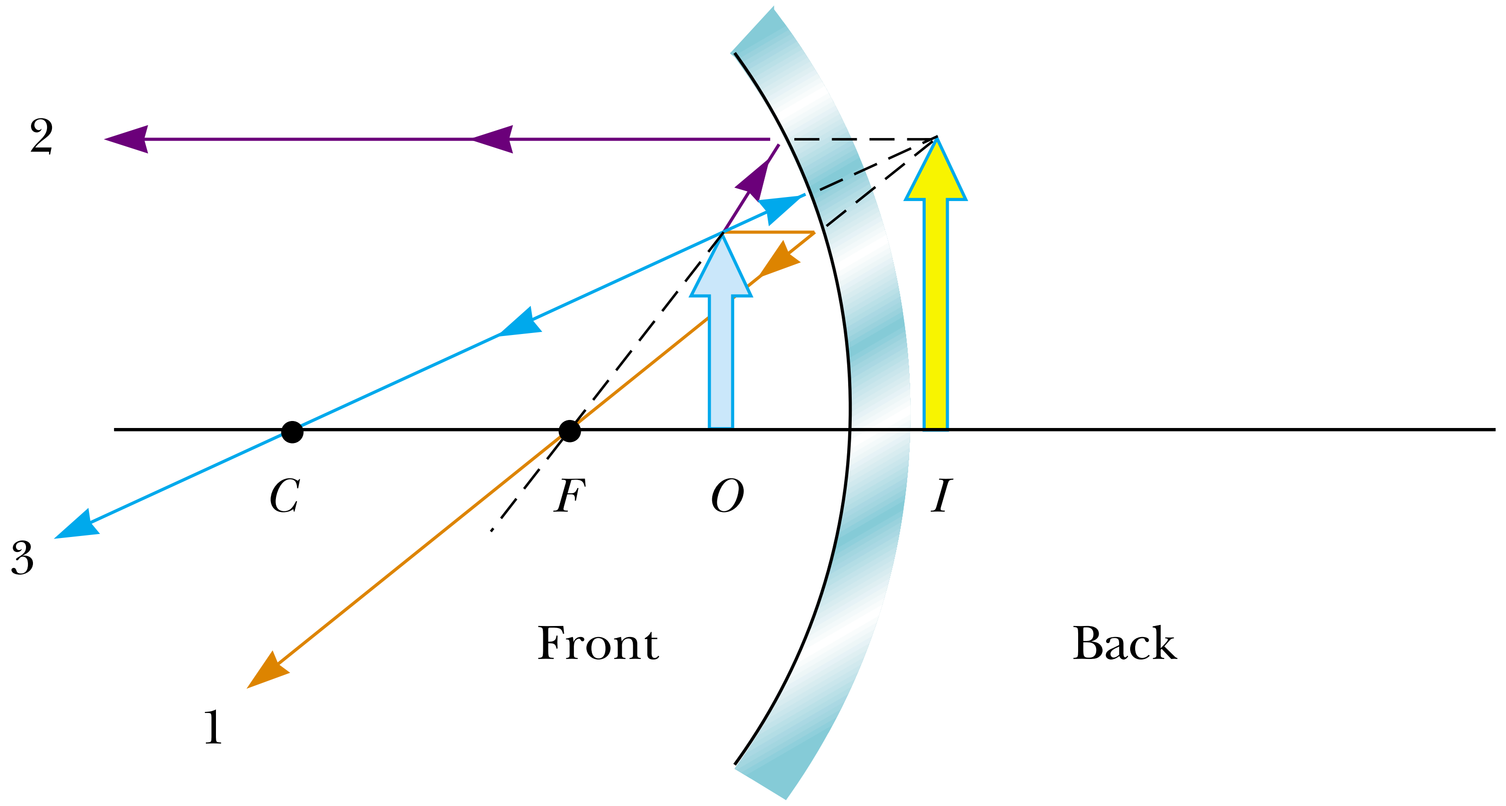


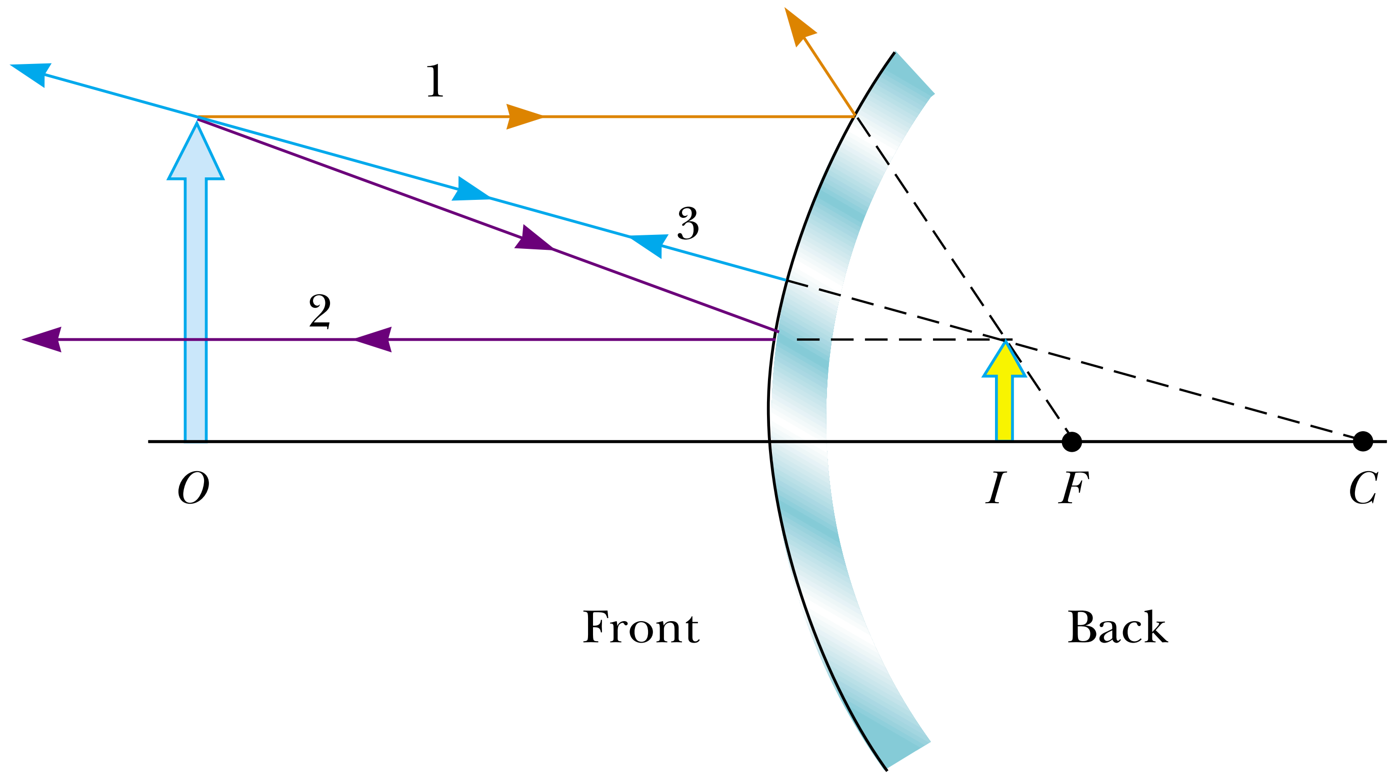
➢ Mirror equation can be expressed in terms of focal length

