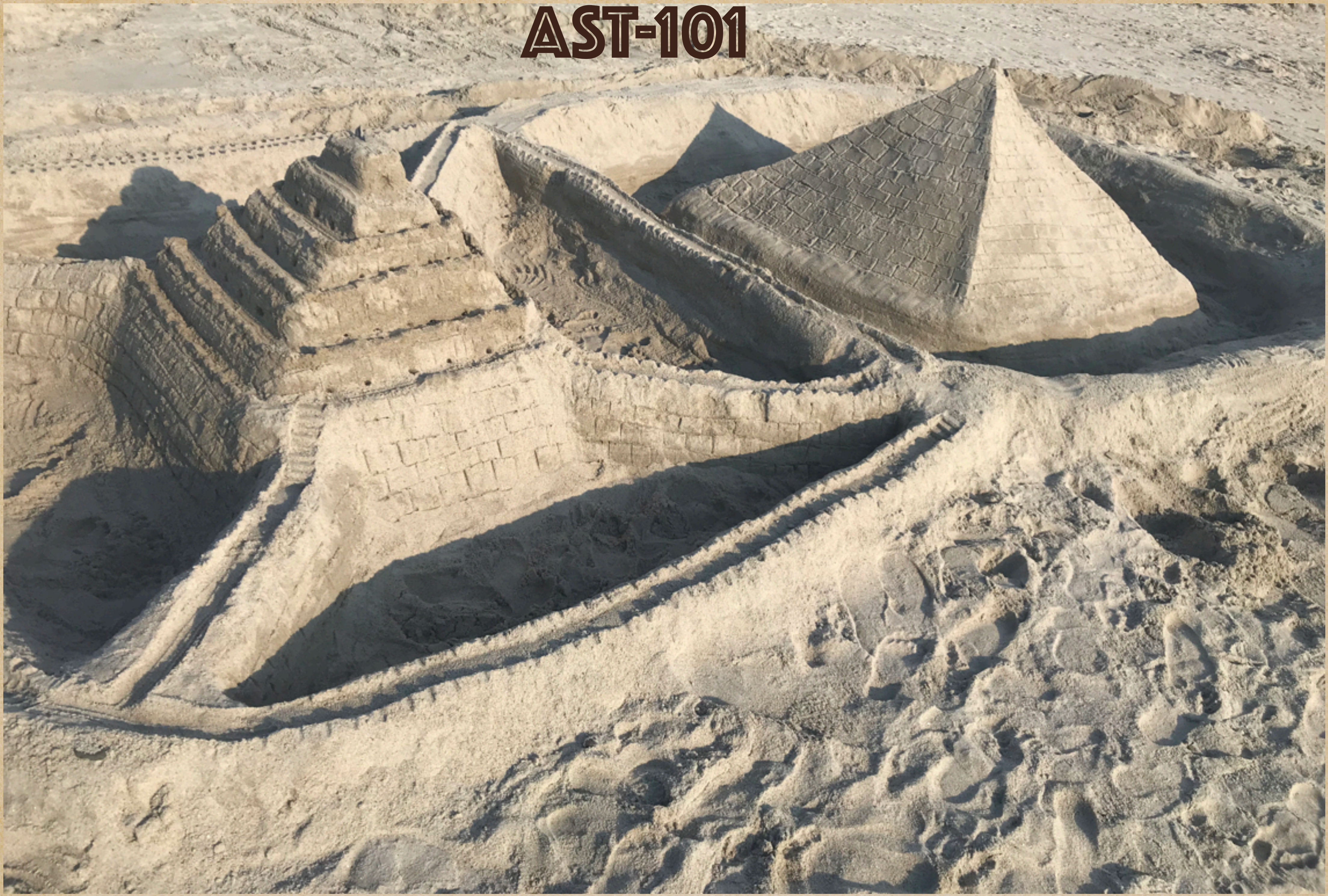


Newtonian Celestial Mechanics

Luis Anchordoqui

Isaac Newton

- Invented the Newtonian Telescope
- Independently developed Calculus
- Derived laws of motion
- Derived a universal law of gravity

Newton's Laws of Motion

- **Physical assumptions mathematically describing the motion of objects**
- **Was able to re-derive Kepler's laws based upon these physical laws**

gave concrete physical insight as to the motion of the planets

- ☑ Newtonian idea of force is based on experimental observation
- ☑ Everything in universe seems to have preferred configuration
- ☑ E.g. ☛ masses attract each other magnets can repel or attract one another
- ☑ Concept of force is introduced to quantify tendency of objects to move towards their preferred configuration
- ☑ If objects accelerate very quickly towards preferred configuration we say that there is a big force acting on them
- ☑ If they don't move (or move at constant velocity) we say there is no force
- ☑ We cannot see a force we can only deduce its existence by observing its effect

- ☑ Forces are defined through Newton's laws of motion:

0. Particle ☛ small mass at some position in space

1. When sum of forces acting on particle is 0 ☛ its velocity is constant

2. Sum of forces acting on particle of constant mass is equal to product of particle's mass and its acceleration ☛ $\sum \vec{F} = m \times \vec{a}$

3. Forces exerted by two particles on each other are equal in magnitude and opposite in direction

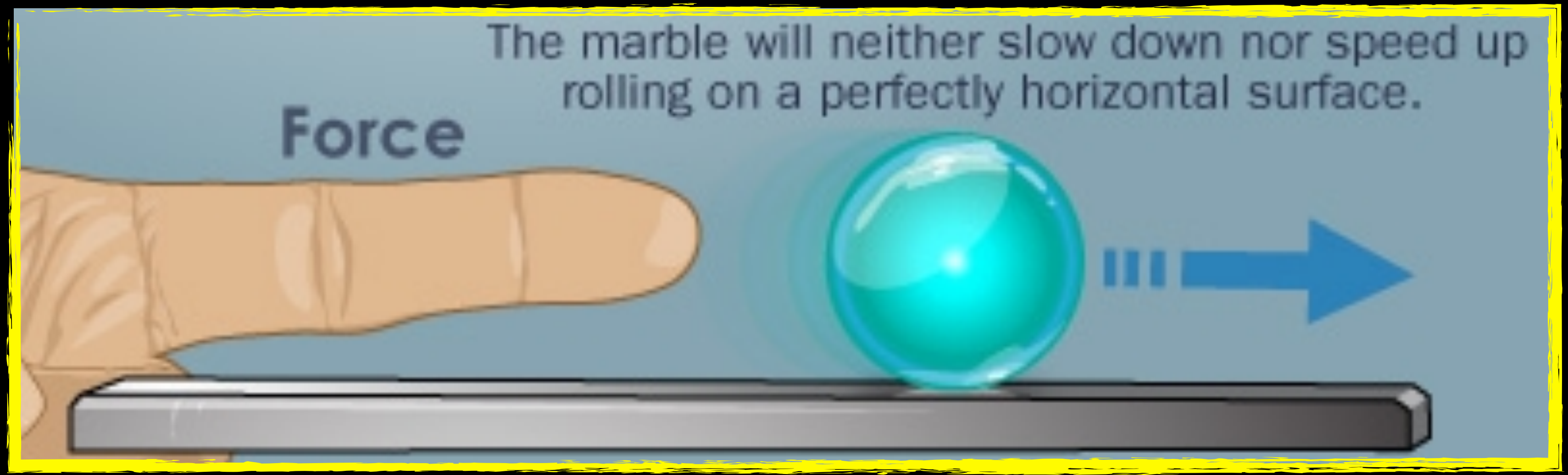
- ☑ Newton is standard unit of force ☛ given symbol N

- ☑ 1N ☛ force needed to accelerate 1kg(kg = 2.2 lb) of mass at rate of 1 m/s^2

- ☑ Forces are vectors ☛ have both magnitude and direction

Every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force...

Newton’s First Law



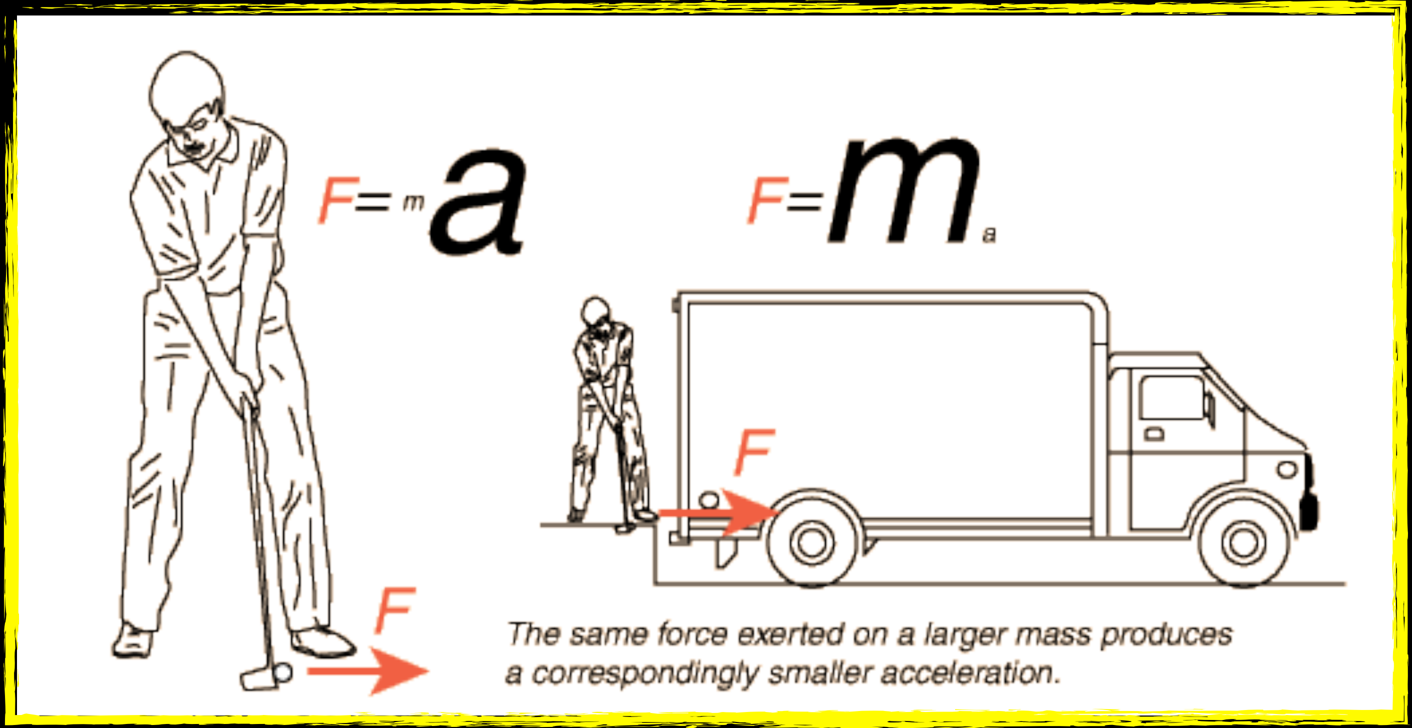
The law of inertia

- Inertia is the property of matter that keeps an object at rest or moving in a straight line at a constant speed unless acted upon by a net external force

A rolling chair will eventually stop due to friction

- Friction is a force that acts in the opposite direction of the motion of the chair therefore slowing it down
- If there was no friction, it would keep moving

