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Problems set # 13 **Physics 167** **Solutions**

1. Counting photons. *(i)* How many photons are emitted by a 100-watt sodium lamp (550 nm) in one second? (1 watt = 1 J/s) Assume the lamp is 100% efficient in converting electrical energy into light. *(ii)* The absolute threshold of the dark-adapted human eye for the perception of light at 510 nm has been measured as 3.5×10^{-17} J. How many photons does this correspond to?

Solution *(i)* The energy of each photon is $E_\gamma = \frac{hc}{\lambda} = \frac{(3\times 10^8~\mathrm{m/s})(6.62618\times 10^{-34}~\mathrm{J\cdot s})}{550\times 10^{-9}~\mathrm{m}} = 3.61 \times 10^{-19}$ J/photon. Using the 100-watt lamp, this is $P/E_\gamma = 100~\mathrm{W}/(3.61 \times 10^{-19}~\mathrm{J/photon}) = 2.77 \times 10^{20}$ photon/s. *(ii)* The energy of each photon now is $E_\gamma = hc/\lambda = 3.90 \times 10^{-19}$ J/photon. Hence, $E/E_\gamma = 3 \times 10^{-17}~\mathrm{J}/(3.90 \times 10^{-19}~\mathrm{J/photon}) = 90$ photons.

2. The temperature of a nuclear blast is 10^7 K. Conclude from this why it is not advised to be in the line of sight of a nuclear blast.

Solution Using Wien's displacement law $\lambda_{\rm max} = \frac{2.90\times 10^{-3}\mathrm{m\cdot K}}{10^7~\mathrm{K}} = 2.910^{-10}$ m = 0.290 nm. This is in the middle of the X-rays. The light emitted by a blackbody at this temperature would have a lot of radiant energy density in the X-rays, which can strip valence electrons. This would not be good for your atoms.

3. Light of wavelength 50 nm strikes a clean metal surface in vacuum, emmiting electrons of maximum kinetic energy 12.4 eV. What is the maximum wavelength of light that can eject electrons from this metal, in nm? [Hint: Use $hc = 1240~\mathrm{eV \cdot nm}$ to do your calculation].

Solution If $\lambda = 50$ nm produces electrons with kinetic energy $E_k = \frac{hc}{\lambda} - \varphi_w = 12.4$ eV, then using longer and longer wavelengthswe will produce less and less energetic electrons until they have barely zero kinetic energy $0 = \frac{hc}{\lambda_{\rm max}} - \varphi_w \Rightarrow \lambda_{\rm max} = hc/\varphi_w$, where φ_w is the work function. Now, using the kinetic energy relation we write $\varphi = \frac{hc}{\lambda} - E_k$ and substituting in the expression for $\lambda_{\rm max}$ we have $\lambda_{\rm max} = \frac{hc}{hc/\lambda - E_k} = 100$ nm.

4. A non-relativistic particle of mass m has a position uncertainty equal to its de Broglie wavelength. What is the minimum fractional uncertainty in its velocity, $\Delta v/v$?

Solution Heisenber uncertainty principle states that $\Delta x~\Delta p \geq \hbar/2$, where $\hbar = h/2\pi$. The de Broglie hypothesis states that $p = h/\lambda \Rightarrow \lambda = h/p$. The non-relativistic momentum is $p = mv \Rightarrow \Delta p = m\Delta v$. The position uncertainty is given in the problem as $\Delta x = \lambda$. Therefore, we have $\Delta x~\Delta p \geq \hbar/2 \Rightarrow \lambda~(m~\Delta v) \geq \hbar/2 \Rightarrow \frac{h}{mv}(m~\Delta v) \geq \hbar/2 \Rightarrow \frac{\Delta v}{v} \geq \frac{1}{4\pi}$

5. *(i)* Stars behave approximately like blackbodies. Use Wien's displacement formula to obtain a rough estimate of the surface temperature of the Sun, assuming that it is an ideal blackbody as