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Ray Diagrams for Mirrors

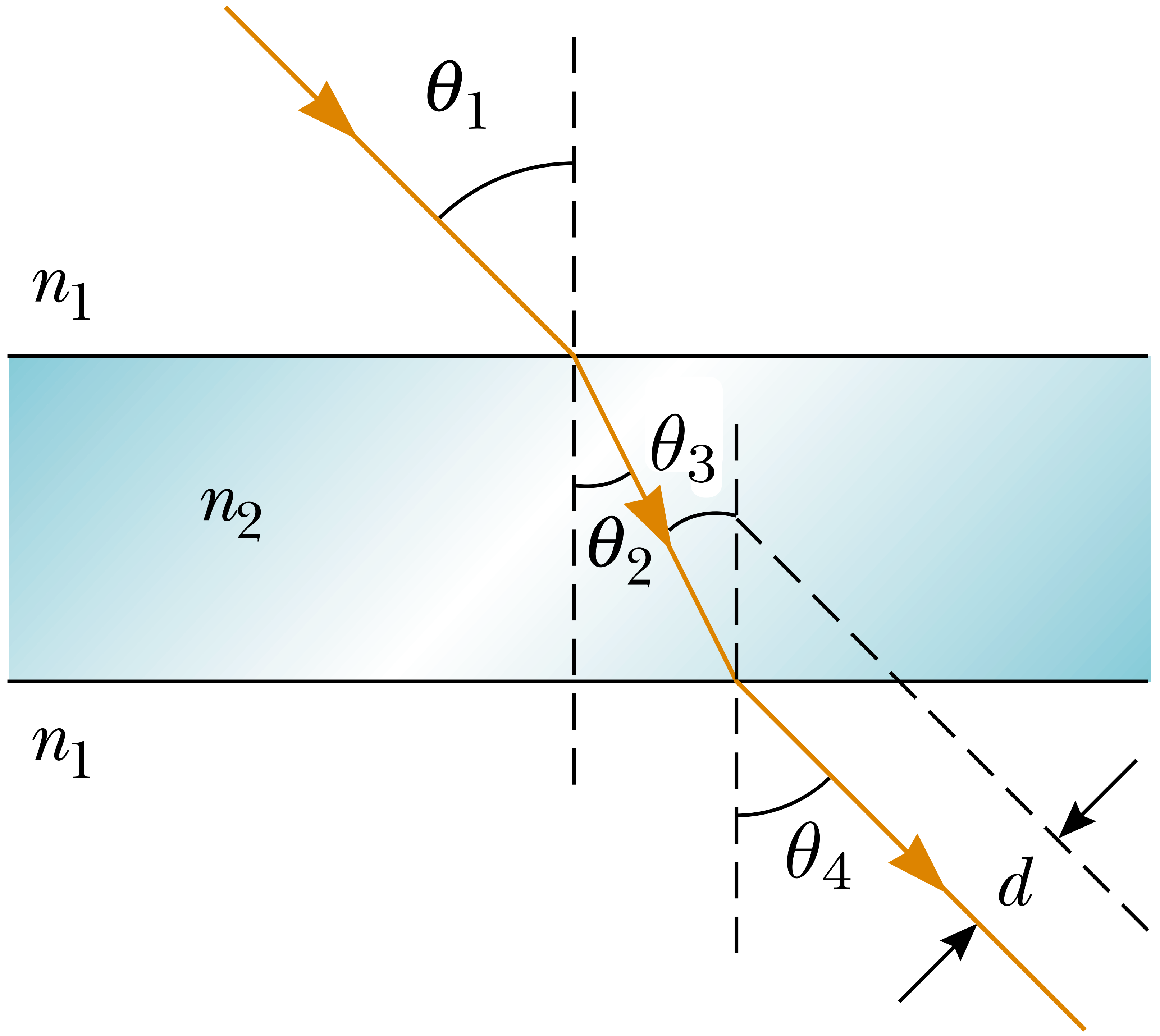






Images Formed by Thin Lenses

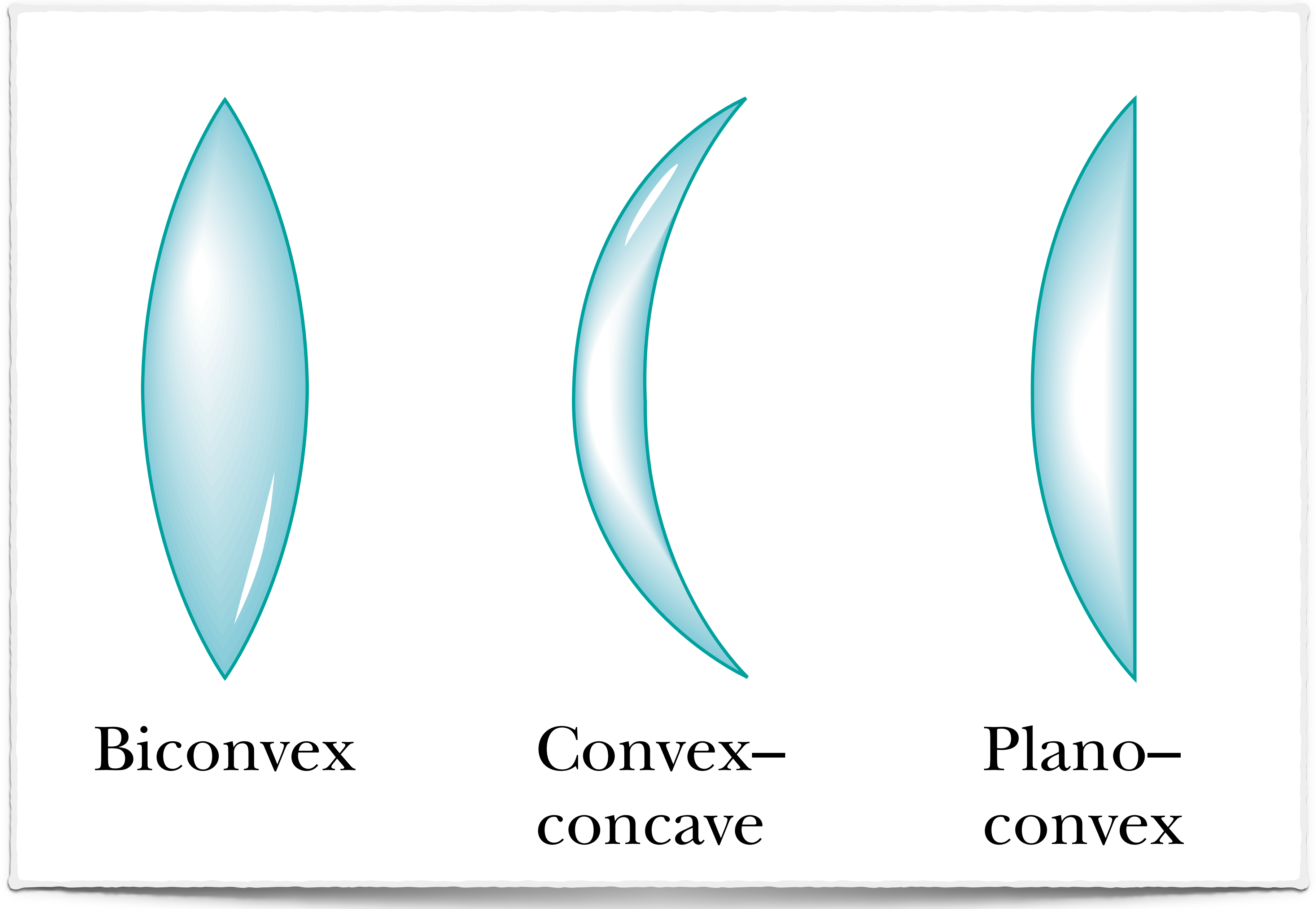
- Geometry tells us (if walls are parallel) that $\theta_2 = \theta_3$
- This means $\sin\theta_2 = \sin\theta_3$
- So $n_1 \sin\theta_{in} = n_2 \sin\theta_2 = n_2 \sin\theta_3 = n_1 \sin\theta_{out}$
- This means (compare far left with far right of equation) $\sin\theta_{in} = \sin\theta_{out}$ ☛ which says $\theta_{in} = \theta_{out}$

