

Conceptual Physics

Luis A. Anchordoqui

Department of Physics and Astronomy
Lehman College, City University of New York

Lesson XI
November 28, 2017

<https://arxiv.org/abs/1711.07445>

Table of Contents

- 1 Spacetime
 - Foundations of Special Relativity
 - Einstein postulates
 - Relativity of simultaneity
 - Time dilation
 - Length contraction
 - What is gravity

Historical Overview

- In 19th century  it was thought that just as water waves must have medium to move across (water) & audible sound waves require medium to move through (e.g. air) so also light waves require a medium which was called “luminiferous” (i.e. light-bearing) “æther”
- If this were the case  as Earth moves in its orbit around Sun flow of æther across Earth could produce detectable “æther wind”
- Unless æther were always stationary with respect to Earth speed of beam of light emitted from source on Earth would depend on magnitude of æther wind and on beam direction
- 1881 Michelson-Morley experiment
to measure speed of light in different directions
became most famous failed experiment to date
and first strong evidence against luminiferous æther

Historical Overview (cont'd)

(1892 -1909)

- To explain nature's apparent conspiracy to hide æther drift
Lorentz developed theory based on two *ad hoc* hypotheses:
 - Longitudinal contraction of rigid bodies
 - slowing down of clocks (time dilation)
 when moving through æther at speed v ↗ both by $(1 - v^2/c^2)^{1/2}$
- This would so affect every apparatus designed to measure æther drift as to neutralize all expected effects

(1898)

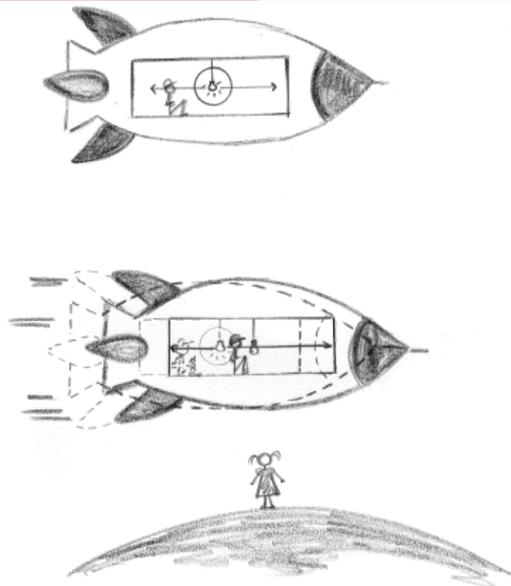
- Poincare argued that æther might be unobservable and suggested concept would be thrown aside as useless
BUT he continued to use concept in later papers of 1908

(1905)

- Einstein advanced principle of relativity

- 1 *All laws of nature are the same
in all uniformly moving reference frames*
- 2 *Speed of light in free space has the same value for all observers
regardless of the motion of source or motion of observer*
 - ☞ *speed of light (in free space) is a constant*



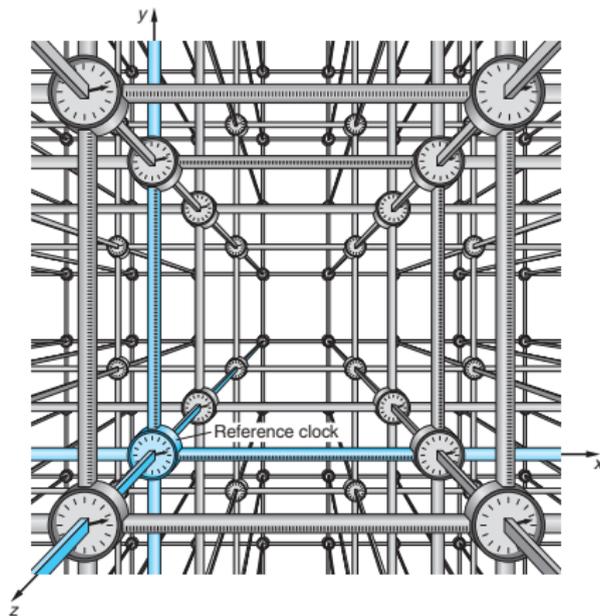


- From Harry's viewpoint light travels equal distances to both ends of rocket → striking both ends simultaneously
- Events of striking front and the end of spacecraft are not simultaneous in Sally's reference frame
- Because of rocket's motion light strikes back end sooner than front end

How does observer in uniformly moving frame describe event?