Newton's Second Law

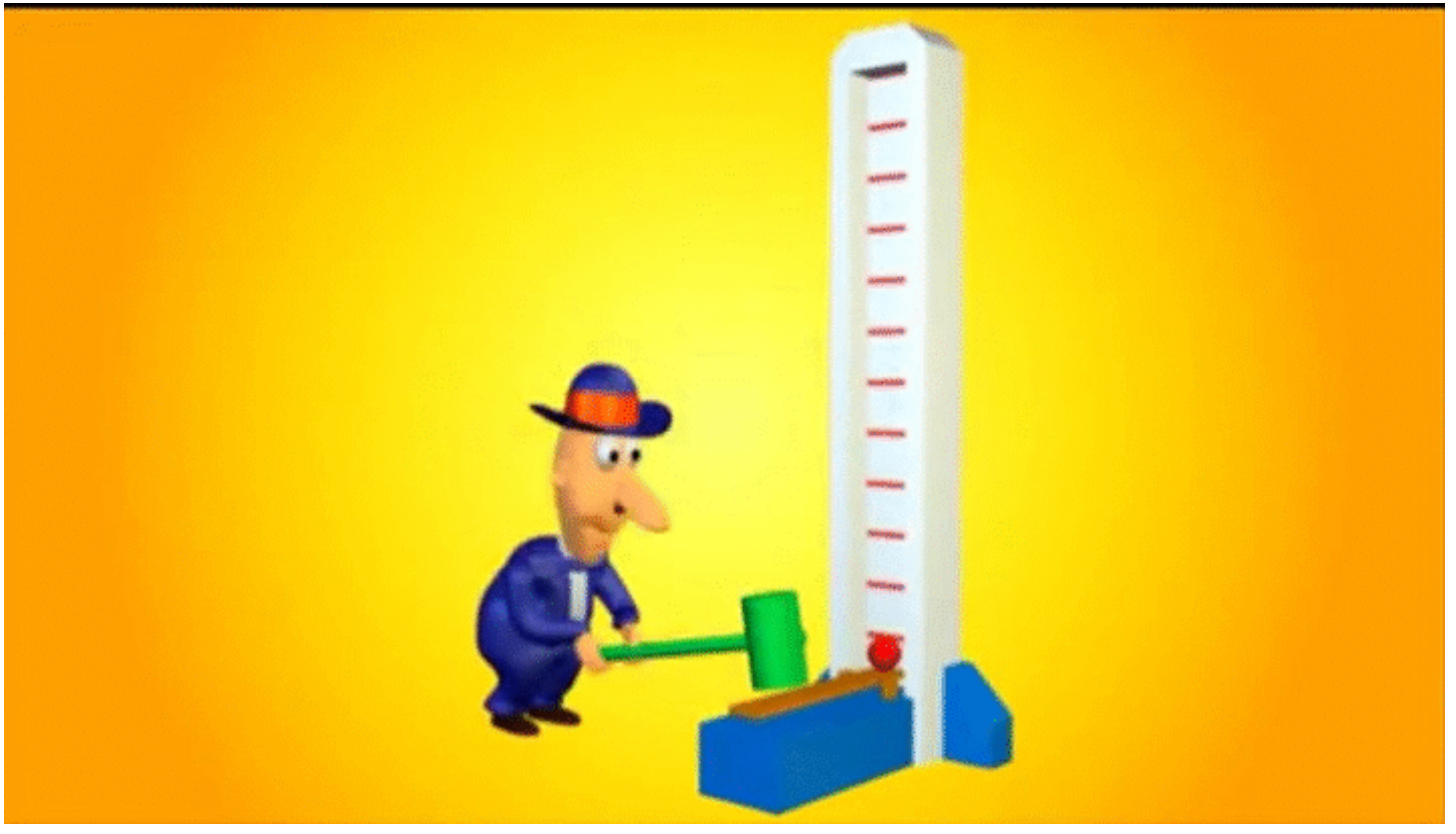


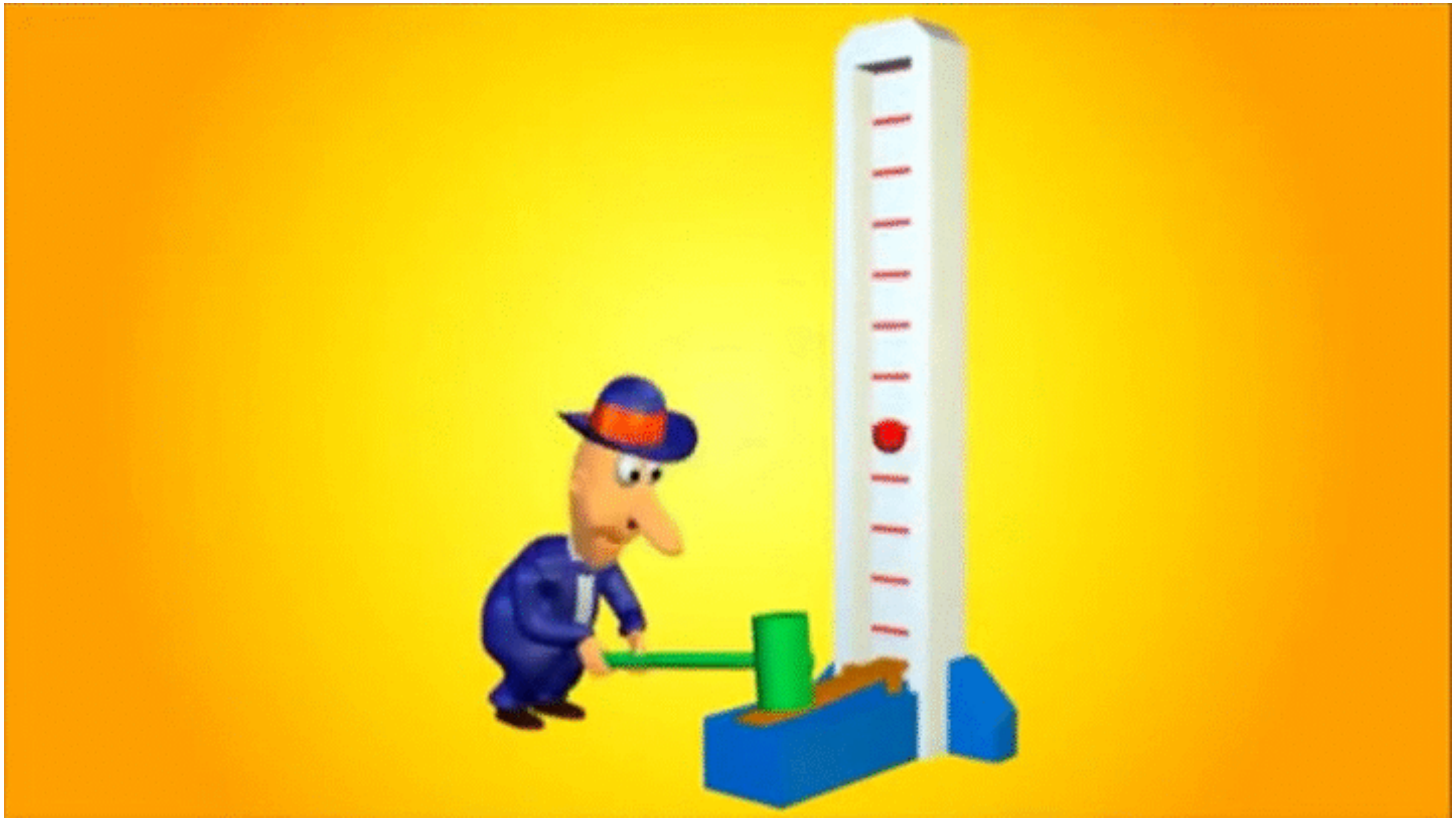
The force law

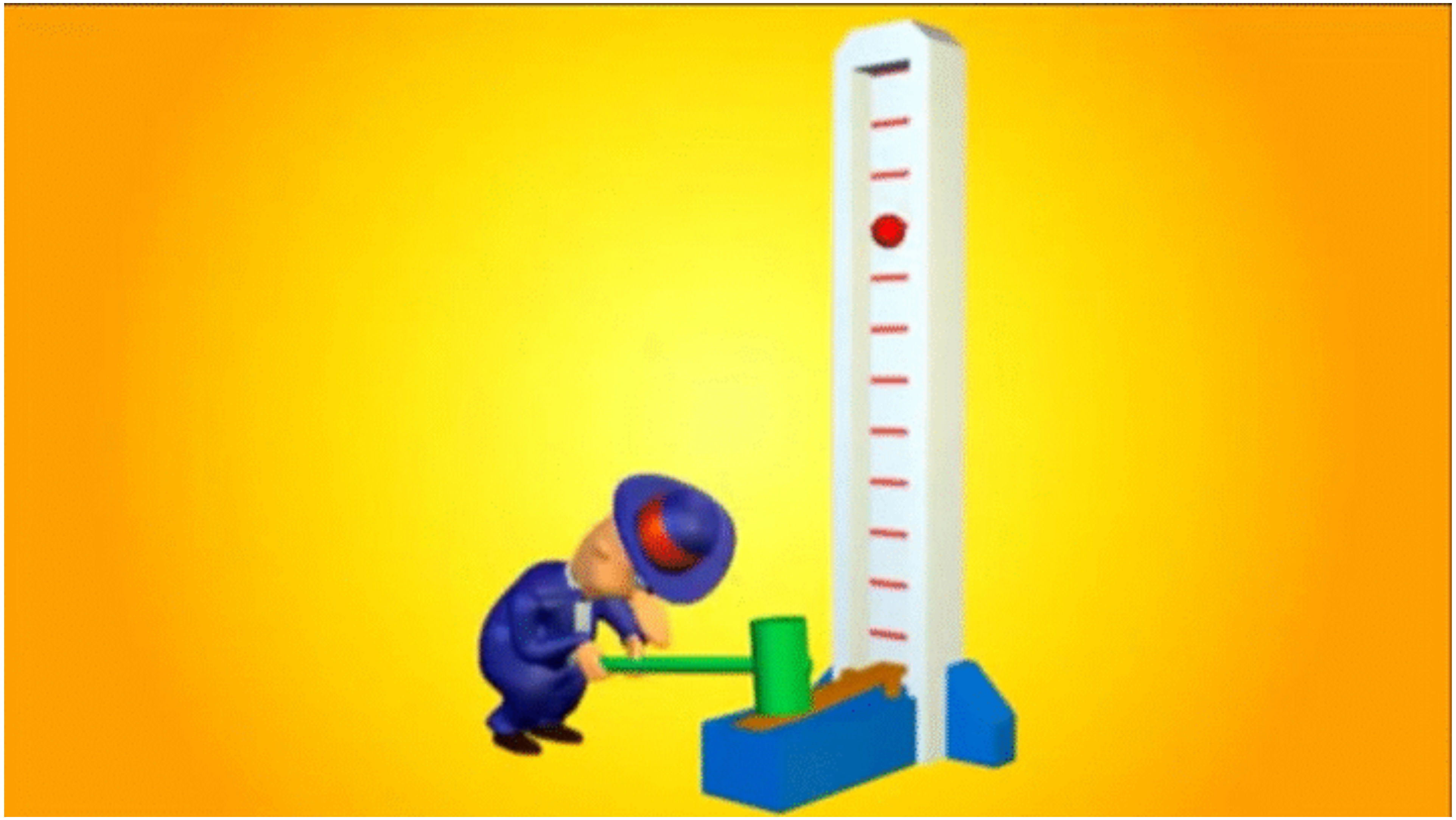
- The acceleration of an object is directly proportional to the net force acting on it and is inversely proportional to its mass

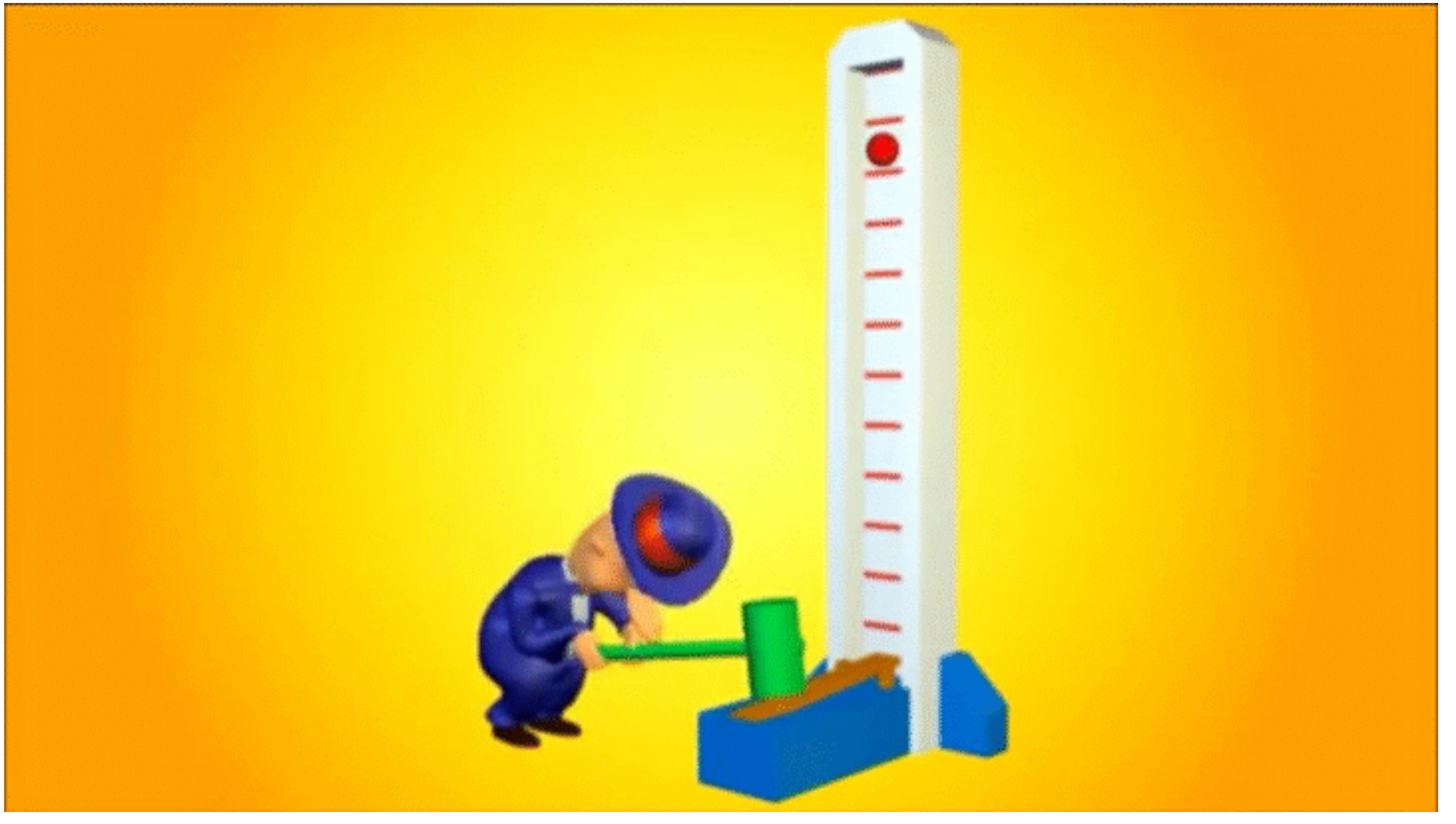
Force = Mass x acceleration

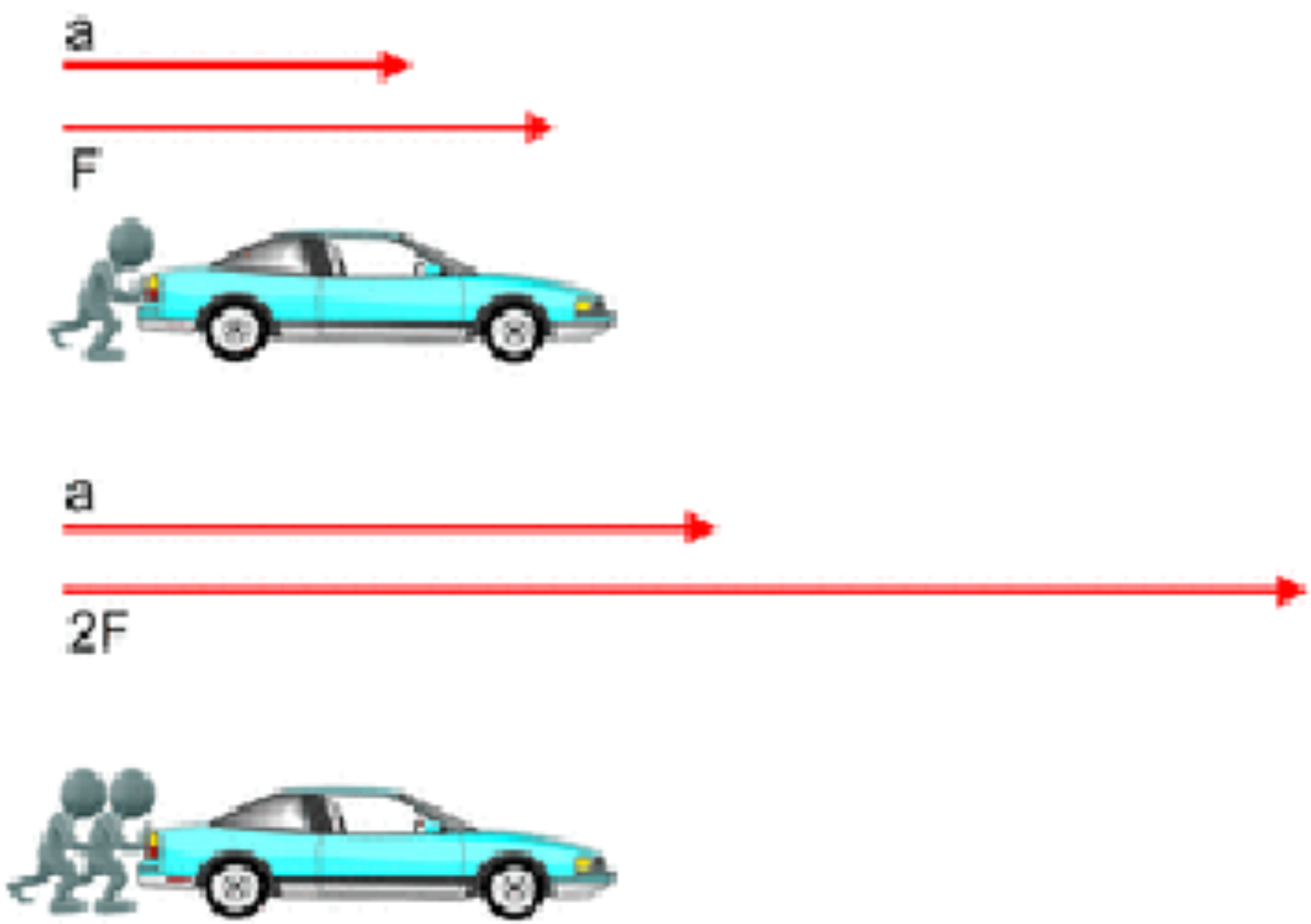
- The harder you push on something the faster it will accelerate
- If a force is applied to an object of large mass it will accelerate more slowly than an object of smaller mass with the same force