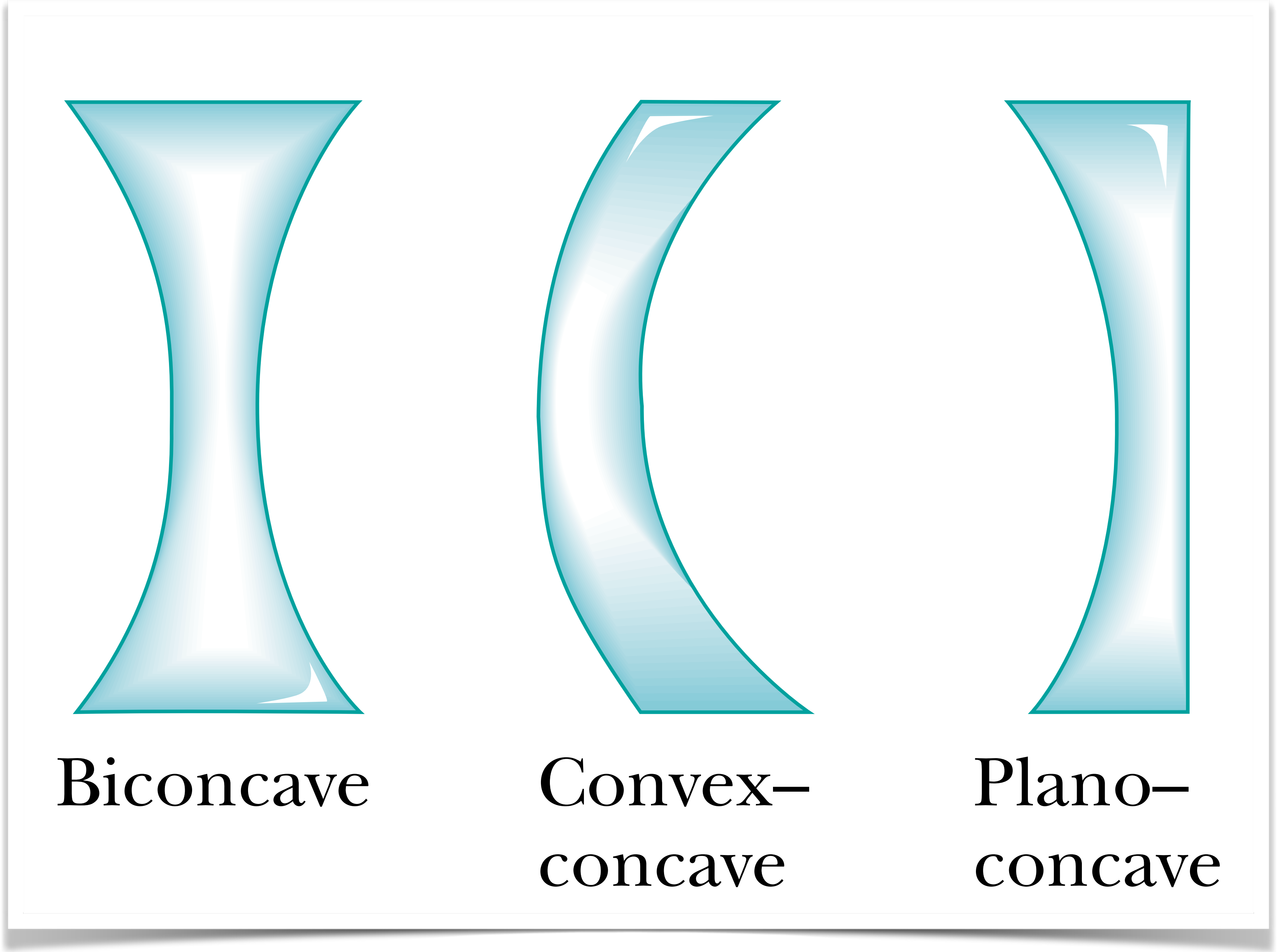


$$\theta_1 = \theta_{in}$$

$$\theta_4 = \theta_{out}$$

- What if you have glass with walls that are not parallel?
- This is idea behind lenses
- As light enters ☛ it is bent and rays come out different depending on where and how they strike
- Focal length of optical system measures of how strongly system converges or diverges light
- For optical system in air ☛ focal length is distance over which initially collimated (parallel) rays are brought to a focus
- Lens geometry usually looks complicated (and it is!) but for thin lenses ☛ result is relatively simple





