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Problems set # 12 **Physics 167** May 9, 2023

1. The escape velocity from Earth is 4×10^4 km/h. What would be the percent decrease in length of a 95.2 m long spacecraft traveling at that speed?

2. At what speed do the relativistic formulas for length and time intervals differ from the classical values by 1%?

3. *(i)* What will be the mean lifetime of a muon as measured in the laboratory if it is traveling at $v = 0.6c$ with respect to the laboratory? A muon's mean lifetime at rest is 2.20 μs. *(ii)* How far does a muon travel in the laboratory, on average, before decaying?

4. Space explorer Harry sets off at a steady $0.95c$ to a distant star. After exploring the star for a short time, he returns at the same speed and gets home after a total absence of 80 yr (as measured by earth-bound observers). How long do Harry's clocks say he was gone, and by how much has he aged as compared to his twin Sally who stayed behind on Earth. [Note: This is the famous "twin paradox." It is fairly easy to get the right answer by judicious insertion of a factor of γ in the right place, but to understand it, you need to recognize that it involves three uniformly moving reference frames: the earth-bound frame S, the frame S' of the outbound rocket, and the frame S'' of the returning rocket. Write down the time dilation formula for the two halves of the journey and then add. Noticed that the experiment is not symmetrical between the two twins: Sally stays at rest in the single uniformly moving frame S, but Harry occupies at least two different frames. This is what allows the result to be unsymmetrical.]

5. A very fast train with a "proper length" of $\ell_{0_{\rm train}} = 500$ m (measured by people at rest on the train) is passing through a tunnel that is 200 m long according to observers on the ground. Let us imagine the train's speed to be so great that the train fits completely within the tunnel as seen by observers on the ground. That is, the engine is just about to emerge from one end of the tunnel at the time the last car disappears into the other end. *(i)* What is the train's speed? *(ii)* Show that the length of the tunnel as measured by observers on the fantasy train of is $\ell_{\rm tunnel} \approx 80$ m. *(iii)* Observers at rest on the Earth see a very fast 200-m-long train pass through a 200-m-long tunnel so that the train momentarily disappears from view inside the tunnel. Observers on the train measure the train's length to be 500 m and the tunnel's length to be only 80 m. Clearly a 500-m-long train cannot fit inside an 80-m-long tunnel. How is this apparent inconsistency explained?