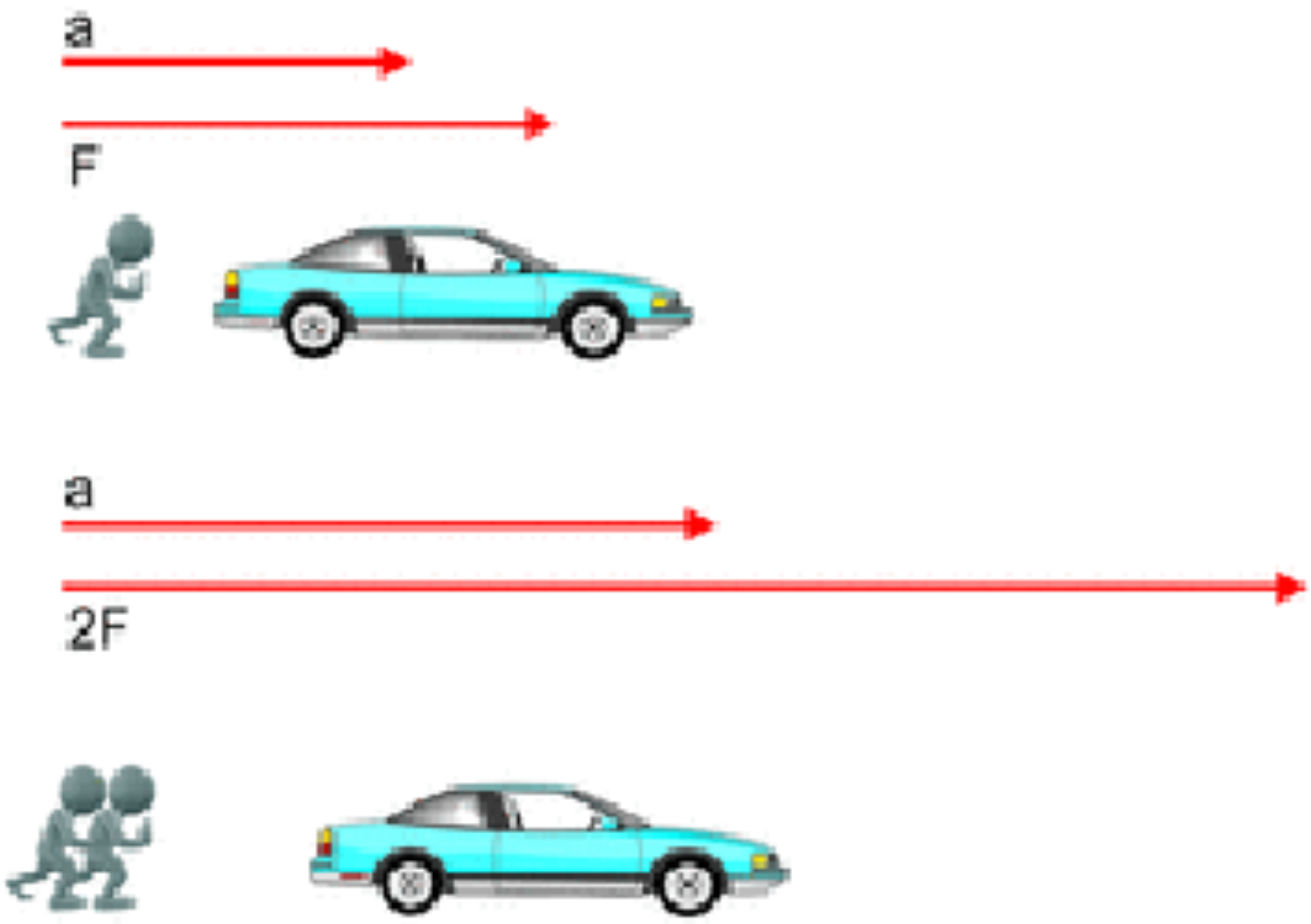


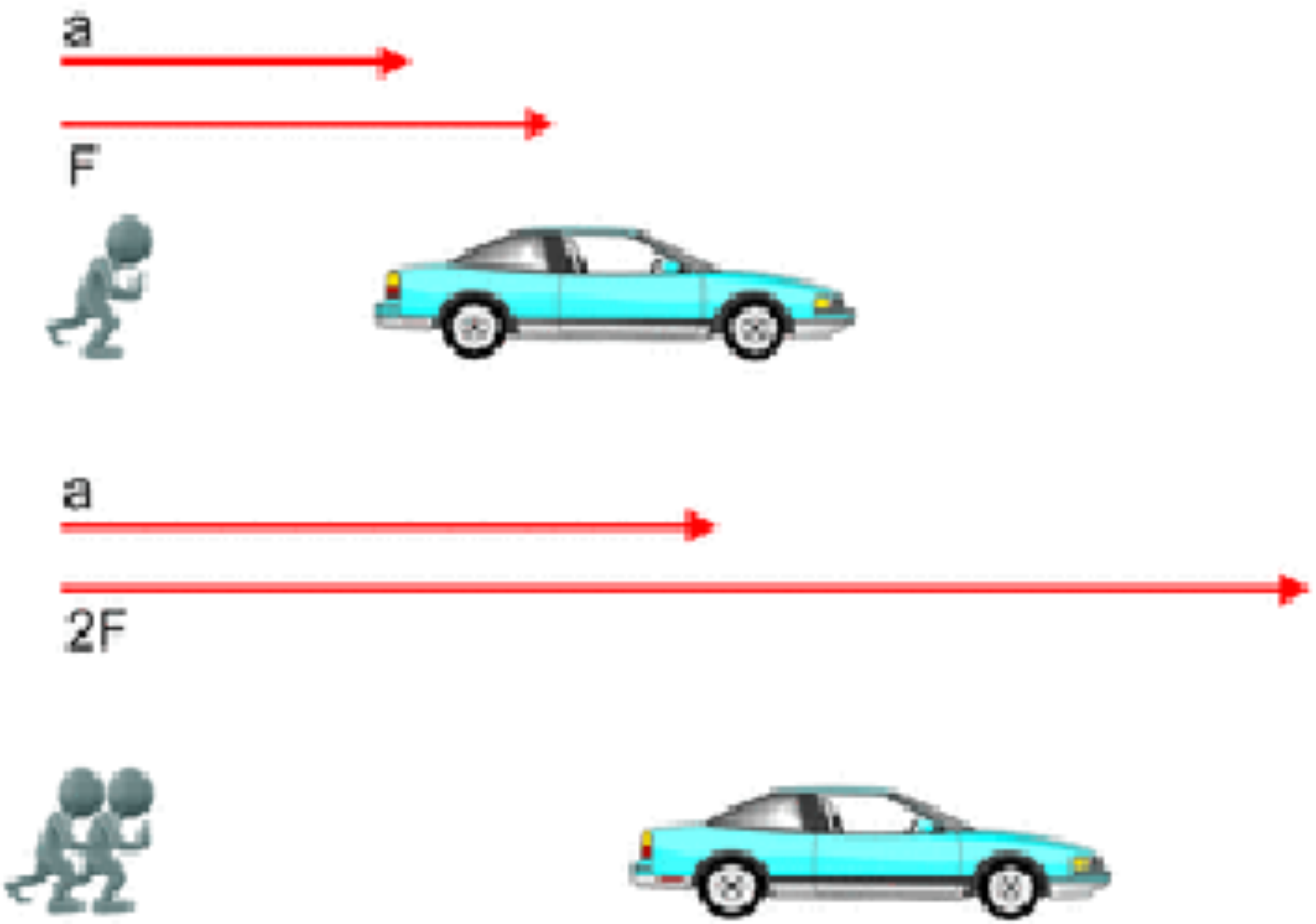


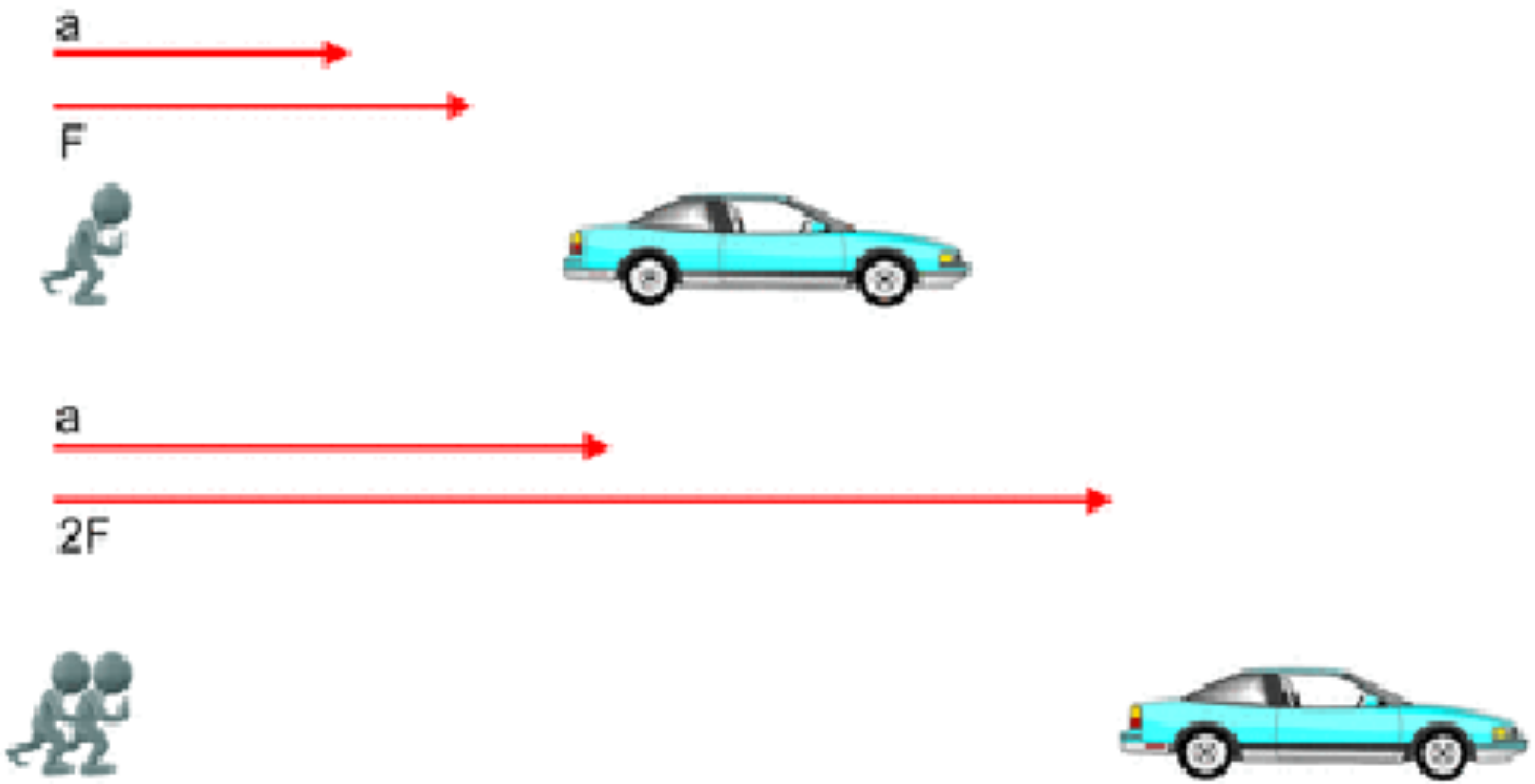




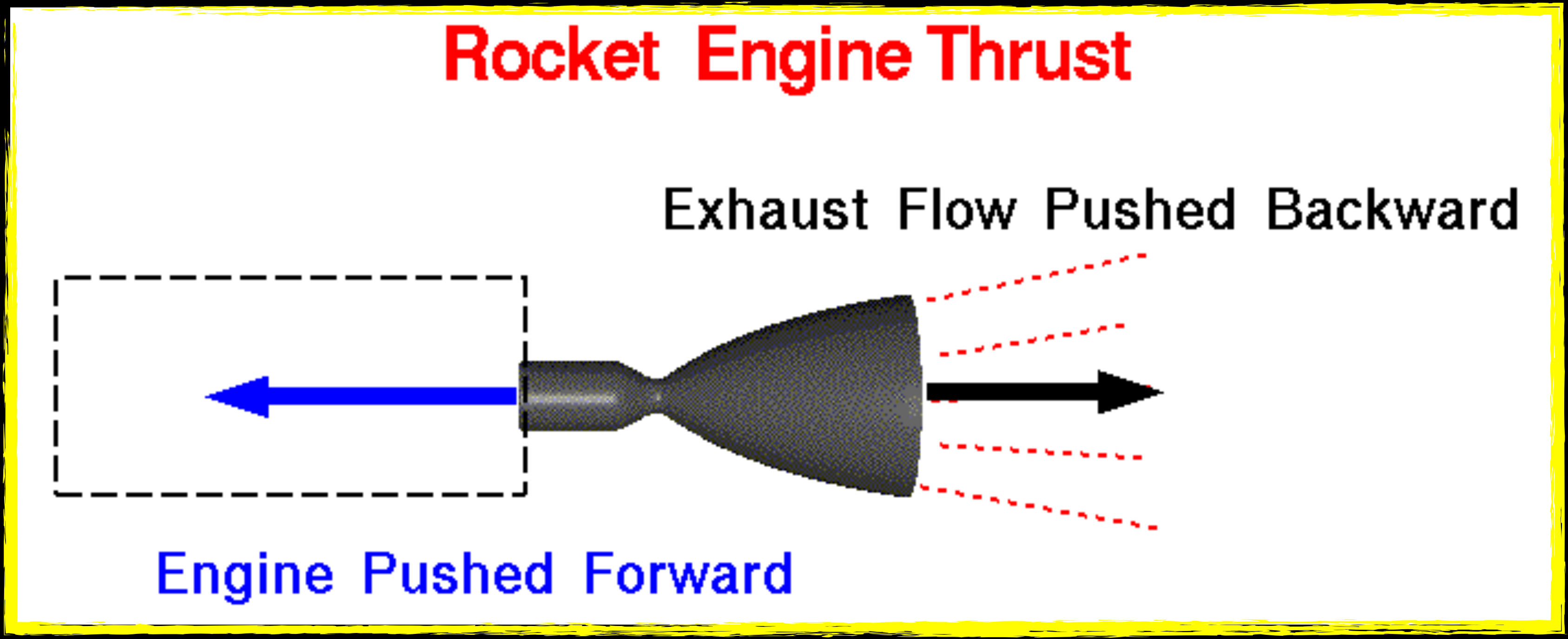

a
F
a
2F


a
F
a
2F


a
F
a
2F


a
F
a
2F

Newton's Third Law of Motion



The law of action and reaction

- Whenever one object exerts a force on a second object, the second object exerts an equal and opposite force on the first object
- Every action has an equal and opposite reaction