Images Formed by Refraction

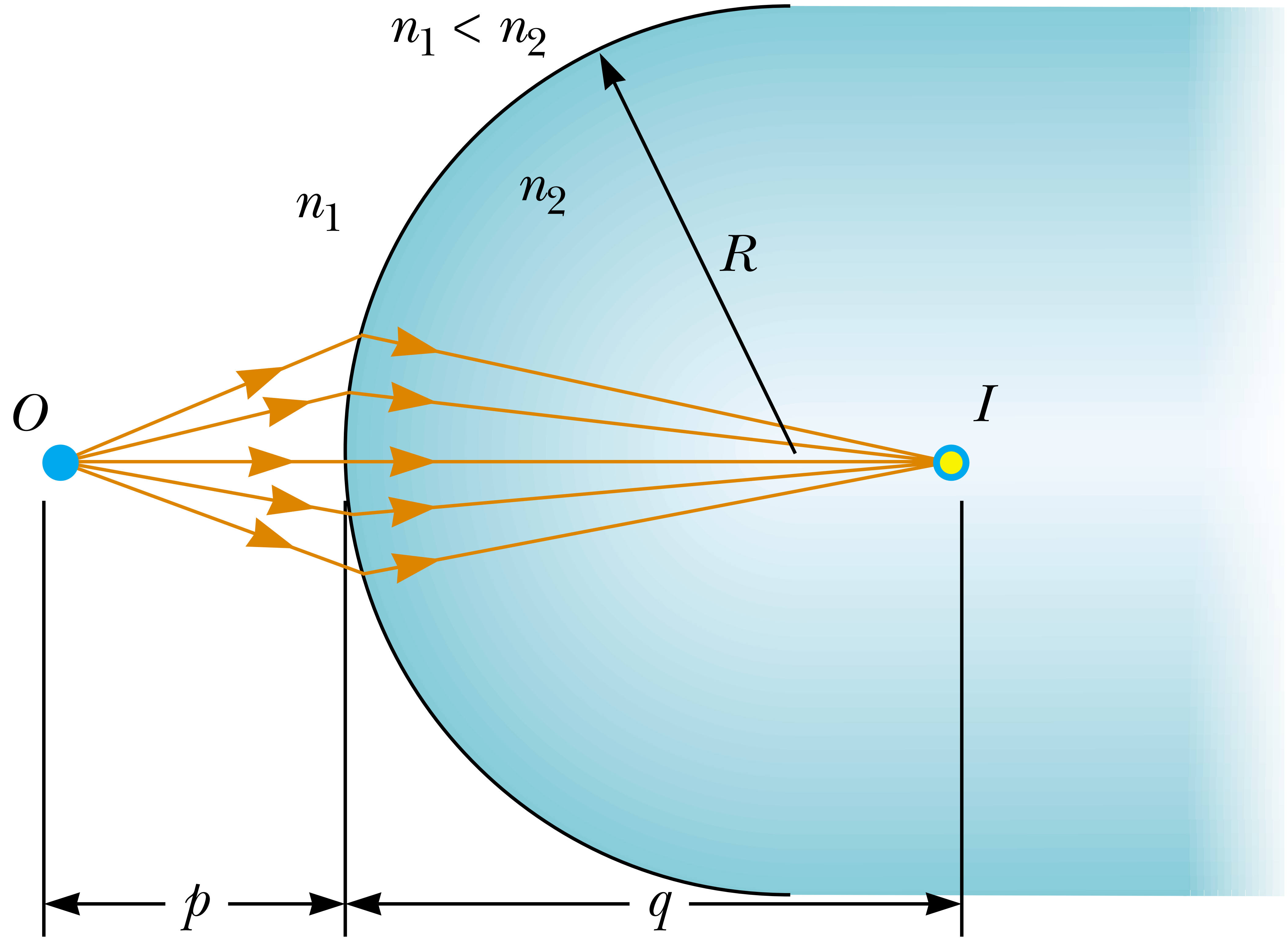
➢ Consider two transparent media having indices of refraction n_1 and n_2

boundary between two media is a spherical surface of radius R

➢ Object at O is in medium for which index of refraction is n_1

➢ Consider rays leaving O all such rays are refracted at spherical surface and focus at single point I

image point



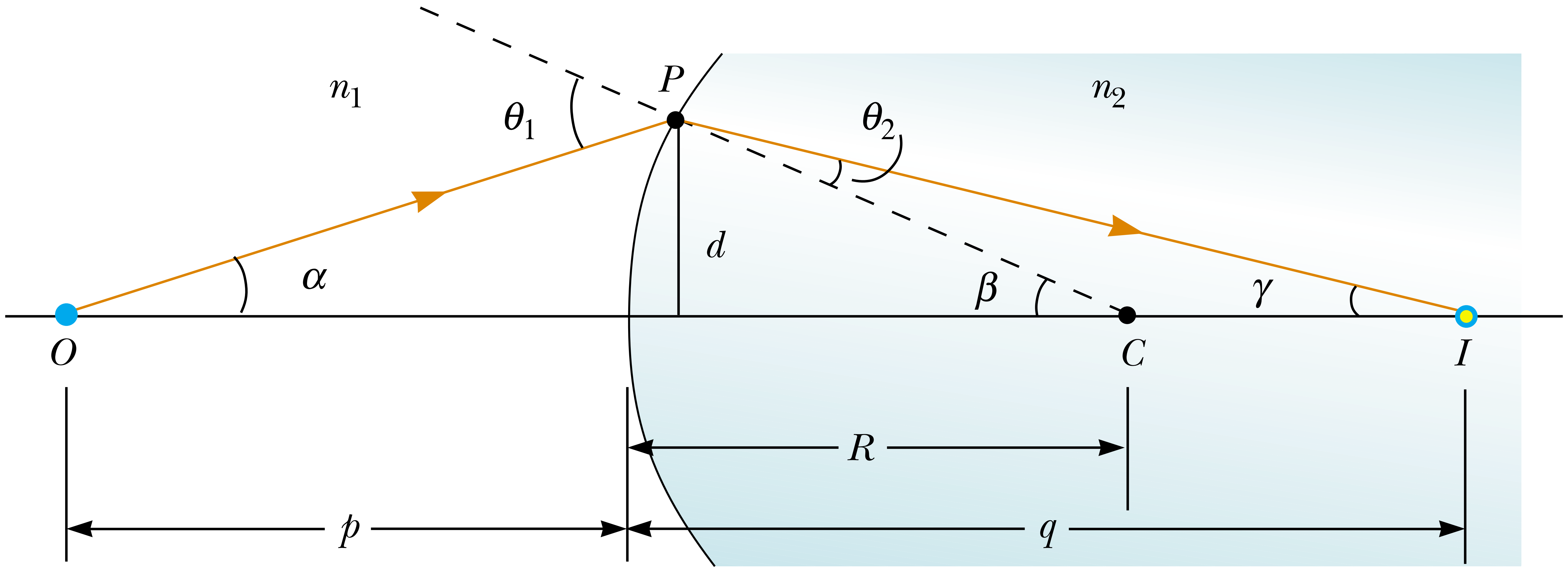
➢ Single ray leaving point O and refracting to point I

➢ Snell's law of refraction applied to this ray gives $n_1 \sin \theta_1 = n_2 \sin \theta_2$

➢ Because θ_1 and θ_2 are assumed to be small we can use small-angle approximation

$$n_1 \theta_1 = n_2 \theta_2$$

➢ An exterior angle of any triangle equals sum of two opposite interior angles



➢ Applying this rule to triangles OPC and PIC gives

$$\theta_1 = \alpha + \beta \qquad \beta = \theta_2 + \gamma$$

- If we combine all three expressions and eliminate θ_1 and θ_2

$$n_1 \alpha + n_2 \gamma = (n_2 - n_1) \beta$$

- In small-angle approximation

$$\tan \theta \approx \theta$$

$$\tan \alpha \approx \alpha \approx \frac{d}{p} \qquad \tan \beta \approx \beta \approx \frac{d}{R} \qquad \tan \gamma \approx \gamma \approx \frac{d}{q}$$