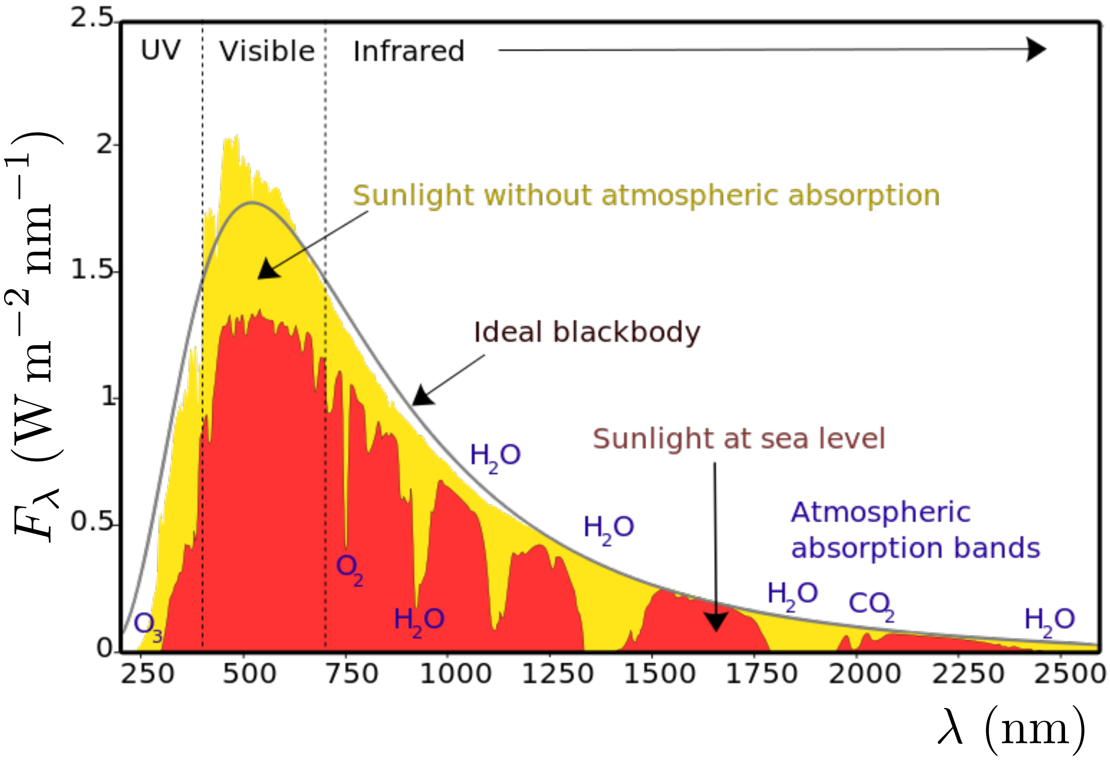

Figure 1: Solar energy incident at Earth's atmosphere and surface. The yellow band is the radiation incident at the top of the atmosphere, while the red band is the radiation at Earth's surface, diminished by the atmospheric absorbers shown. The radiation approximates a blackbody curve. These data are from the American Society for Testing and Materials (ASTM) Terrestrial Reference Spectra.

suggested by the ASTM data shown in Fig. 1 and that evolution on Earth worked well (i.e., that the human eye uses optimal the light from the Sun). *(ii)* The solar constant (radiant flux at the surface of the Earth) is about 1.365 kW/m^2. Find the effective surface temperature of the Sun *(iii)* Assuming that the surface of Neptune and the thermodynamics of its atmosphere are similar to those of the Earth estimate the surface temperature of Neptune. Neglect any possible internal source of heat. [*Hint:* Astronomical data which may be helpful: radius of Sun $R_\odot = 7 \times 10^5$ km; radius of Neptune $R_{\rm N} = 2.2 \times 10^4$ km; mean Sun-Earth distance $r_{\rm SE} = 1$ AU $= 1.5 \times 10^8$ km; mean Sun-Neptune distance $r_{\rm SN} = 4.5 \times 10^9$ km.]

Solution We can obtain a first estimate of the surface temperature of the Sun from the sensitivity of the human eye to light in the range $400 - 700$ nm. Assuming that the evolution worked well, i.e. that the human eye uses optimal the light from the Sun, and that the atmosphere is for all frequencies in the visible range similarly transparent, we identify the maximum in Wien's law with the center of the frequency range visible for the human eye. Thus we set $\lambda_{\max,\odot} \approx 550$ nm, and obtain $T_\odot \approx 5270$ K for the surface temperature of the Sun. *(ii)* The bolometric luminosity L of a star is given by the product of its surface $A = 4\pi R^2$ and the radiation emitted per area σT^4, i.e., $L = 4\pi R^2 \sigma T^4$. The radiant flux is defined by $F = L/A$, so that we recover the well known inverse-square law for the energy flux at the distance $r > R$ outside of the star, $F = L/(4\pi r^2)$. The validity of the inverse-square law $F(r) \propto r^{-2}$ relies on the assumptions that no radiation is absorbed and that relativistic effects can be neglected. The later condition requires, in particular, that the relative velocity of observer and source is small compared to the velocity of light. The energy flux received from the Sun at the distance of the Earth, $r_{\rm SE} = 1$ AU, is equal to $F = 1365$ W/m^2. The solar luminosity follows then as $L_\odot = 4\pi d^2 F = 4 \times 10^{33}$ erg s^{-1}, and serves as a convenient unit in stellar astrophysics. The Stefan-Boltzmann law can then be used to define, with $R_\odot \approx 7 \times 10^{10}$ cm, the effective temperature of the Sun, $T_\odot \approx 5780$ K. *(iii)* The average temperature on the surface of Neptune is $T_{\rm N} = L_\odot/(4\pi r_{\rm N}^2 \sigma) = 73$ K.