- Event  an occurrence characterized by: three space coordinates and one time coordinate
- Events are described by observers who do belong to particular uniformly moving frames of reference
- Different observers in different uniformly moving (u.m.) frames would describe same event with different spacetime coordinates
- Observer's rest frame is also known as proper frame
- Up until now it was enough for us to have a measuring stick for each reference frame a rigid body that defined units of a coordinate system
- But we could all depend on just one clock a master timepiece that was used by all observers
- Now what we need is a measuring stick with clocks all along it so that when something happens we can record both time and place

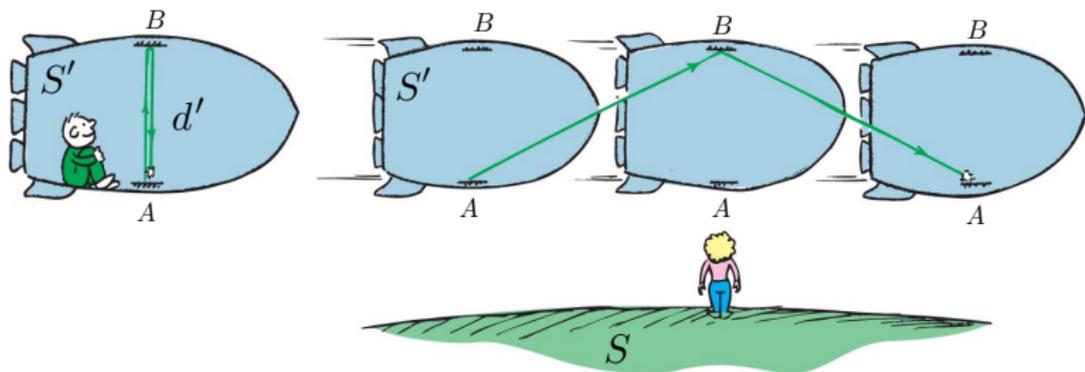
Confederate scheme for coordinatizing any event

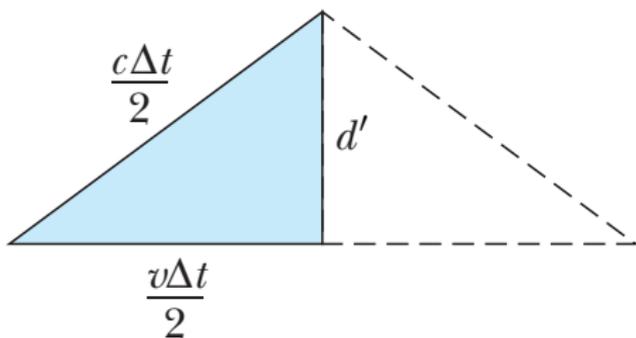


- Observer establishes lattice of confederates with identical synchronized clocks
- Label of any event in spacetime is reading of clock and location of nearest confederate to event

Einstein's thought experiment

- Idealized clock
 - light wave is bouncing back and forth between two mirrors
- Clock “ticks” when light wave makes a round trip
 - from mirror A to mirror B and back
- Assume mirrors A and B are separated a distance d' in rest frame
- Light wave will take $\Delta t' = 2d' / c$ for round trip $A \rightarrow B \rightarrow A$





Time dilation

Since light has velocity c in all directions

$$d'^2 + \left(v \frac{\Delta t}{2}\right)^2 = \left(\frac{c\Delta t}{2}\right)^2$$

$$\Delta t = \frac{2d'}{\sqrt{c^2 - v^2}} = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}$$

Ticking of clock in Hary's frame

which moves @ v wrt Sally in direction \perp to separation of mirrors

is slower by $\gamma = (1 - v^2/c^2)^{-1/2}$

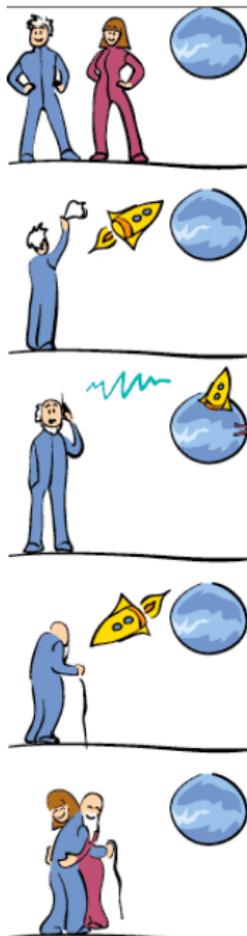
Twin Paradox

- Consider two synchronized standard clocks A and B at rest at point P of uniformly moving frame S
- Let A remain @ P while B is briefly accelerated to some velocity v with which it travels to distant point Q
- There it is decelerated and made to return with velocity v to P
- If one of two twins travels with B while other remains with A
 - ☞ B twin will be younger than A twin when meet again

Can't B claim with equal right it was her who remained where she was while A went on round-trip ☞ A should be younger when meet again?

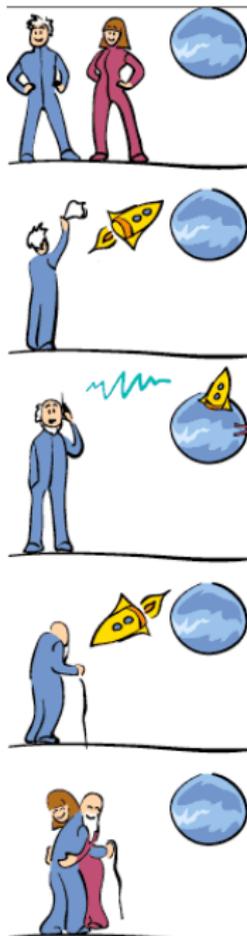
Answer is NO  this solves paradox

- *A* remained at rest in single u.m. frame while *B* accelerated out of his rest frame: @*P*, @*Q*, and once again @*P*
- Accelerations recorded on *B*'s accelerometer she can be under no illusion that it was her who remain at rest
- Two accelerations at *P* are not essential (age comparison could be made in passing) but acceleration in *Q* is vital