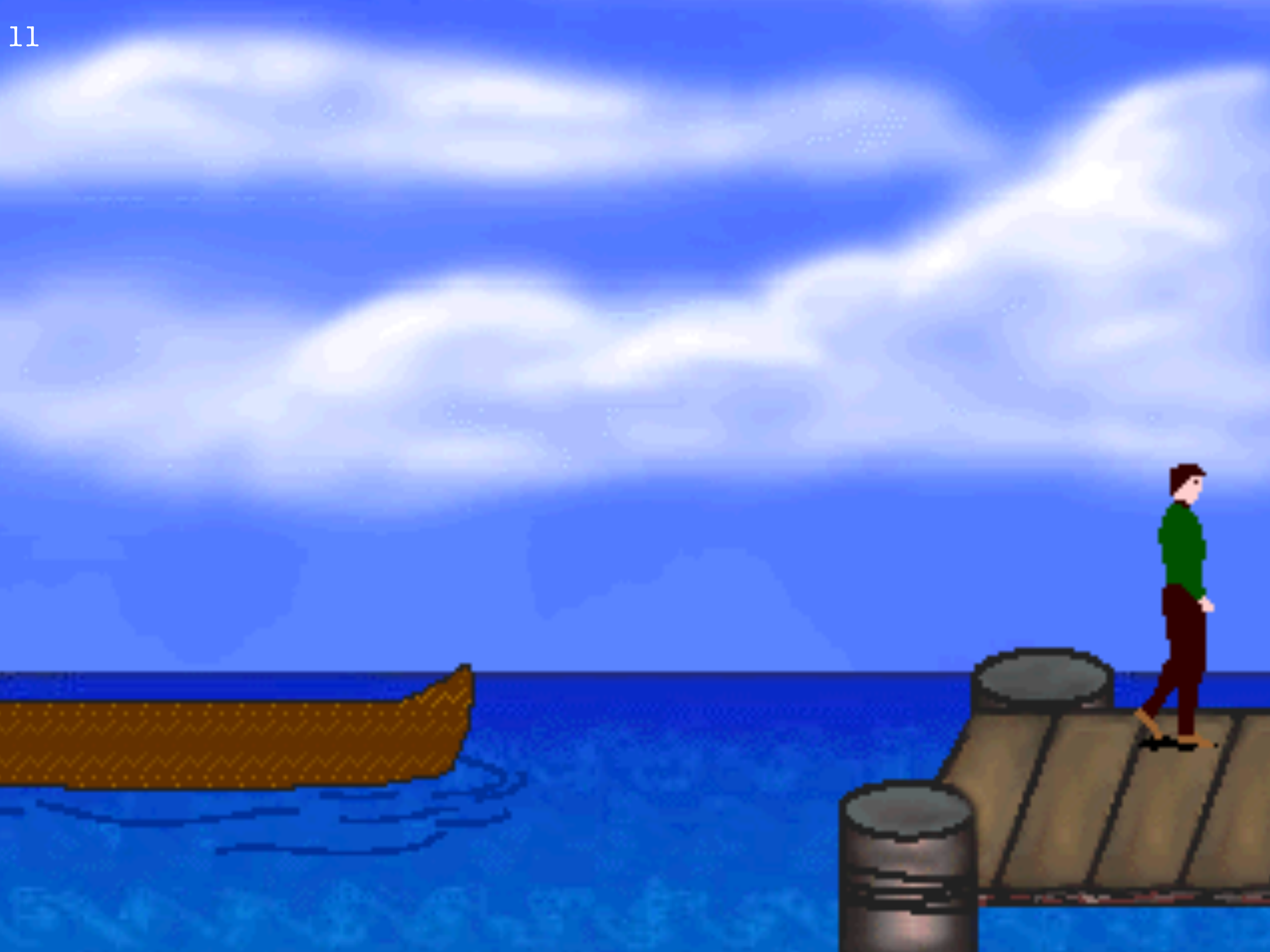
Example

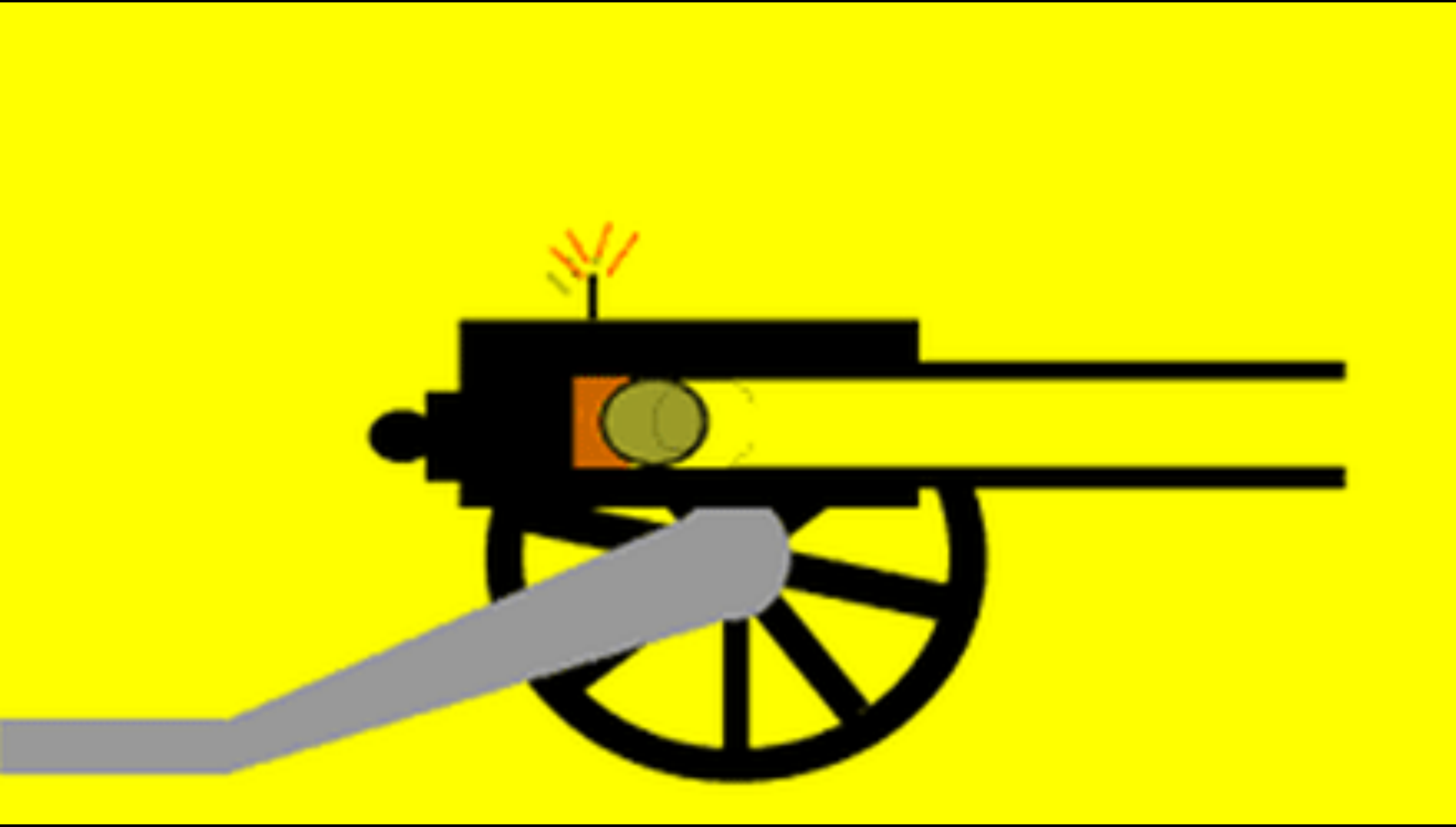
- Step off a small boat, it moves in the opposite direction to which you are stepping

