Prof. Anchordoqui

Problems set # 6 **Physics 167** **Solutions**

1. Find the current I in the circuit shown Fig. 1.

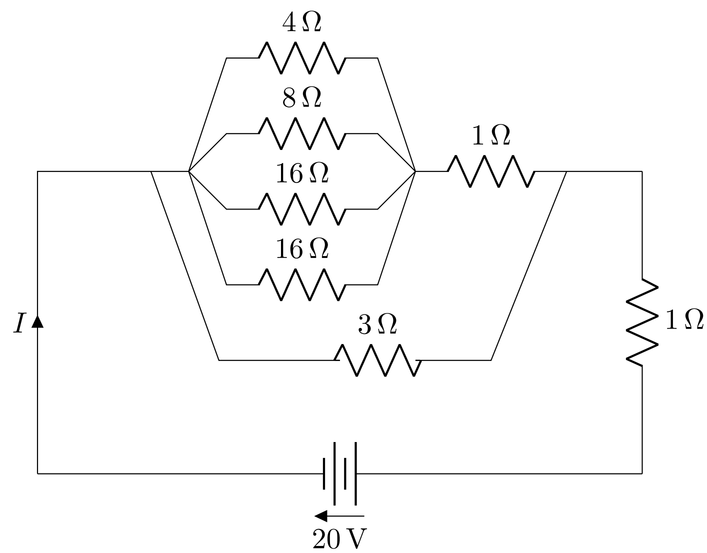
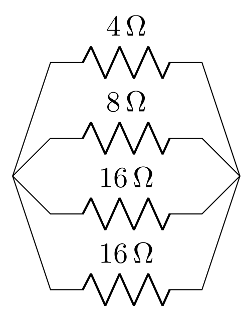


Figure 1: Problem 1.

Solution: The equivalent resistance, $R^1_{\rm eq}$ for



is $\frac{1}{R^1_{eq}} = \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16}\right)\ \Omega^{-1} = \frac{1}{2}\ \Omega^{-1}$; therefore, $R^1_{\rm eq} = 2\ \Omega$. This is in series with the $1\ \Omega$ resistor. Hence, $R^2_{\rm eq} = R^1_{\rm eq} + 1\ \Omega = 3\ \Omega$. Now, $R^2_{\rm eq}$ is in parallel with the $3\ \Omega$ resistance, $\frac{1}{R^3_{\rm eq}} = \left(\frac{1}{3} + \frac{1}{3}\right)\ \Omega^{-1}$; therefore, $R^3_{\rm eq} = 1.5\ \Omega$ The total equivalent resistance of the circuit is $R_{\rm eq} = (1.5 + 1)\ \Omega = 2.5\ \Omega$ From Ohm's law, $V = IR$, we get $I = \frac{V}{R_{\rm eq}} = \frac{20\ {\rm V}}{2.5\ \Omega} = 8$ A.

2. In the circuit shown in Fig. 2, the power produced by bulb$_1$ and bulb$_2$ is 1 kW and 50 W, respectively. Which light has the higher resistance? (Assume the resitance of the light bulb remains constant with time.)

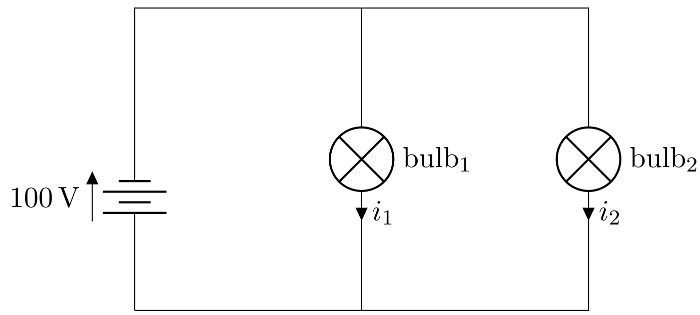


Figure 2: Problem 2.

Solution: The power dissipated by light bulb one and two, respectively, is $P_1 = I_1^2 R_1 = I_1 V$ and $P_2 = I_2^2 R_2 = I_2 V$. Thus, we have $I_1 = P_1/V$ and $I_2 = P_2/V$. This implies that $P_1 = \left(\frac{P_1}{V}\right)^2 R_1 \Rightarrow R_1 = \frac{V^2}{P_1} = 10\ \Omega$. Likewise $R_2 = \frac{V^2}{P_2} = 200\ \Omega$.