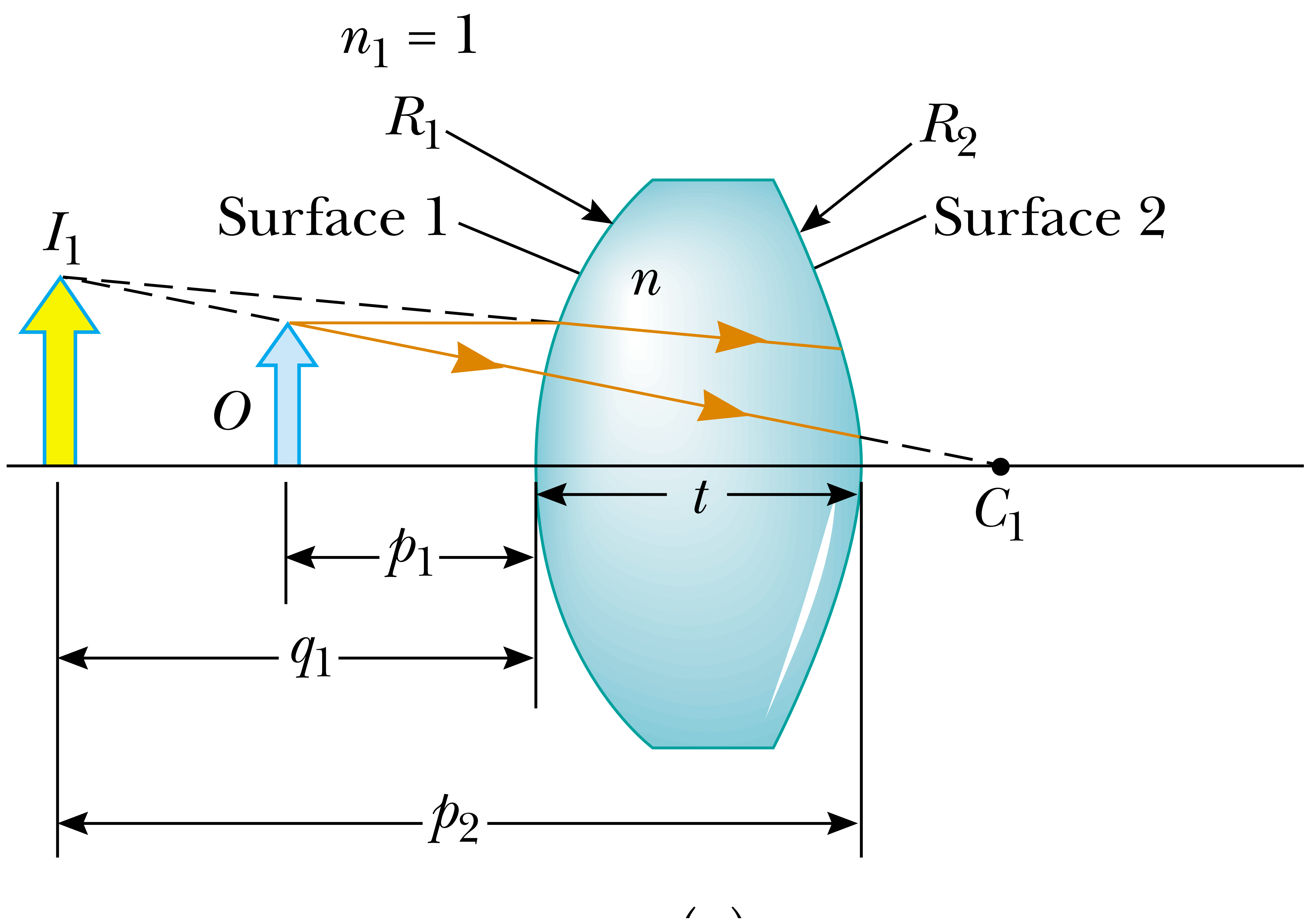
- Substitute these expressions and divide through by d to give valuable equation

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \qquad \text{Eq. (✣)}$$

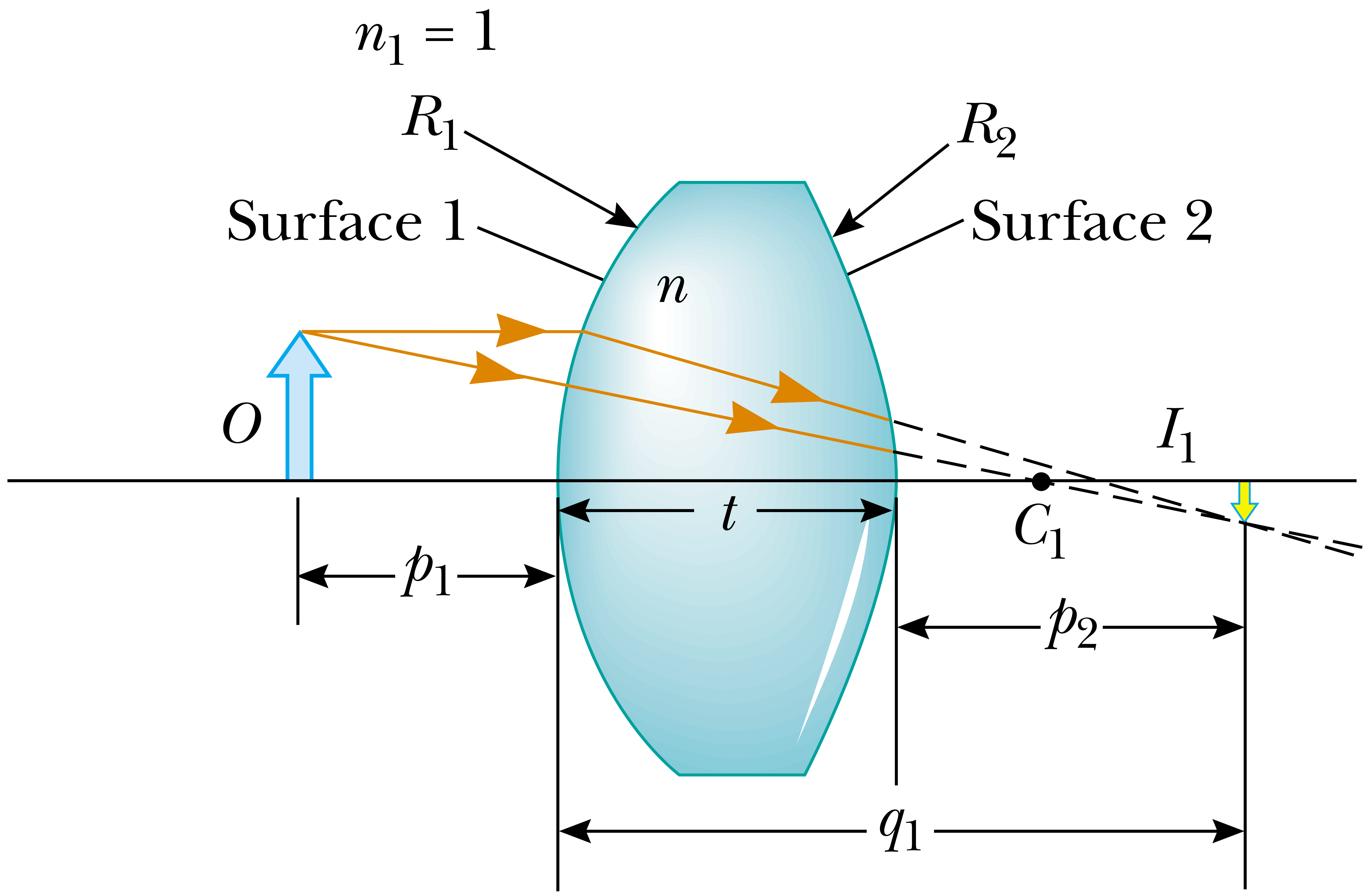
- For a fixed object distance p image distance q is independent of angle that ray makes with axis

