

Figure 1: Problem 7.

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Problems set \# 7

Physics 167
Solutions

1. For the circuit shown in Fig. 1, calculate (i) the current in the $2.00 \Omega$ resistor and (ii) the potential difference between points $a$ and $b$.
(i) We name the currents $i_{1}, i_{2}$, and $i_{3}$ as shown in the figure, and so $i_{1}=i_{2}+i_{3}$. Going countercolckwise around the loop we get, $12.0 \mathrm{~V}-2.00 \Omega i_{3}-4.00 \Omega i_{1}=0$. Traversing the bottom loop we have $8 \mathrm{~V}-6.00 \Omega i_{2}+2.00 \Omega i_{3}=0$. We can solve these last two equations for $i_{1}$ and $i_{2}$, yielding $i_{1}=3.00 \mathrm{~A}-\frac{1}{2} i_{3}$ and $i_{2}=\frac{4}{3} \mathrm{~A}+\frac{1}{3} i_{3}$. This means that $i_{3}=909 \mathrm{~mA}$. (ii) $V_{a}-0.909 \mathrm{~A} \cdot 2.00 \Omega=V_{b}$, then $V_{b}-V_{a}=-1.82 \mathrm{~V}$.
2. The heating element of an electric oven is designed to produce 3.3 kW of heat when connected to a $240-\mathrm{V}$ source. What must be the resistance of the element?

Solution: Since $P=V^{2} / R$ we have $R=V^{2} / P=17 \Omega$.
3. A heater coil connected to a $240-\mathrm{V}_{\mathrm{rms}} \mathrm{AC}$ line has a resistance of $38 \Omega$. (i) What is the average power used? (ii) What are the maximum and minimum values of the instantaneous power?

Solution: (i) The average power used can be found from the resistance and the rms voltage $\bar{P}=$ $V_{\text {rms }}^{2} / R=1516 \mathrm{~W}$. (ii) The maximum power is twice the average power, $P_{\text {max }}=2 \bar{P}=3 \times 10^{3} \mathrm{~W}$, and the minimum power is $P_{\min }=0$.
4. At a point high in the Earth's atmosphere, $\mathrm{He}^{2+}$ ions in a concentration of $2.4 \times 10^{12} \mathrm{~m}^{-3}$ are moving due north at a speed of $2.0 \times 10^{6} \mathrm{~m} / \mathrm{s}$. In addition, a $7.0 \times 10^{11} \mathrm{~m}^{-3}$ concentration of
$\mathrm{O}_{2}^{-}$ions is moving due south at a speed of $6.2 \times 10^{6} \mathrm{~m} / \mathrm{s}$. Determine the magnitude and direction of the net current passing through unit area $A / \mathrm{m}^{2}$.

Solution: We are given a net charge, a concentration, and a speed (like the drift speed) for both types of ions. We can determine the current per unit area, $I / A=n e v_{d}$. Both currents are in the same direction in terms of conventional current -positive charge moving north has the same effect as negative charge moving south- so they can be added: $I / A=\left(n e v_{d}\right)_{\mathrm{He}}+\left(n e v_{d}\right)_{\mathrm{O}}=2.2 \mathrm{~A} / \mathrm{m}^{2}$, north.
5. Consider the circuit shown in Fig. 2, with the start up switch $T_{1}$ open (for a long time). Now, close the switch and wait for a while. What is the change in the total charge of the capacitor if $V_{2}>V_{1}$ ?


Figure 2: Problem 5.
Solution: While the switch remains open the charge in the capacitor is $q_{i}=V_{2} C$. If we closed the switch and wait for a while we have $V_{1}-V_{2}+I R_{2}+I R_{1}=0$ and so $I=\frac{V_{2}-V_{1}}{R_{1}+R_{2}}$. The voltage across the capacitor is $V_{C}=V_{1}+I R_{1}=V_{1}+\left(\frac{V_{2}-V_{1}}{R_{1}+R_{2}}\right) R_{1}$. Hence, the change in the charge is $\Delta q=C\left(V_{1}-V_{2}+\frac{V_{2}-V_{1}}{R_{1}+R_{2}} R_{1}\right)$.