3. A regular tetrahedron is a pyramid with a triangular base. Six $R = 10.0\ \Omega$ resistors are placed along its six edges, with junctions at its four vertices, as shown in Fig. 3. A 12.0-V battery is connected to any two of the vertices. Find *(i)* the equivalent resistance of the tetrahedron between these vertices and *(ii)* the current in the battery.

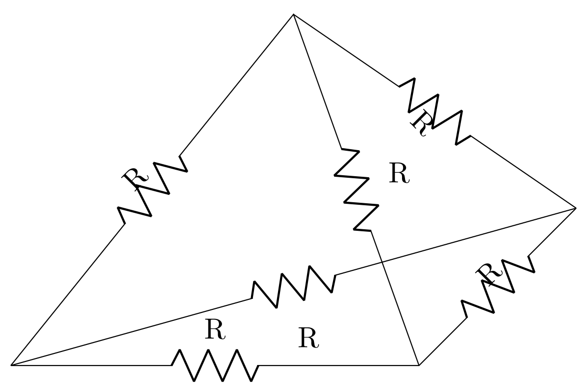


Figure 3: Problem 3.

Solution: *(i)* First let us flatten the circuit on a 2-D plane as shown in Fig. 4; then reorganize it to a format easier to read. Note that the voltage $V_{AB} = 0$ in Fig. 5 and so the middle resistor can be removed without affecting the circuit. The remaining resistors over the three parallel branches have equivalent resistance $\frac{1}{R_{\text{tot}}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{2R} = \frac{2}{R} \Rightarrow R_{\text{eq}} = 5\ \Omega$. *(ii)* The current through the battery isn $\frac{\Delta V}{R_{\text{eq}}} = \frac{12.0\ \text{V}}{5\ \Omega} = 2.40$ A.

4. Determine the magnitude and directions of the currents through $R_1 = 22\ \Omega$ and $R_2 = 15\ \Omega$ in the circuit of Fig. 6. The batteries have an internal resistance of $r = 1.2\ \Omega$.