Thin lenses

- ➢ Light passing through a lens experiences refraction at two surfaces
- ➢ **Image formed by one refracting surface serves as the object for second surface**
- ➢ Analyze thick lens first and then let thickness of lens be approximately zero



Virtual image



Real image

➢ Using Eq. (✣) and assuming $n_1 = 1$ because lens is surrounded by air we find that image I_1 formed by surface 1 satisfies

$$\frac{1}{p_1} + \frac{n}{q_1} = \frac{n - 1}{R_1}$$

➢ Apply Eq. (✣) to surface 2 taking $n_1 = n$ and $n_2 = 1$

➢ Taking p_2 as object distance for surface 2 and q_2 as image distance gives

$$\frac{n}{p_2} + \frac{1}{q_2} = \frac{1 - n}{R_2}$$

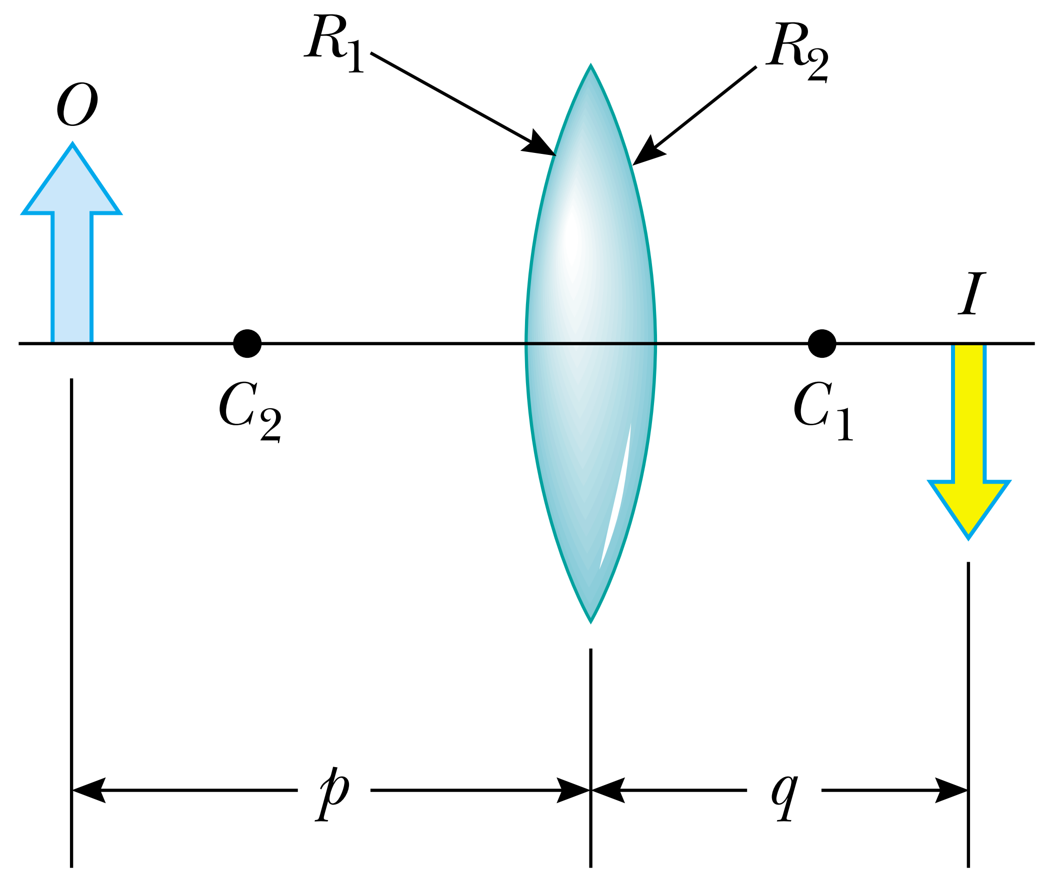
➢ Introduce mathematically fact that image formed surface 1 acts as object for 2

Virtual image ☛ $p_2 = -q_1 + t \quad (q_1 \text{ is negative})$

Real image ☛ $p_2 = -q_1 + t \quad (q_1 \text{ is positive})$

t ☛ **thickness of lens**

➢ For thin lens (one whose thickness is small compared to radii of curvature) we can neglect t



➢ In this approximation $p_2 = -q_1$ for either type of image from surface 1

➢ If image from surface 1 is real ☛ image acts as a virtual object so p_2 is negative

$$-\frac{n}{q_1} + \frac{1}{q_2} = \frac{1-n}{R_2}$$

➢ Substituting $-\frac{n}{q_1}$ from surface 1 equation and rearranging terms gives

$$\frac{1}{p_1} + \frac{1}{q_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

➢ For a thin lens ☛ we can omit subscripts on q and p and call object distance p and image distance q

$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

➢ **Focal length** f of thin lens is image distance that corresponds to infinite object distance

☛ inverse of focal length for thin lens gives

➢ Letting p approach ∞ and q approach f

Lens makers' equation ☛ $$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ ☛ just as with mirrors

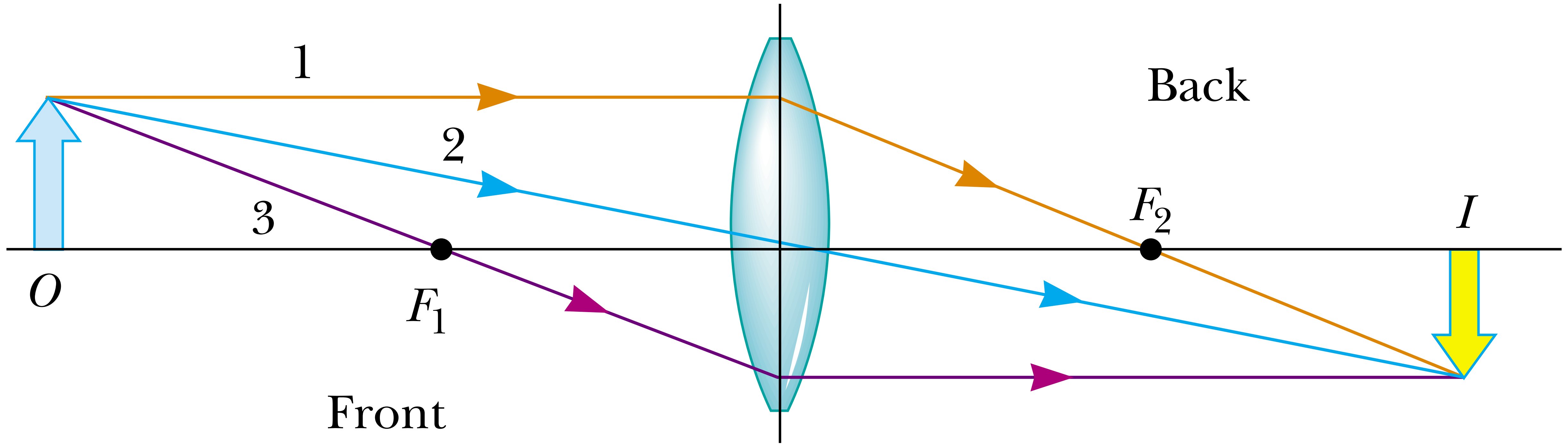
➢ If index of refraction and radii of curvature of lens are given