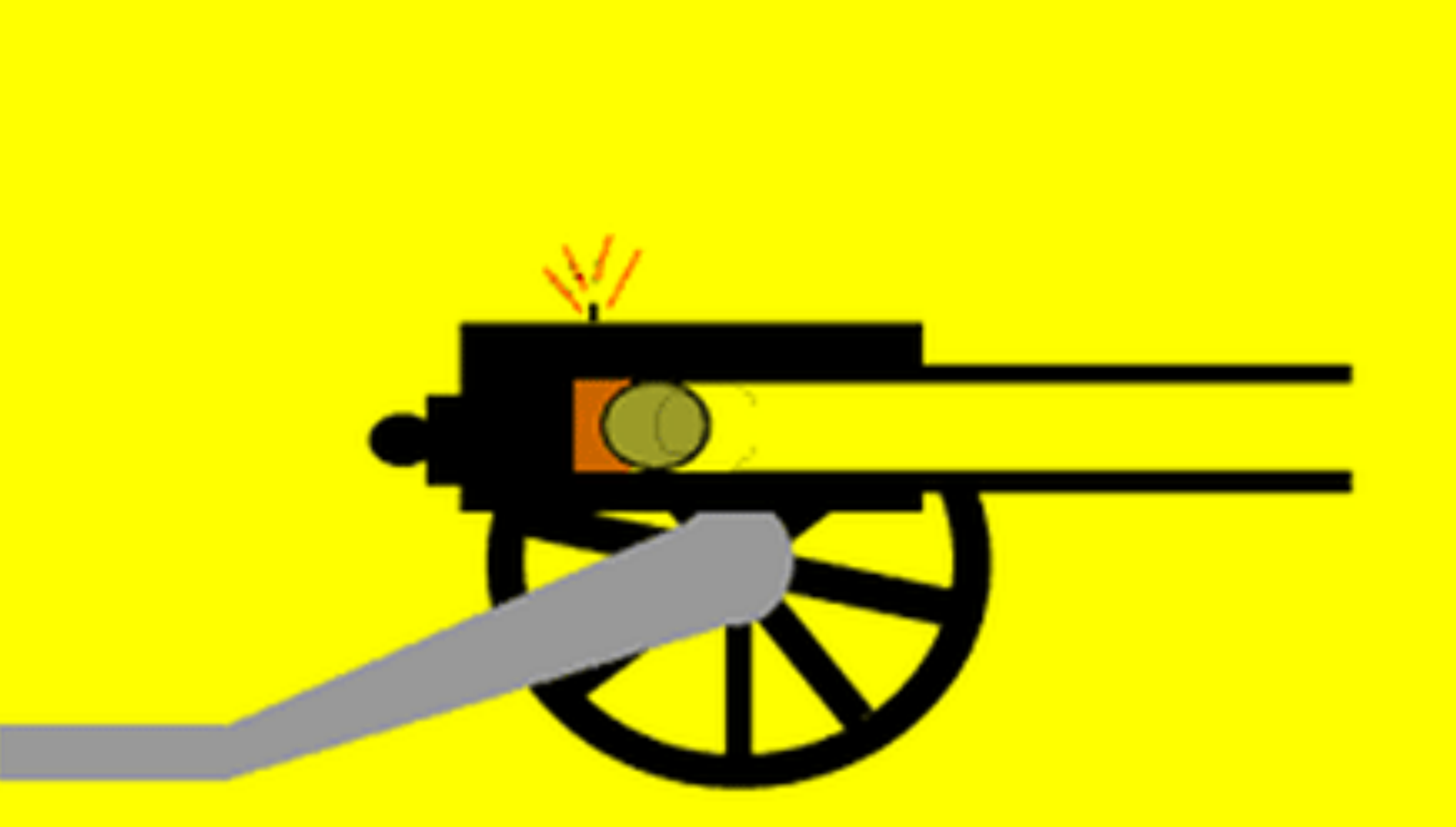

11

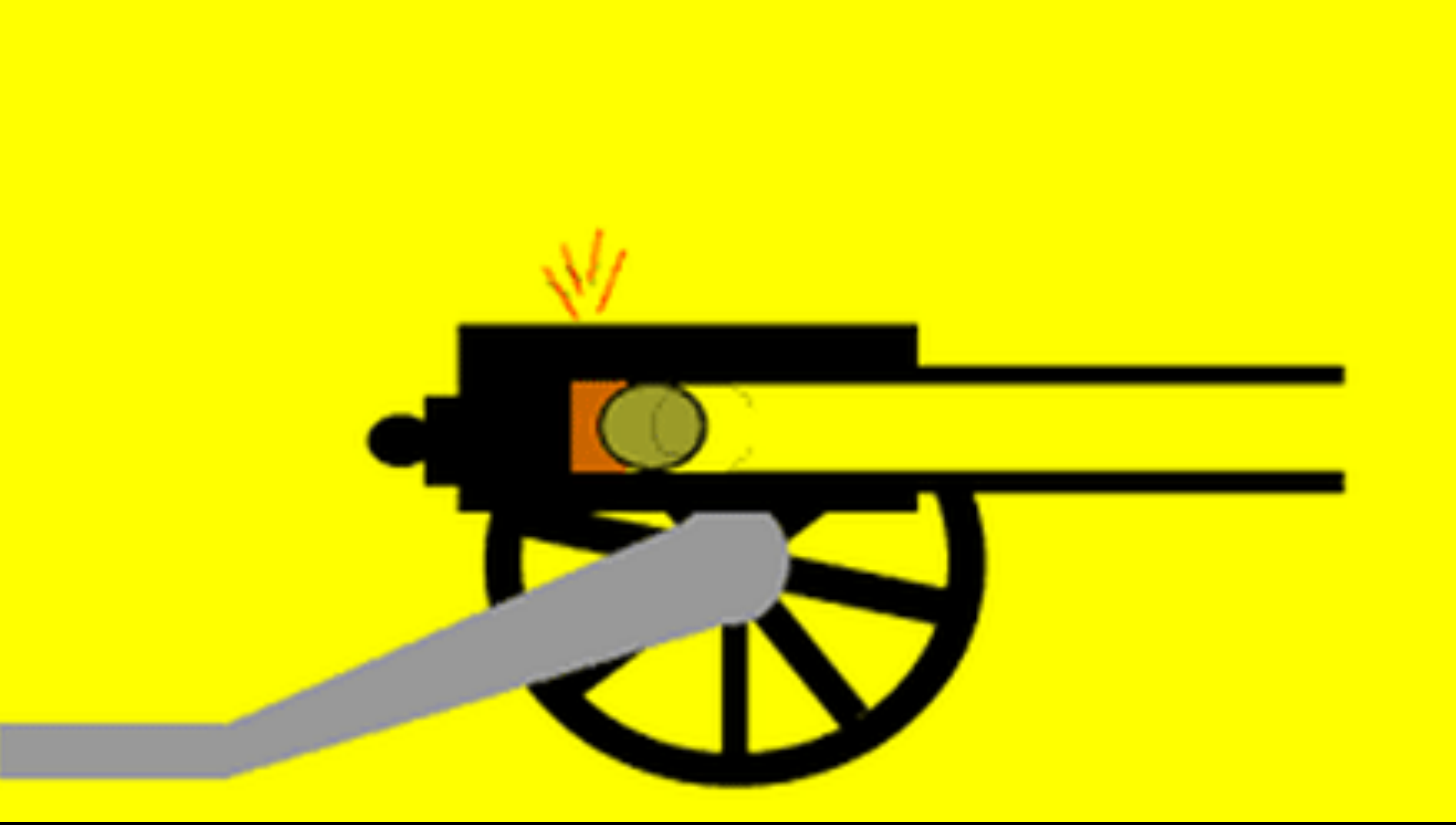

11

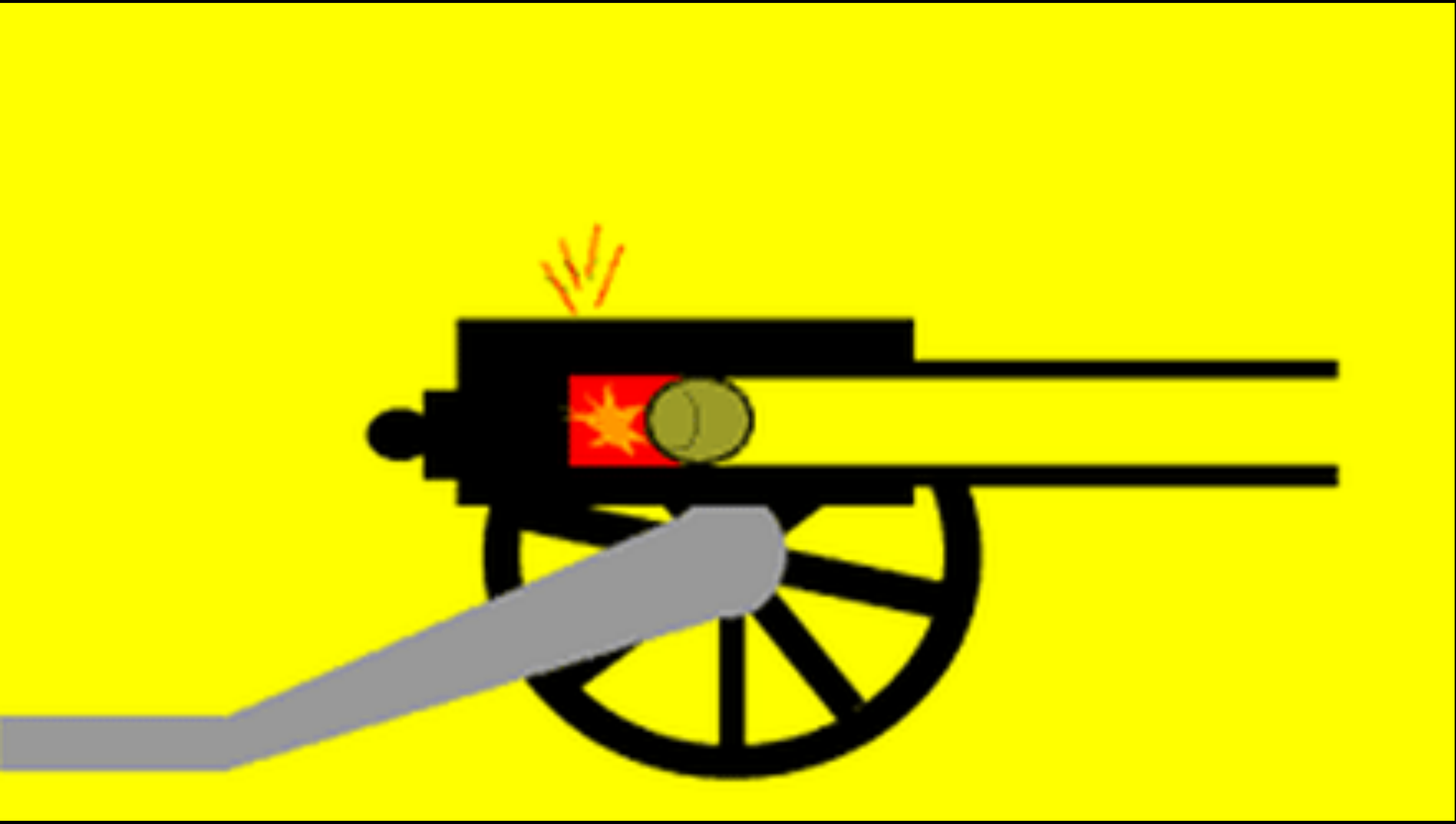

11

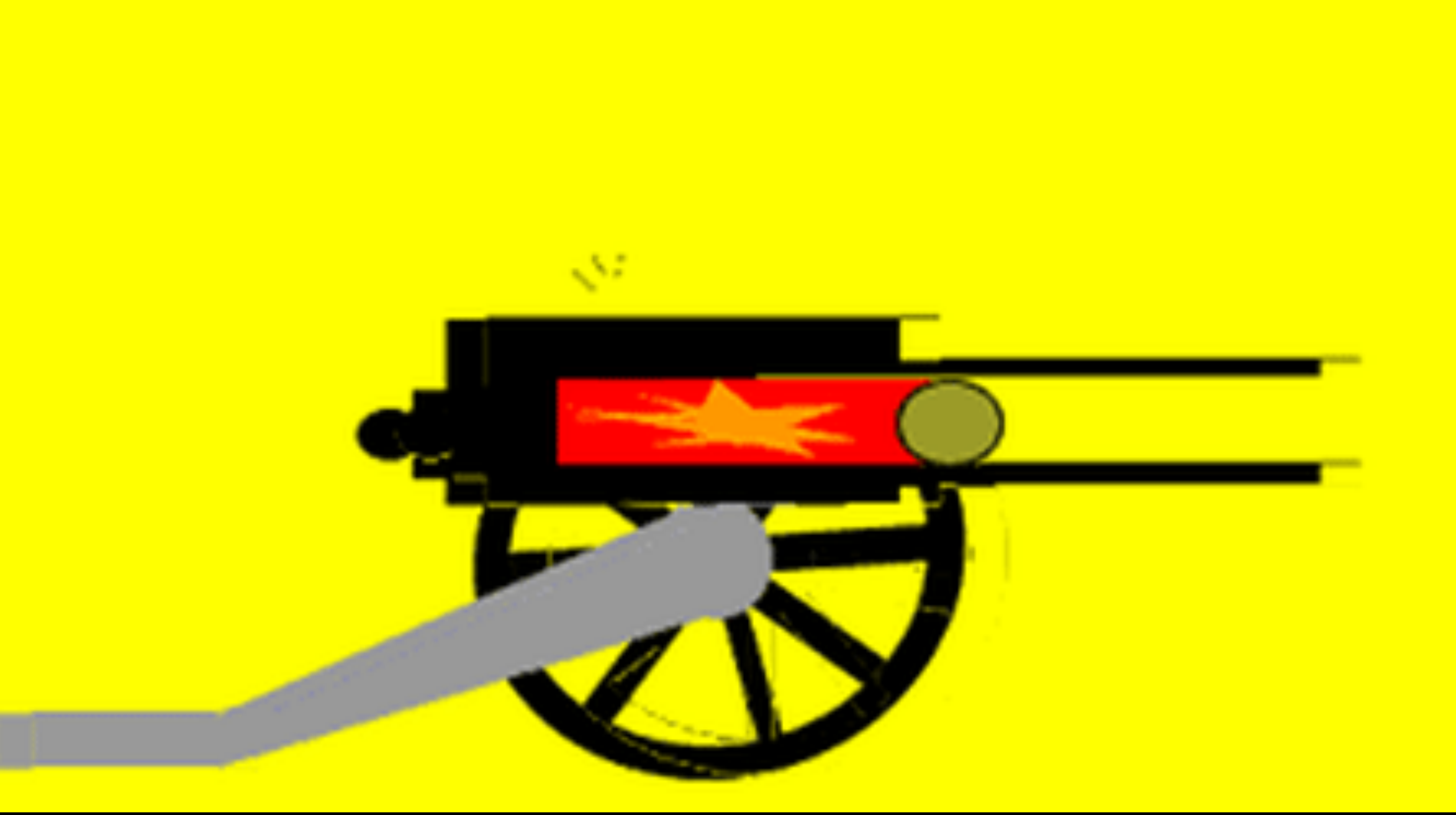


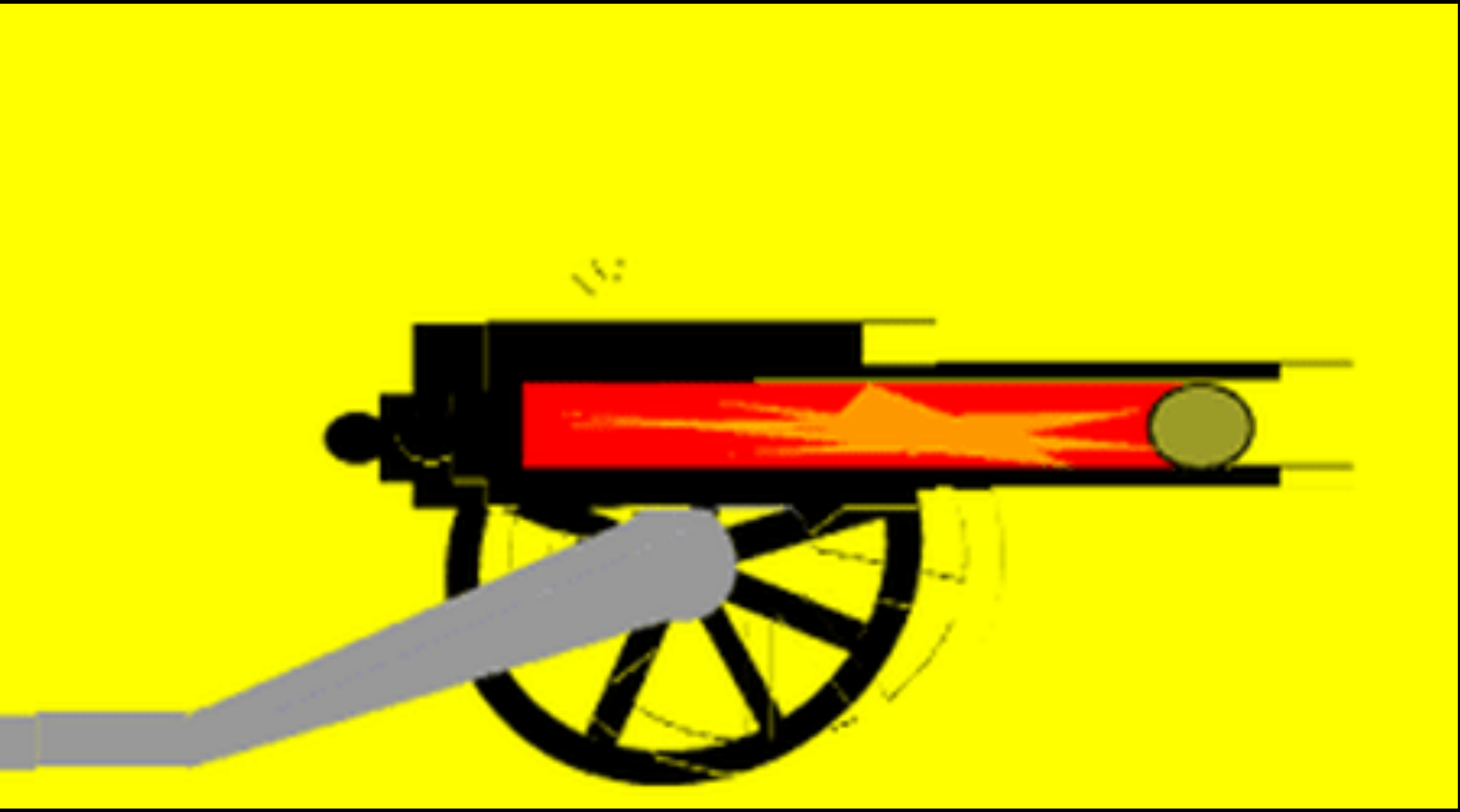


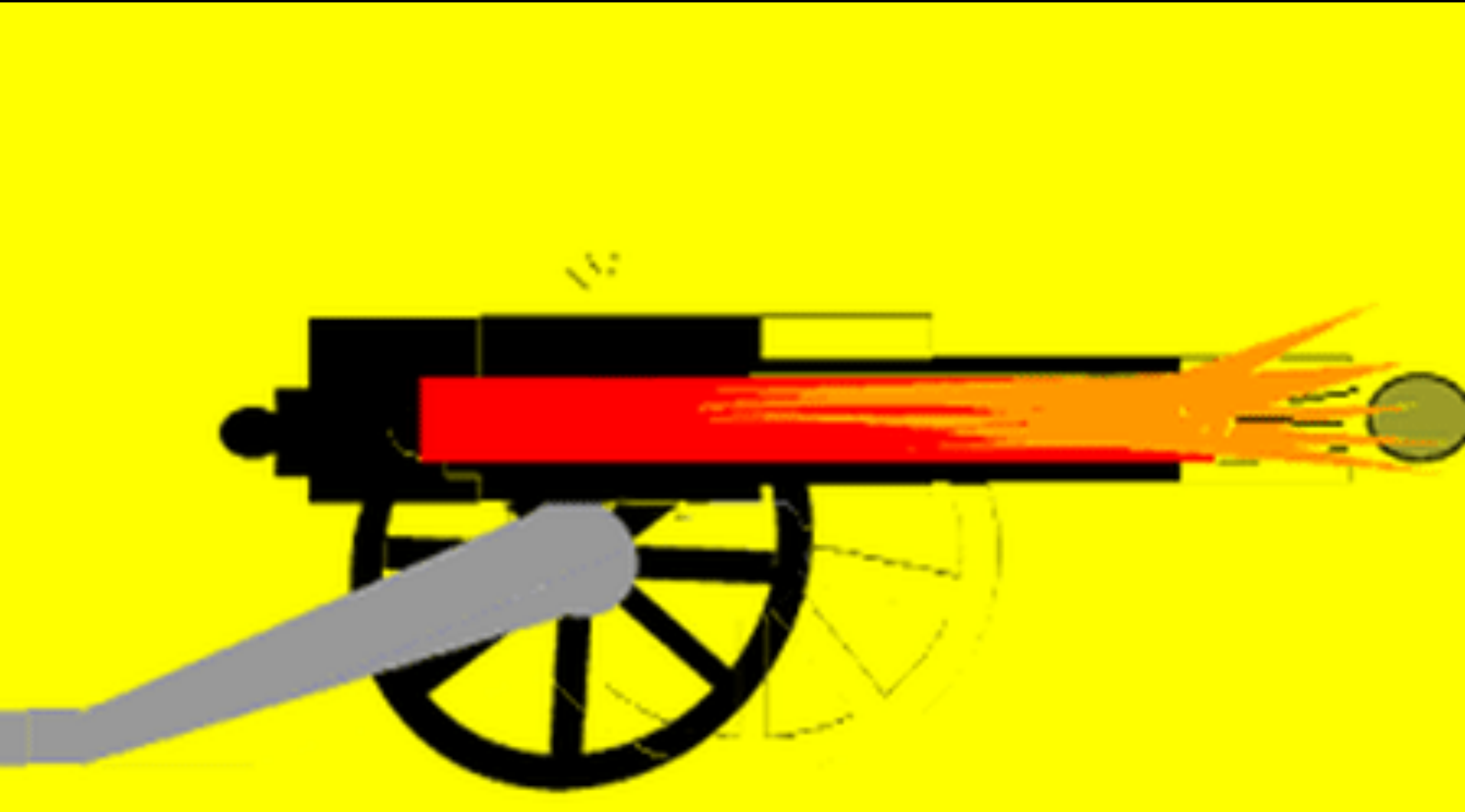


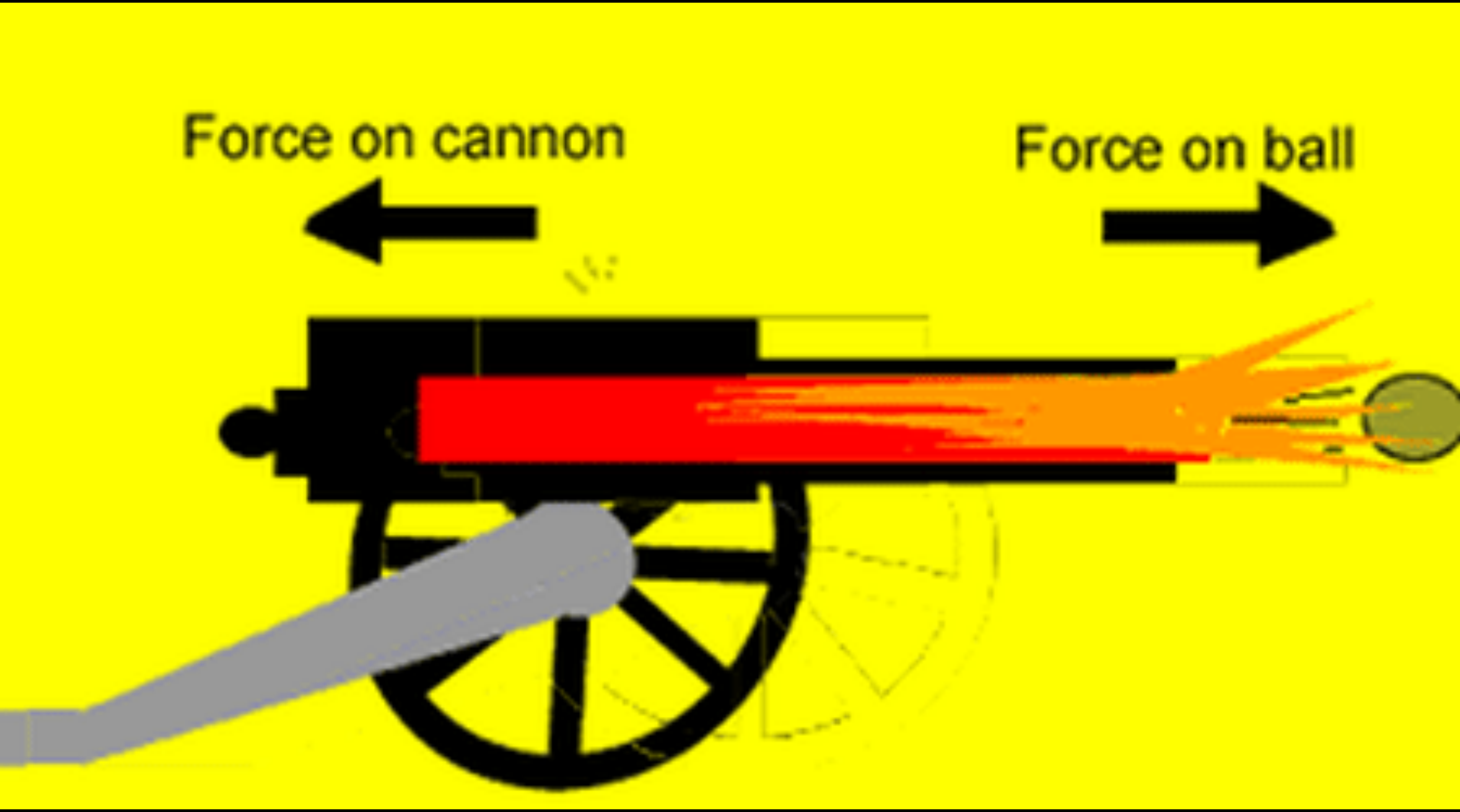


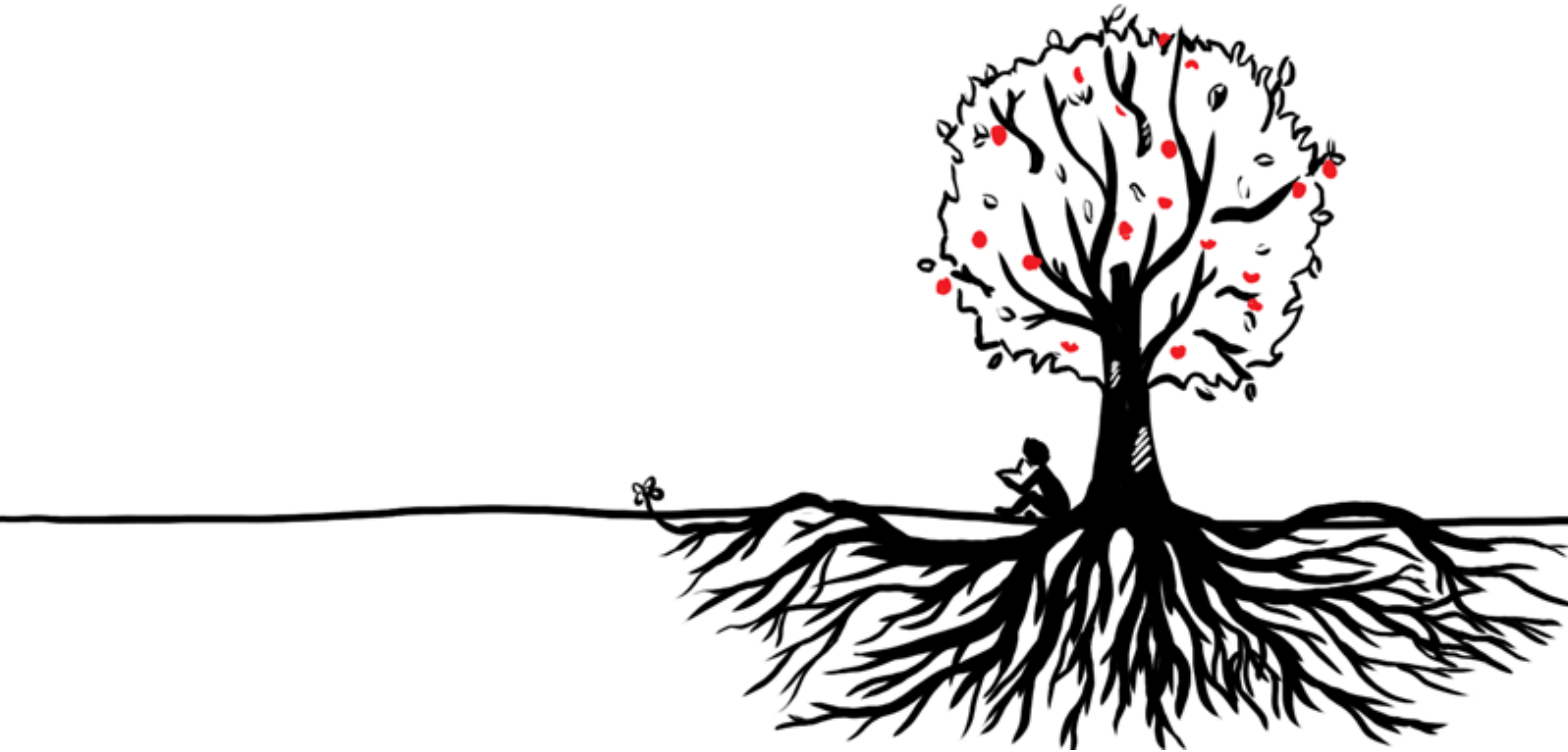


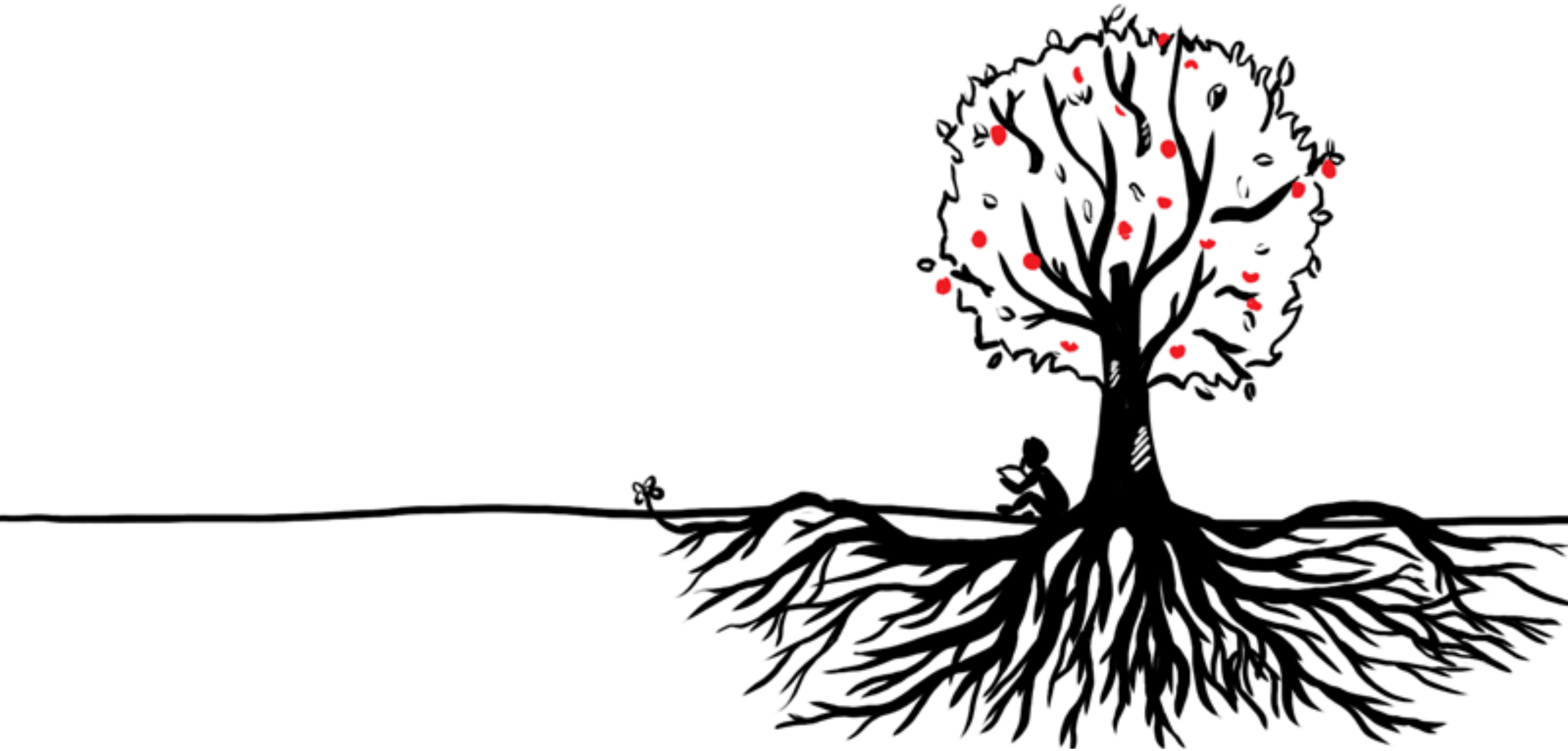


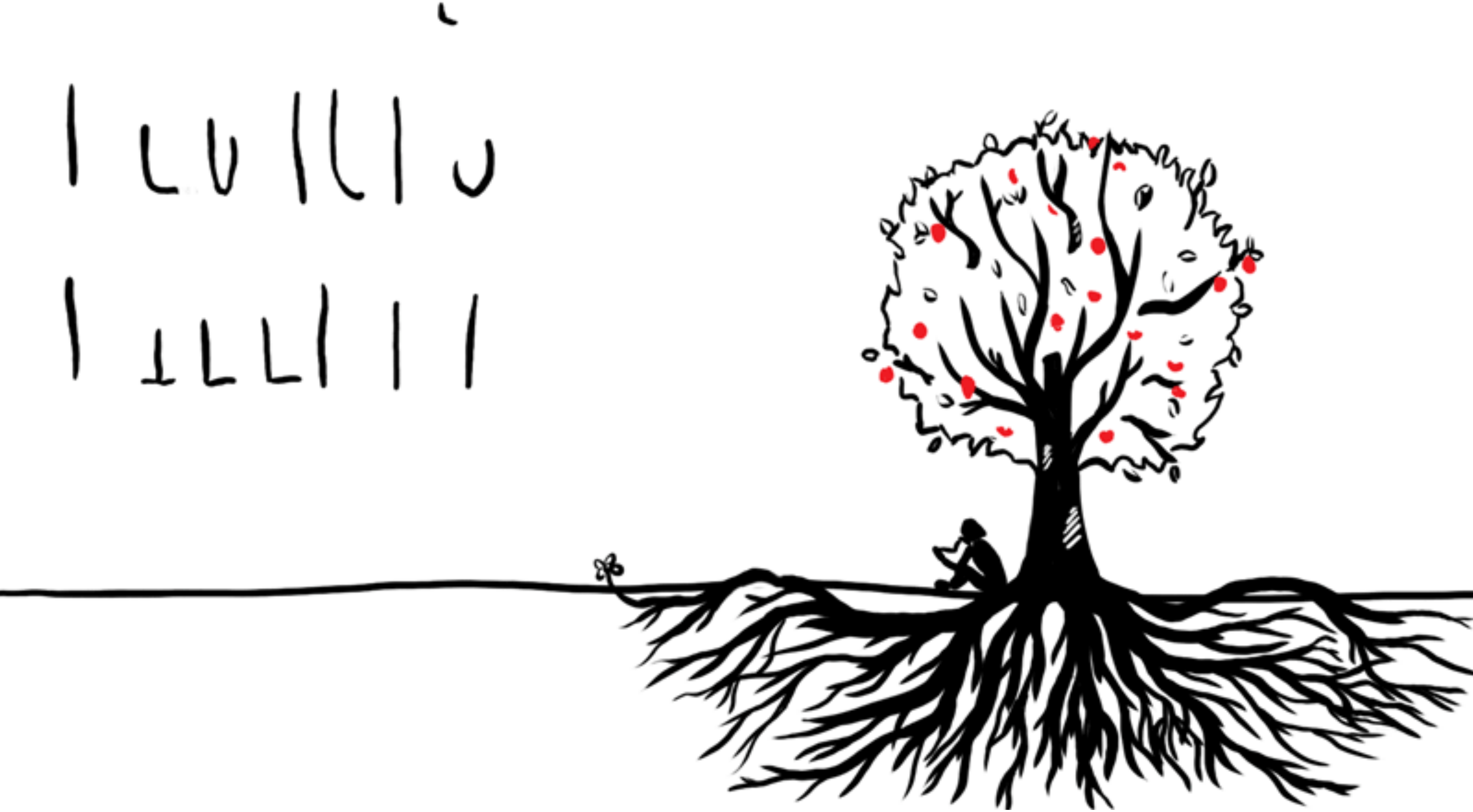


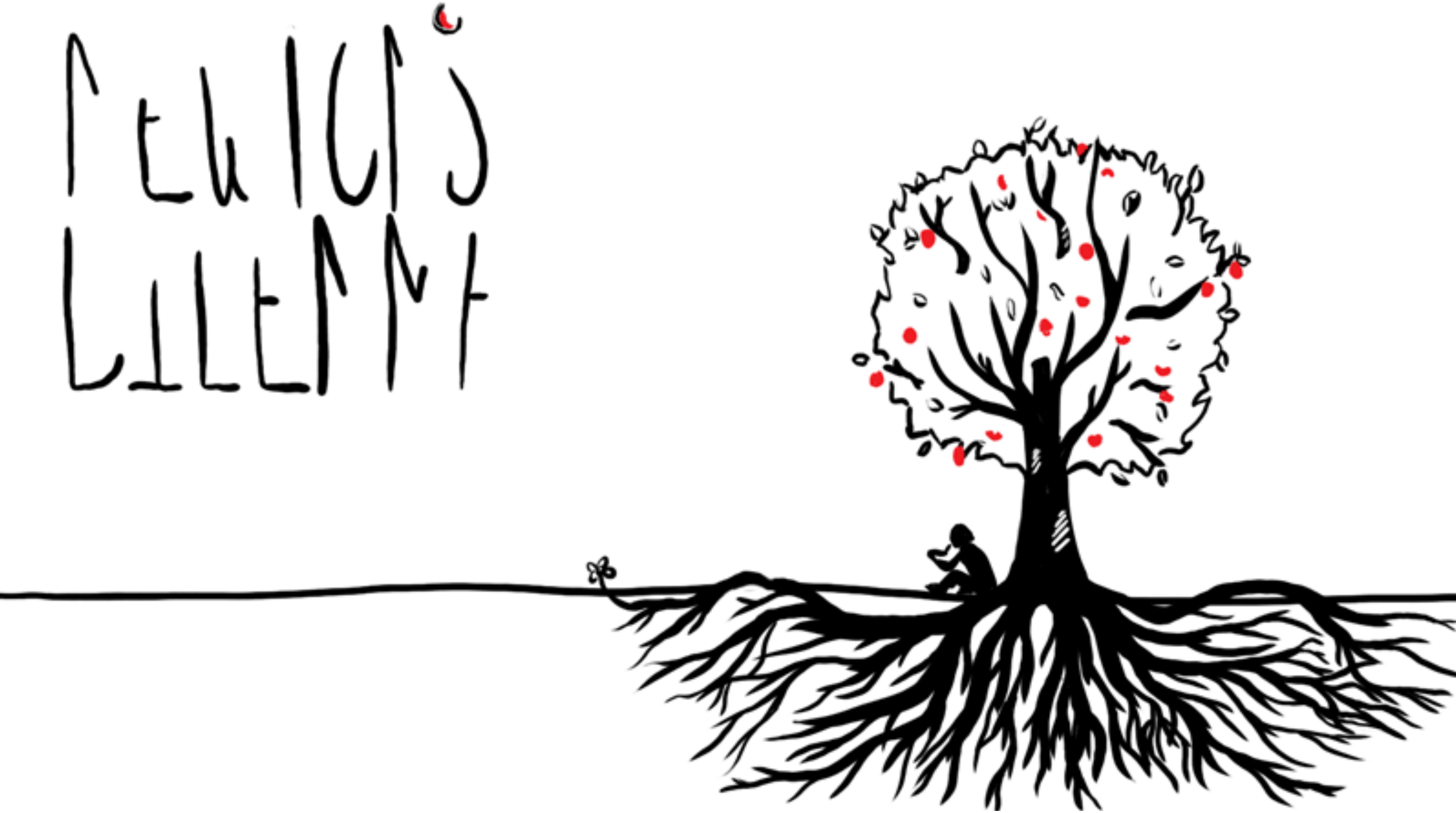



Force on cannon
Force on ball









NEWTON'S DILEMMA

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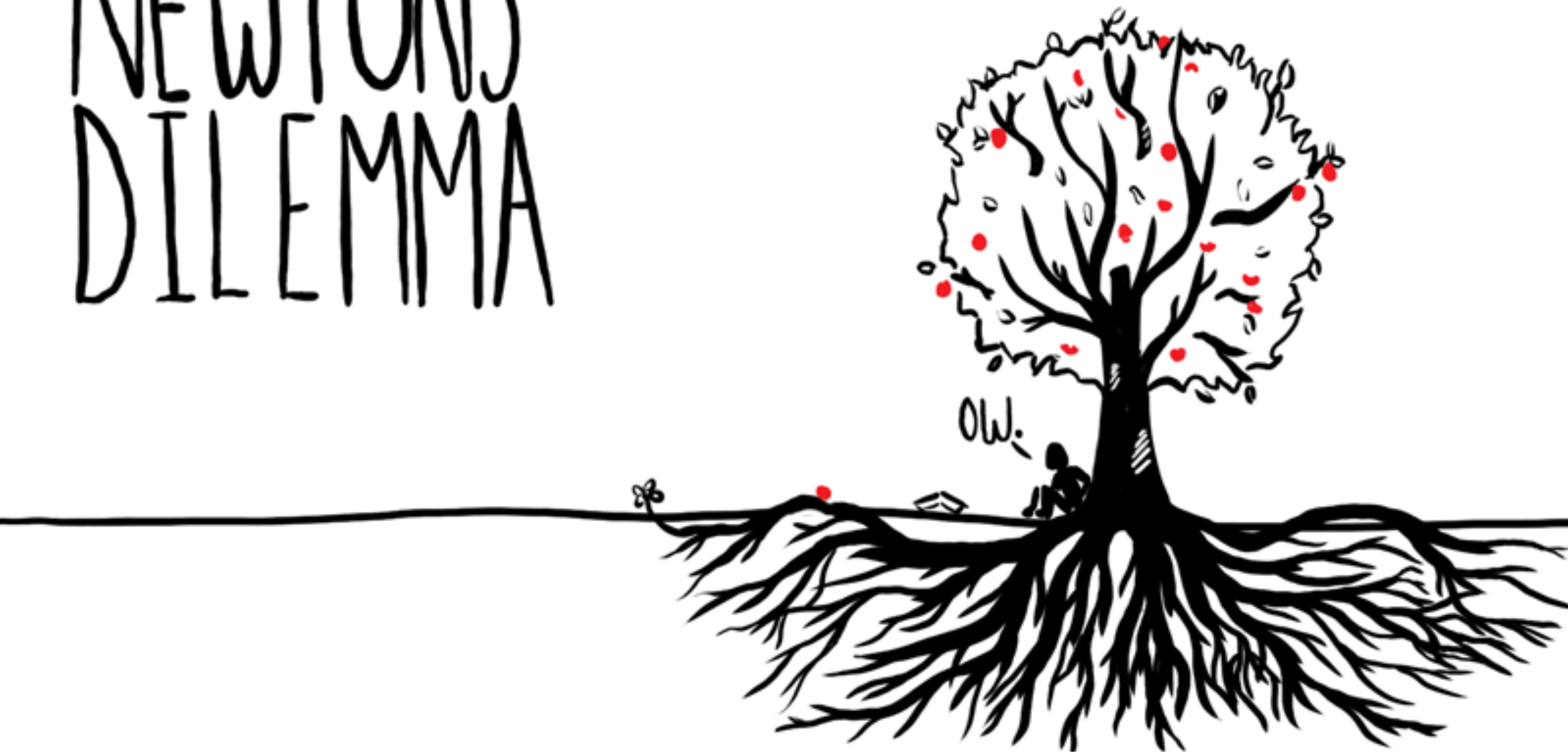


Newton's Law of Universal Gravitation

