Prof. Anchordoqui

Problems set # 12	Physics 167	Solutions

1. The escape velocity from Earth is 4×10^4 km/h. What would be the percent decrease in length of a 95.2 m long spacecraft traveling at that speed?

Solution The fractional decrease in length of the spacecraft is $\delta d' = (d' - d)/d'$, where d' is the length measured by observers relative to whom the vessel is at rest and d is the contracted length of the vessel along the direction of its motion. For a spacecraft moving at speed of 1.1×10^4 m/s, we have $\delta d' = 1 - d/d' = 1 - \sqrt{1 - v^2/c^2} = 7 \times 10^{-10}$, where we have used the length contraction relation. This corresponds to $7 \times 10^{-8}\%$.

2. At what speed do the relativistic formulas for length and time intervals differ from the classical values by 1%?

Solution For a 1% change, $\sqrt{1 - v^2/c^2} = 0.99$, which gives $v = 0.14c$.

3. *(i)* What will be the mean lifetime of a muon as measured in the laboratory if it is traveling at $v = 0.6c$ with respect to the laboratory? A muon's mean lifetime at rest is 2.20 μs. *(ii)* How far does a muon travel in the laboratory, on average, before decaying?

If an observer were to move along with the muon (the muon would be at rest to this observer), the muon would have a mean life of 2.20×10^{-6} s. To an observer in the lab, the muon lives longer because of time dilation. *(i)* We find the mean lifetime using with the time dilation formula $\Delta t = \Delta t_0/\sqrt{1 - v^2/c^2} = 2.8 \times 10^{-6}$ s. *(ii)* Relativity predicts that a muon with speed 1.80×10^8 m/s would travel an average distance $d = v\Delta t = 500$ m, and this is the distance that is measured experimentally in the laboratory. Note that at a speed of 1.8×10^8 m/s, classical physics would tell us that with a mean life of 2.2 ms, an average muon would travel $d = vt = 400$ m. This is shorter than the distance measured. The measurement of the muon lifetime constitutes compelling evidence of relativistic time dilation.

4. Space explorer Harry sets off at a steady $0.95c$ to a distant star. After exploring the star for a short time, he returns at the same speed and gets home after a total absence of 80 yr (as measured by earth-bound observers). How long do Harry's clocks say he was gone, and by how much has he aged as compared to his twin Sally who stayed behind on Earth. [Note: This is the famous "twin paradox." It is fairly easy to get the right answer by judicious insertion of a factor of γ in the right place, but to understand it, you need to recognize that it involves three uniformly moving reference frames: the earth-bound frame S, the frame S' of the outbound rocket, and the frame S'' of the returning rocket. Write down the time dilation formula for the two halves of the journey and then add. Noticed that the experiment is not symmetrical between the two twins: Sally stays at rest in the single uniformly moving frame S, but Harry occupies at least two different frames. This is

what allows the result to be unsymmetrical.]

Solution For $v/c = 0.95$, the Lorentz factor for both the outward and return trip is $\gamma = (1 - v^2/c^2)^{-1/2} = 3.20$. The times for the two halves of the journey satisfy $\Delta t_{\rm S}^{\rm out} = \gamma \Delta t_{\rm H}^{\rm out}$ and $\Delta t_{\rm S}^{\rm back} = \gamma \Delta t_{\rm H}^{\rm back}$, so by addition, the times for the whole journey satisfy the same relation. Thus $\Delta t_{\rm H} = \Delta t_{\rm S}/\gamma = 25$ yr, which is the amount by which Harry has aged.

5. A very fast train with a "proper length" of $\ell_{0_{\rm train}} = 500$ m (measured by people at rest on the train) is passing through a tunnel that is 200 m long according to observers on the ground. Let us imagine the train's speed to be so great that the train fits completely within the tunnel as seen by observers on the ground. That is, the engine is just about to emerge from one end of the tunnel at the time the last car disappears into the other end. *(i)* What is the train's speed? *(ii)* Show that the length of the tunnel as measured by observers on the fantasy train of is $\ell_{\rm tunnel} \approx 80$ m. *(iii)* Observers at rest on the Earth see a very fast 200-m-long train pass through a 200-m-long tunnel so that the train momentarily disappears from view inside the tunnel. Observers on the train measure the train's length to be 500 m and the tunnel's length to be only 80 m. Clearly a 500-m-long train cannot fit inside an 80-m-long tunnel. How is this apparent inconsistency explained?

Solution *(i)* Since the train just fits inside the tunnel, its length measured by the person on the ground is $\ell_{\rm train} = 200$ m. The length contraction formula, $\ell_{\rm train} = \ell_{0\rm train}\sqrt{1 - v^2/c^2}$ can thus be used to solve for $v = 0.92c$. *(ii)* The proper length of the tunnel is $\ell_{0_{\rm tunnel}} = 200$ m. To obtain the length of the tunnel as seen by observers in the train we use again the length contraction formula $\ell_{\rm tunnel} = \ell_{0_{\rm tunnel}}\sqrt{1 - v^2/c^2} = 78 \text{ m} \approx 80$ m. *(iii)* Events simultaneous in one reference frame may not be simultaneous in another. Let the engine emerging from one end of the tunnel be event A, and the last car disappearing into the other end of the tunnel event B. To observers in the Earth frame, events A and B are simultaneous. To observers on the train, however, the events are not simultaneous. In the train's frame, event A occurs before event B. As the engine emerges from the tunnel, observers on the train observe the last car as still 500 m − 80 m = 420 m from the entrance to the tunnel.