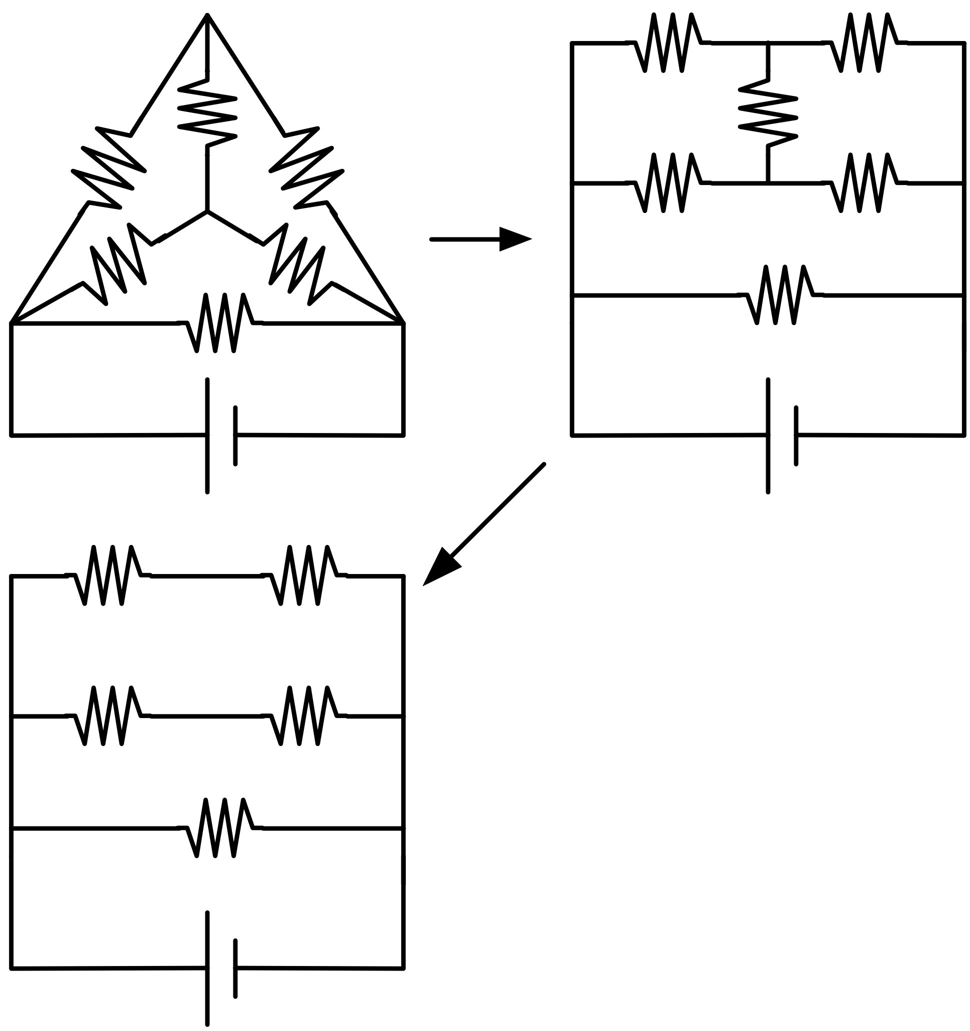


Figure 4: Solution of problem 3.

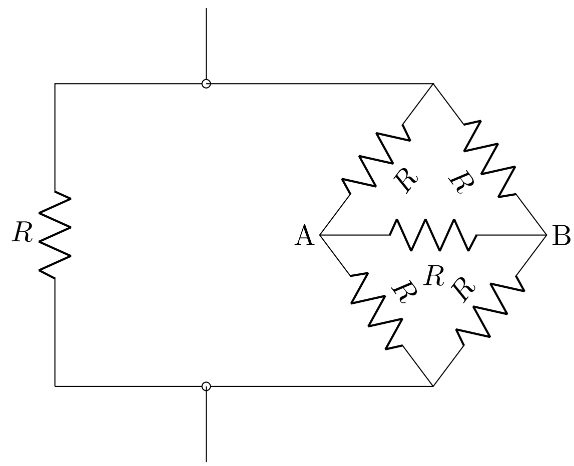


Figure 5: More on the solution of problem 3.

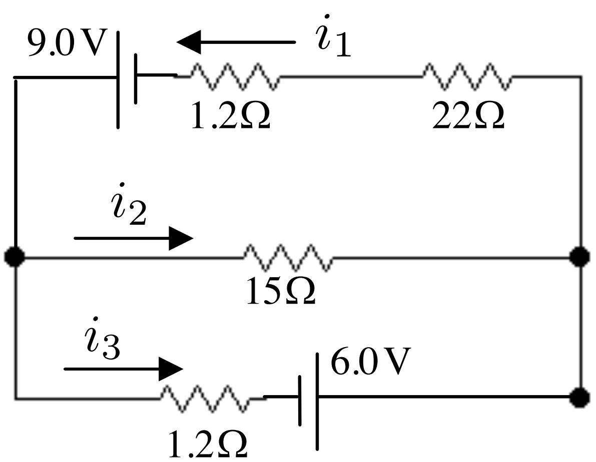


Figure 6: Problem 4.

Solution: There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the left of the circuit: $i_1 = i_2 + i_3$. Another equation comes from Kirchhoff's loop rule applied to the outer loop, starting at the lower left corner, and progressing counterclockwise

$$-i_3(1.2\ \Omega) + 6\ \text{V} - i_1(22\ \Omega) - i_1(1.2\ \Omega) + 9\ \text{V} = 0 \Rightarrow 15 = 23.2 i_1 + 1.2 i_3\ .$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, strating at the lower left corner, and progressing counterclockwise:

$$-i_3(1.2\ \Omega) + 6\ \text{V} + i_2(15\ \Omega) = 0 \Rightarrow 6 = -15 i_2 + 1.2 i_3\,.$$