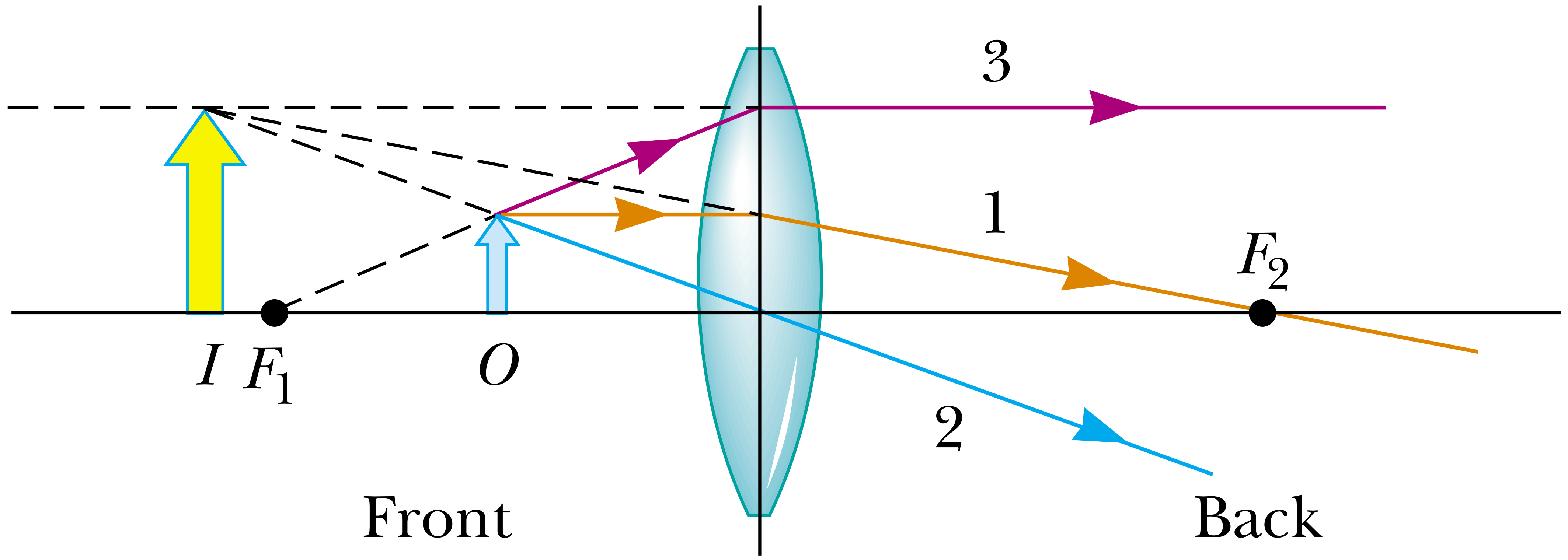
lens makers' equation enables calculation of focal length

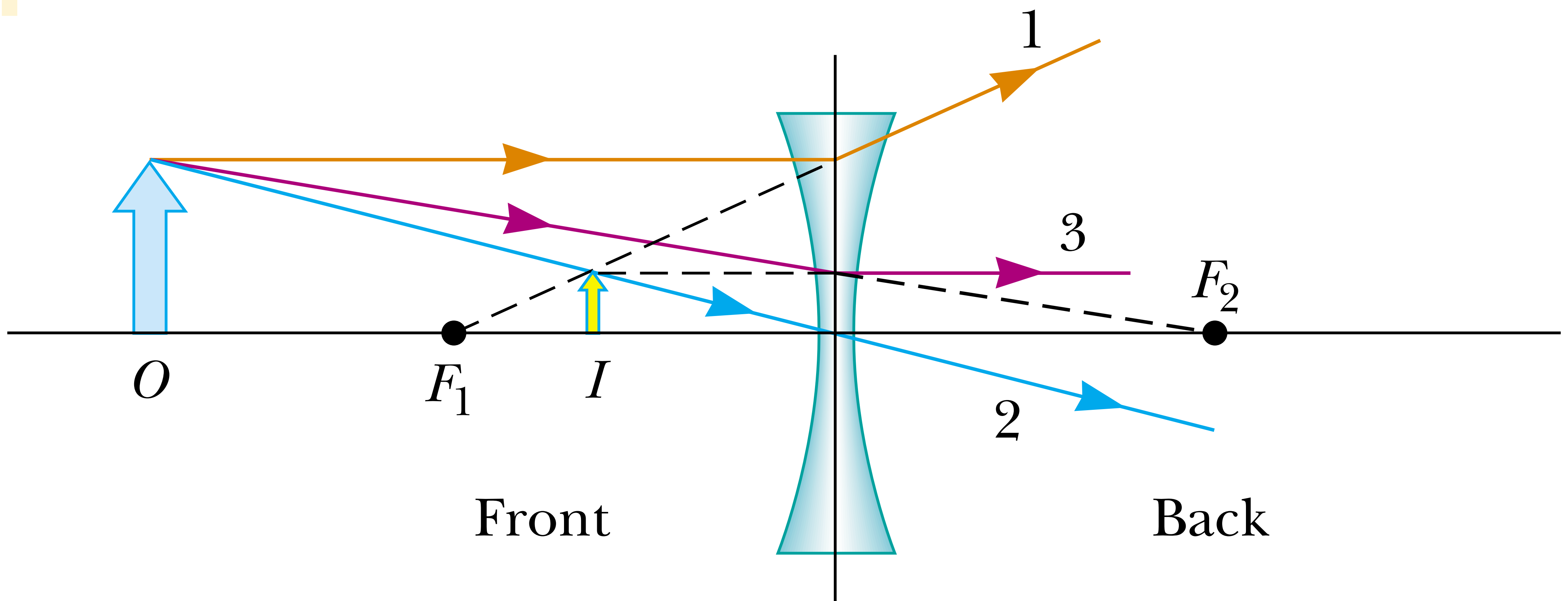
Thin lens equation ☛ $$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