$$F = G\frac{Mm}{r^2}$$

- Two objects attract each other with a force (F)
  - directly proportional to the product of their masses (M and m)
  - inversely proportional to the square of the distance (r) between them
- Weight on Earth = Gravitational Force
- m- your mass in unit of kg
- M- mass of Earth = 5.97×10^{24} kg
- G- universal gravitational constant = 6.67×10^{-11} m^3 kg^{-1} s^{-2}
- r- Earth's radius = 6.37×10^6 m

➢ Gravitational force ☛ force that attracts any two objects with mass

* Magnitude of gravitational force $= F_g = GMm/r^2$

* Force direction ☛ along line joining objects

* $G = 6.673 \times 10^{-11} Nm^2/kg$ ☛ proportionality constant

* Near Earth's surface

$$\text{gravitational acceleration} = g = \frac{GM_\oplus}{R_\oplus^2} \approx 9.8\, m/s^2$$

$$M_\oplus = 1.3 \times 10^{25} \text{lb} \qquad R_\oplus = 3,959 \text{ miles}$$

➢ Centripetal force ☛ force that keeps object moving on circular path

* Object can move around in a circle with constant speed yet still be accelerating because its direction is constantly changing

* Centripetal acceleration ☛ directed toward center of circle

* Magnitude of centripetal acceleration = $(\text{speed})^2/\text{radius}$

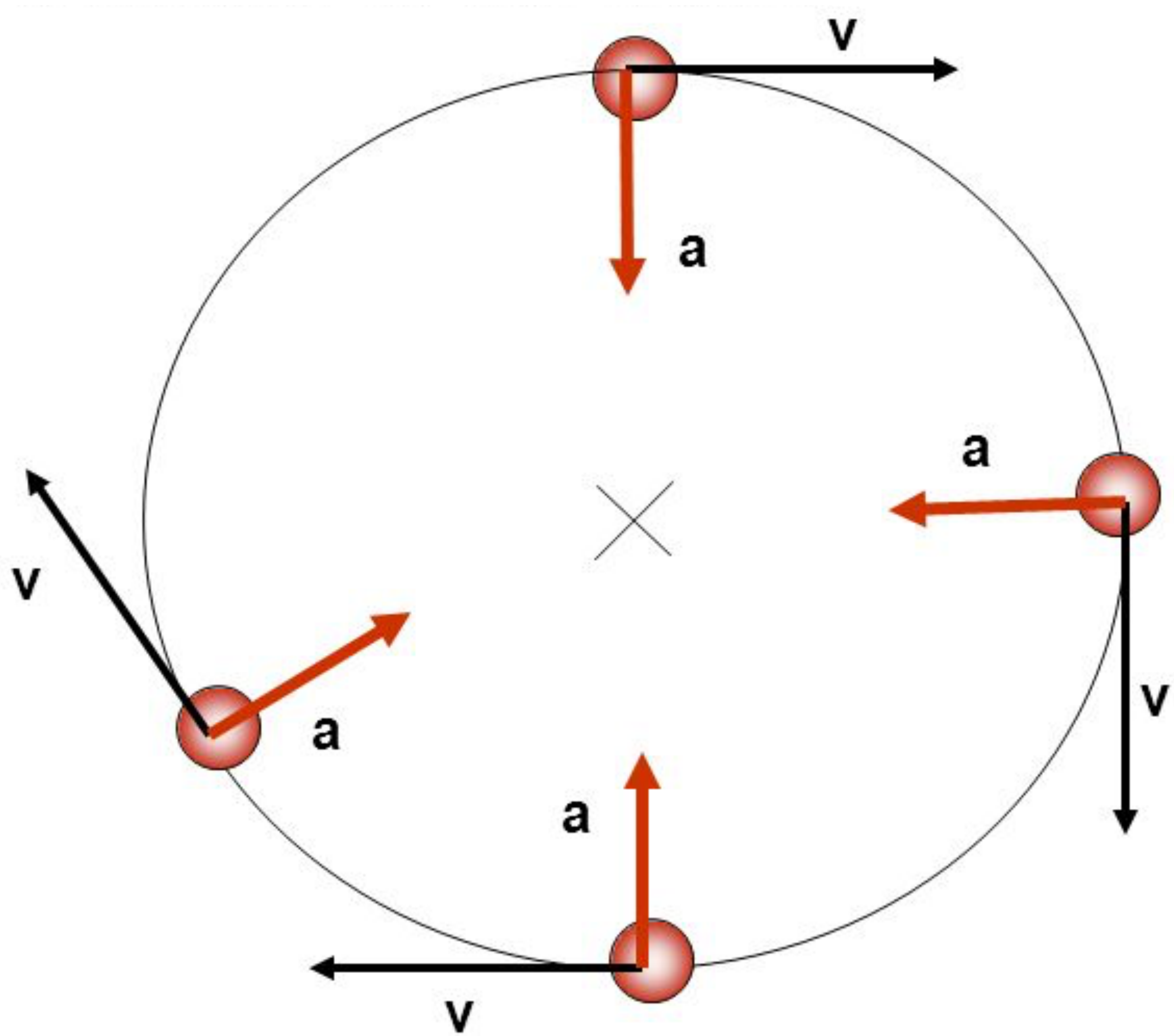
Moon continuously falls toward Earth due to gravity
but does not get any closer to Earth because its motion is an orbit