Substitute $i_1 = i_2 + i_3$ into the loop equation, so that there are two equations with two unknowns

$$15 = 23.2 i_1 + 1.2 i_3 = 23.2(i_2 + i_3) + 1.2 i_3 = 23.2 i_2 + 24.4 i_3$$

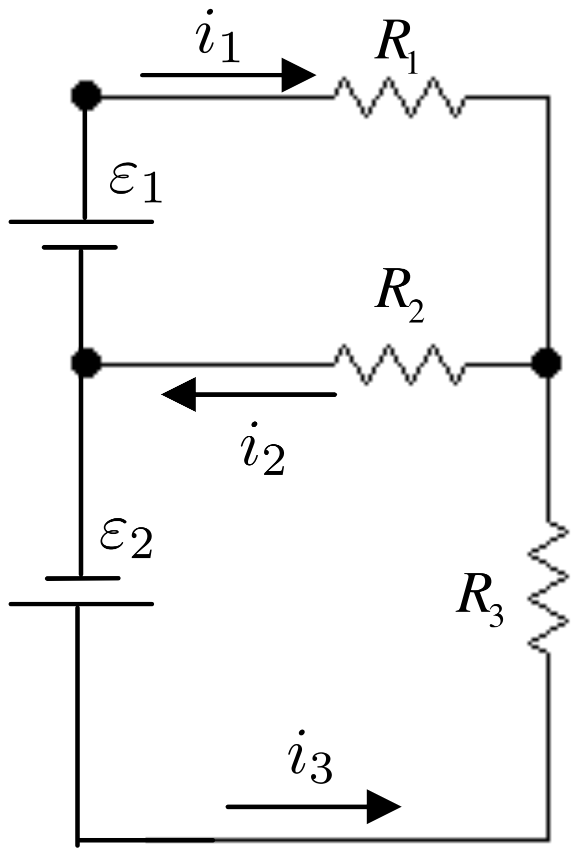


Figure 7: Problem 5.

and

$$6 = -15i_2 + 1.2i_3$$

. Solve the bottom loop equation for i_2 and substitute into the loop equation, resulting in an equation with only one unknown, which can be solved

$$6 = -15i_2 + 1.2i_3 \Rightarrow i_2 = \frac{-6 + 1.2i_3}{15}$$

$$15 = 23.2i_2 + 24.4i_3 = 23.2\left(\frac{-6 + 1.2i_3}{15}\right) + 24.4i_3 \Rightarrow i_3 = 363/393.84 = 0.917 \text{ A};$$

$$i_2 = \frac{-6 + 1.2i_3}{15} = -0.33 \text{ A, left.}$$

$$i_1 = i_2 + i_3 = 0.6 \text{ A, left.}$$

5. Determine the magnitude and directions of the currents in each resistor shown in Fig. 7. The batteries has emfs of $\varepsilon_1 = 9$ V and $\varepsilon_2 = 12$ V and the resistors have values of $R_1 = 25\ \Omega$, $R_2 = 18\ \Omega$, and $R_3 = 35\ \Omega$.

Solution: There are three currents involved, and so there must be three independent equations to determine those currents. One comes from Kirchhoff's junction rule applied to the junction on the three branches on the right of the circuit

$$i_2 = i_1 + i_3 \Rightarrow i_1 = i_2 - i_3.$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the battery and progressing clockwise

$$\varepsilon_1 - i_1 R_1 - i_2 R_2 = 0 \Rightarrow 9 = 25 i_1 + 18 i_2.$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise

$$\varepsilon_2 - i_3 R_3 - i_2 R_2 = 0 \Rightarrow 12 = 35 i_3 + 18 i_2\,.$$

Substitute $i_1 = i_2 - i_3$ into the loop equation, so that there are two equations with two unknowns:

$$9 = 25 i_1 + 18 i_2 = 25(i_2 - i_3) + 18 i_2 = 43 i_2 - 25 i_3$$

and

$$12 = 35 i_3 + 18 i_2\,.$$

Solve the bottom loop equation for i_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved

$$12 = 35 i_3 + 18 i_2 \Rightarrow i_2 = \frac{12 - 35 i_3}{18}$$

$$9 = 43 i_2 - 25 i_3 = 43\left(\frac{12 - 35 i_3}{18}\right) - 25 i_3 \Rightarrow 162 = 516 - 1505 i_3 - 450 i_3 \Rightarrow i_3 = \frac{354}{1955} = 0.18 \text{ A, up};$$

$$i_2 = \frac{12 - 35 i_3}{18} = 0.31 \text{ A, left}$$

and

$$i_1 = i_2 - i_3 = 0.13 \text{ A, right.}$$