➢ Magnification of Images $$M = \frac{h'}{h} = -\frac{q}{p}$$

Ray diagrams for Thin Lenses

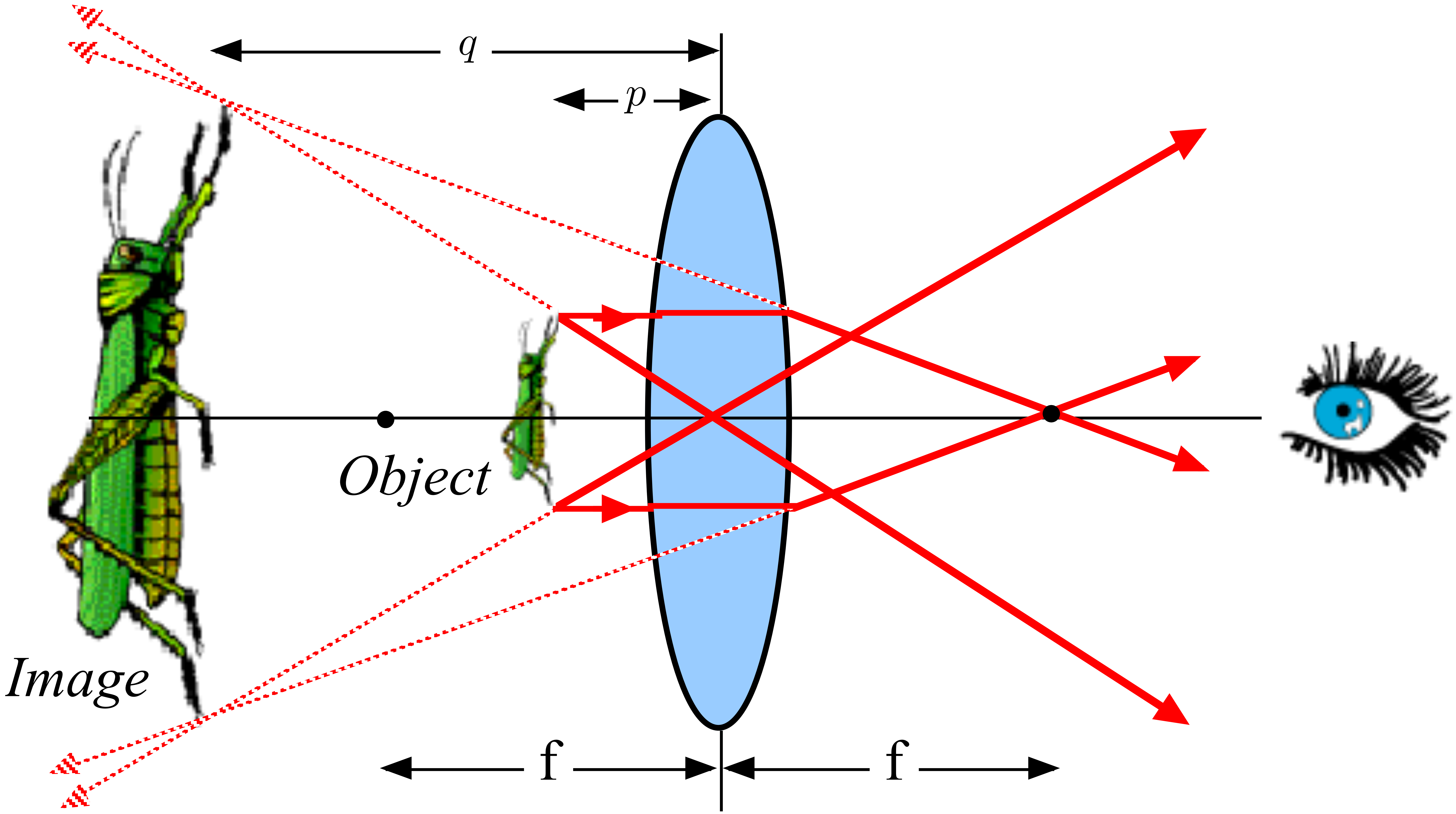




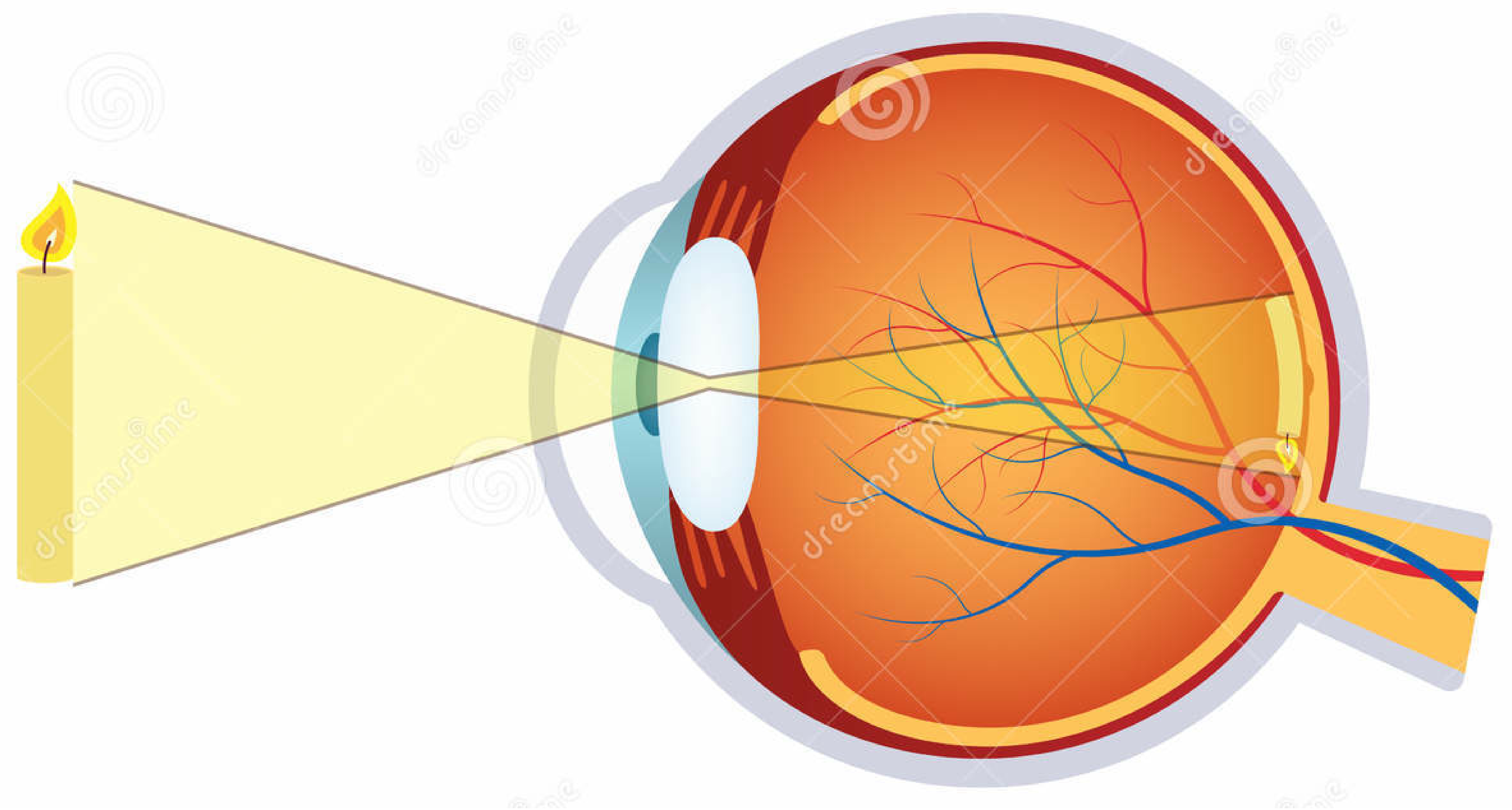


Magnifying glass

➢ Convex lens can be used as a magnifying glass…



- How do you know where objects are?
- How do you see them?
- You deduce direction and distance in complicated ways but arises from angle and intensive of bundle of light rays that make it into your eye
- Eye is adaptive optical system
- Crystalline lens of eye changes its shape to focus light from objects over a great range of distances

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