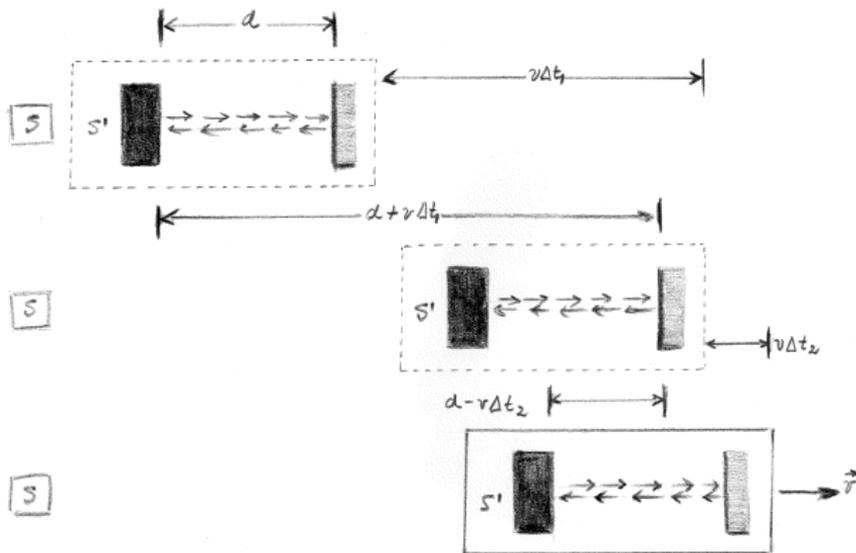


Answer is NO ☞ this solves paradox

- Experiment involves 3 u.m. frames:
 - 1 earth-bound frame S
 - 2 S' of outbound rocket
 - 3 S'' of returning rocket
- Experiment not symmetrical between twins:
 - A stays at rest in single uniformly moving frame S
 - but B occupies at least two different frames
- This allows result to be unsymmetrical



Rotate clock by 90° before setting it in motion



A-B

$$d + v \Delta t_1 = c \Delta t_1$$

$$\Delta t_1 = \frac{d}{c - v}$$

B-A

$$d - v \Delta t_2 = c \Delta t_2$$

$$\Delta t_2 = \frac{d}{c + v}$$

Length contraction

- 1 Interval between two consecutive ticks in the moving frame is

$$\begin{aligned}\Delta t &= \Delta t_1 + \Delta t_2 = \frac{2d}{c(1 - v^2/c^2)} \\ &= \left(\frac{d}{d'}\right) \frac{\Delta t'}{1 - v^2/c^2}\end{aligned}$$

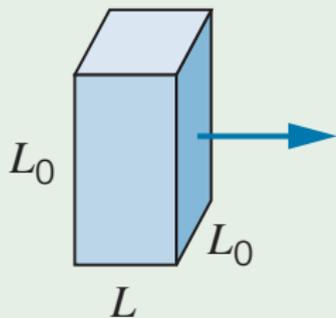
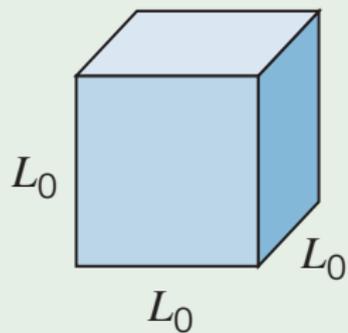
- 2 Because of time dilation

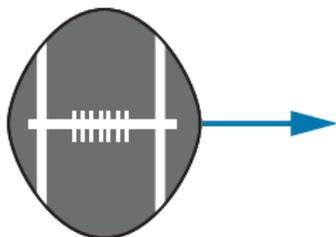
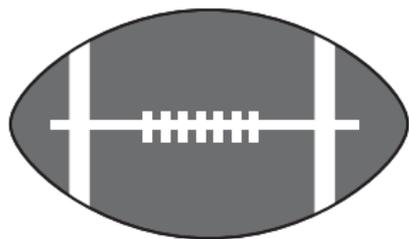
$$\Delta t' = \Delta t \sqrt{1 - v^2/c^2}$$

we get

$$d = \left(1 - \frac{v^2}{c^2}\right)^{1/2} d'$$

Example





- Newton's law is not compatible with special relativity
- It shouldn't be possible to send messages faster than speed of light
- Newton's relation would allow us to do so using gravity
- Newton stated that gravitational force depends on separation between objects at given instant of time
- To solve this problem ➡ introduce concept of gravitation field (akin to electric field)
- Einstein used this idea to develop general theory of relativity in which gravitational field is related to geometry of spacetime
- Major forecast of general relativity:
when two massive objects crash into each other
there should be a release of gravitational waves
which transport energy as gravitational radiation