Centripetal acceleration

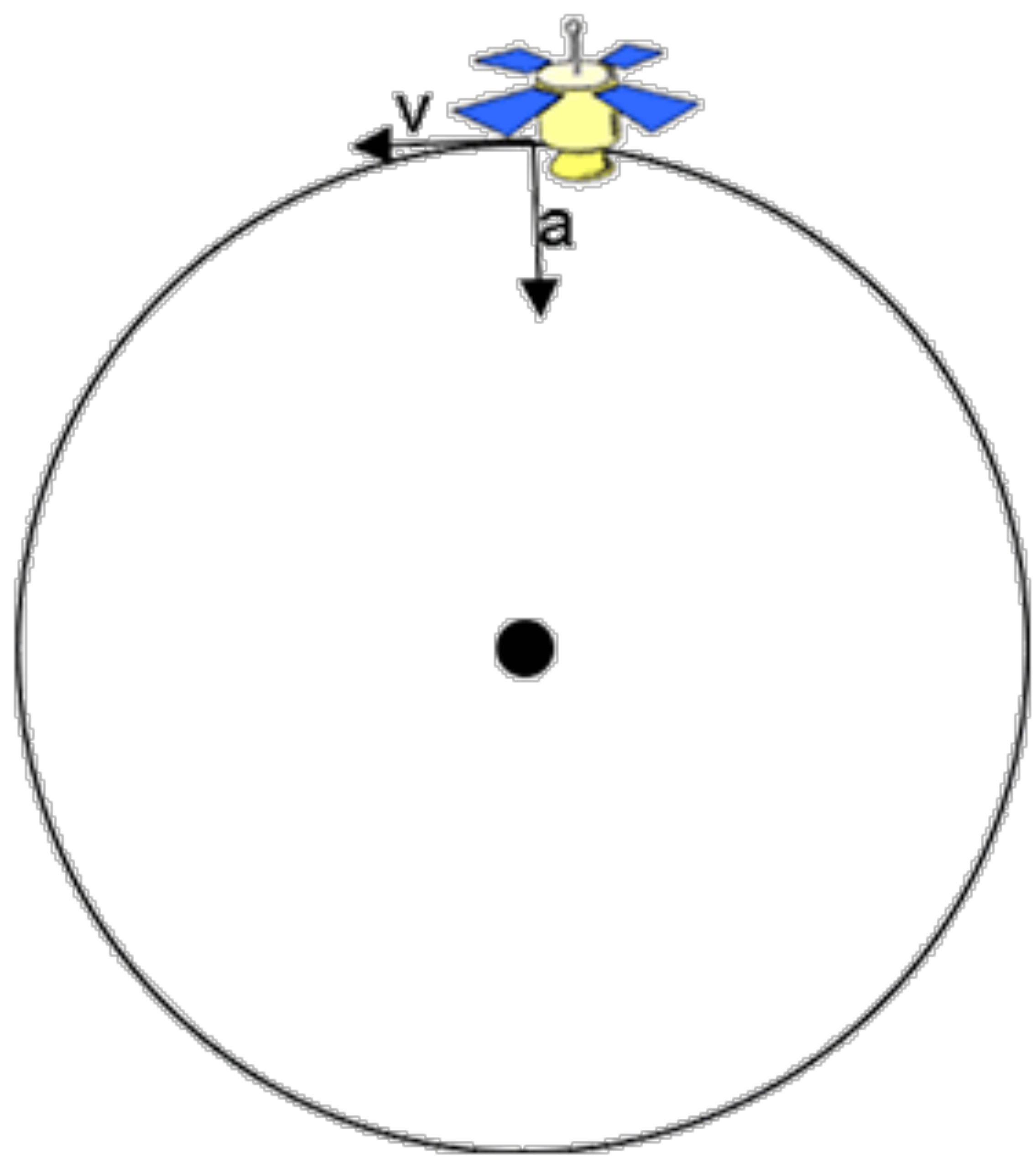
Directed toward center of curvature

$$a_c = \frac{v^2}{r}$$

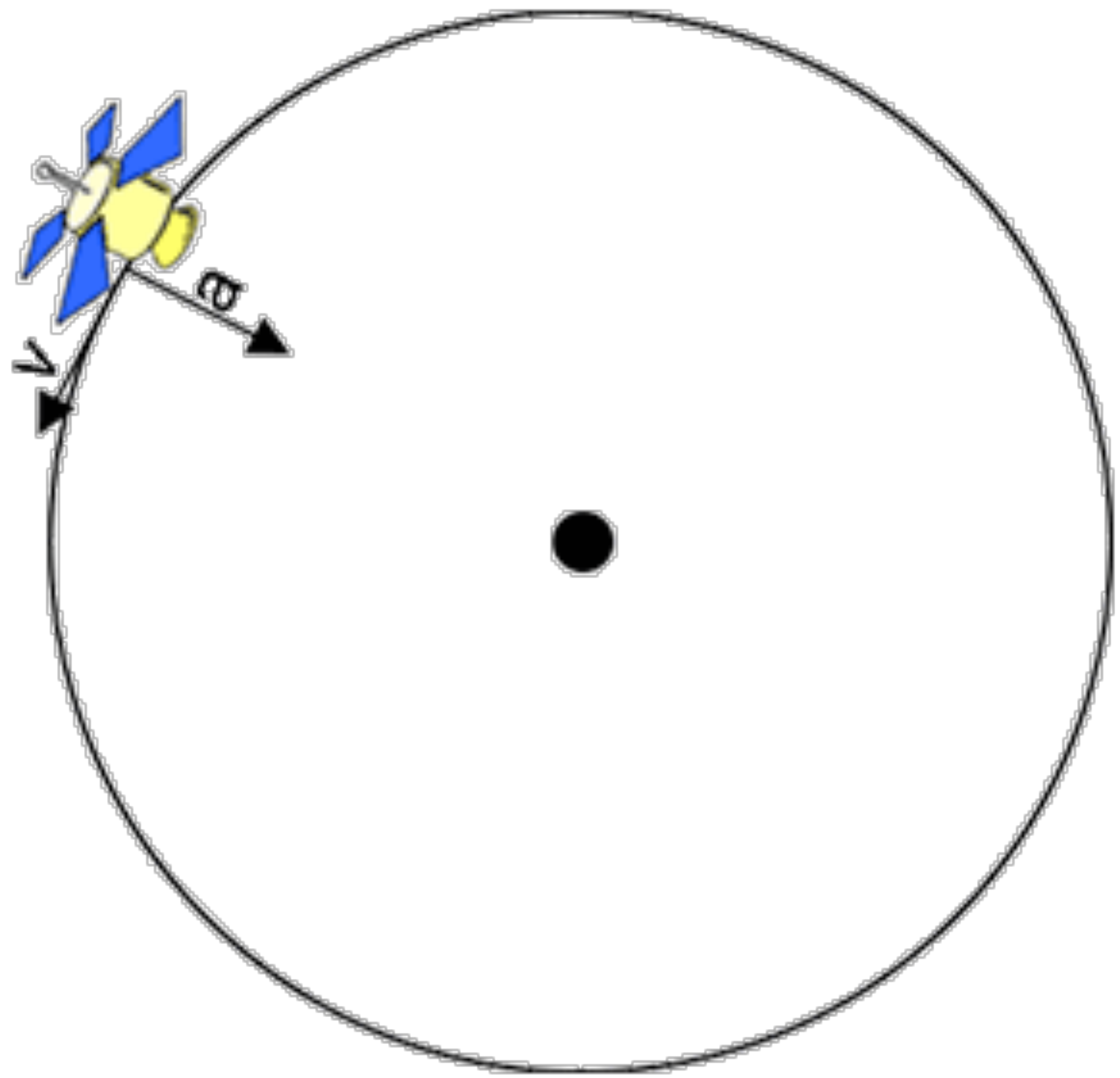
What can you infer about the net force on this object?



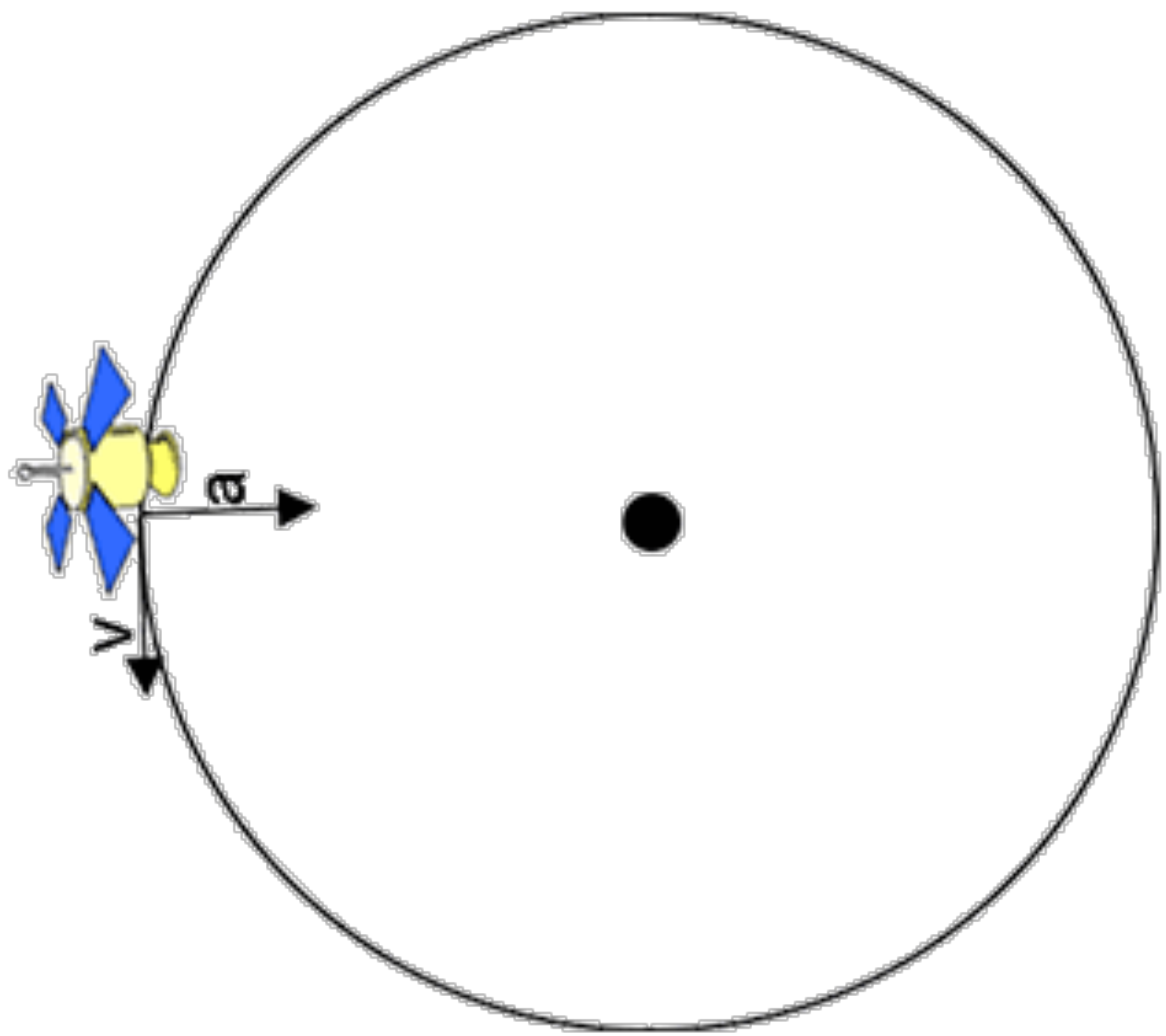
Uniform Circular Motion



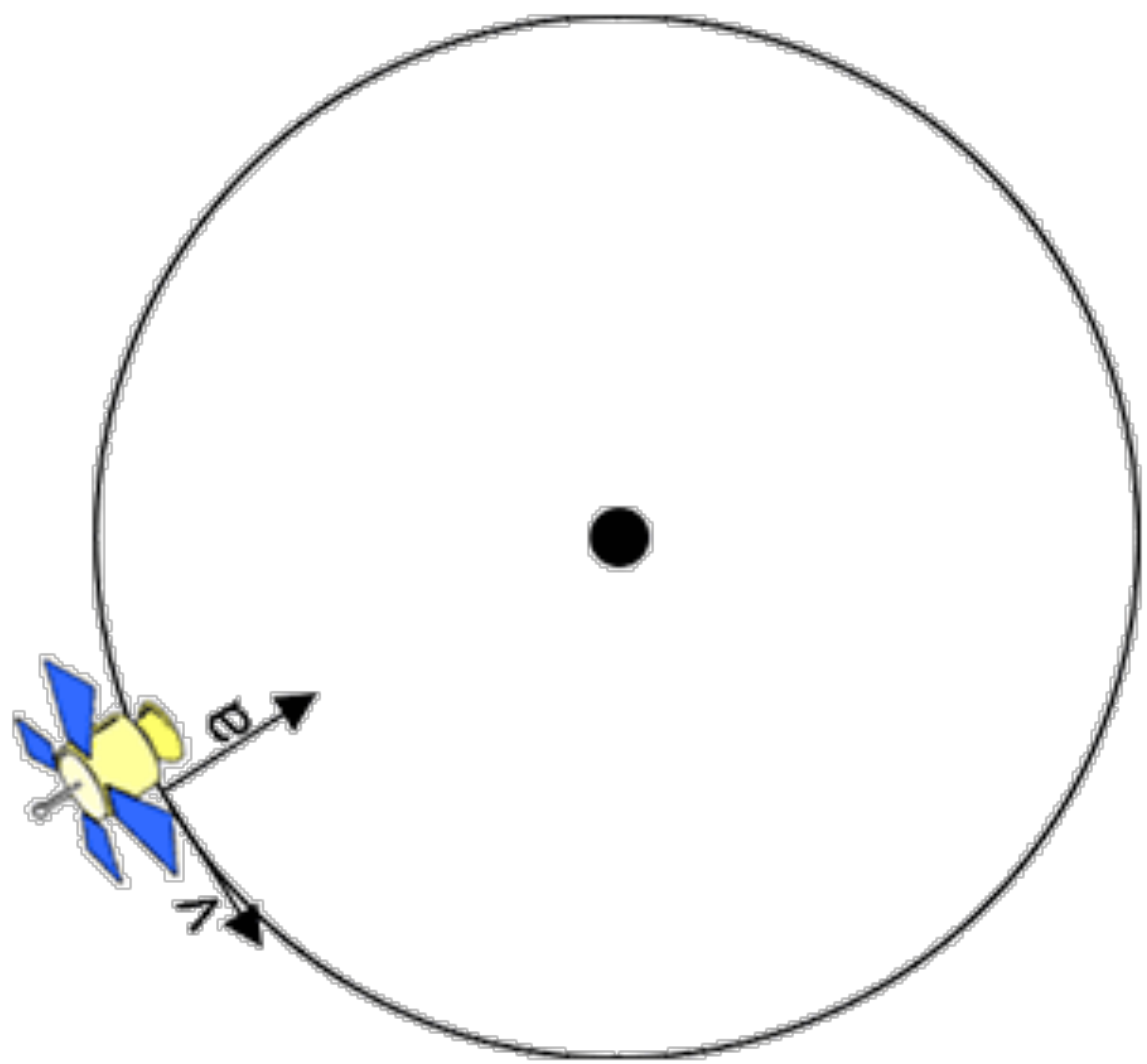
Uniform Circular Motion



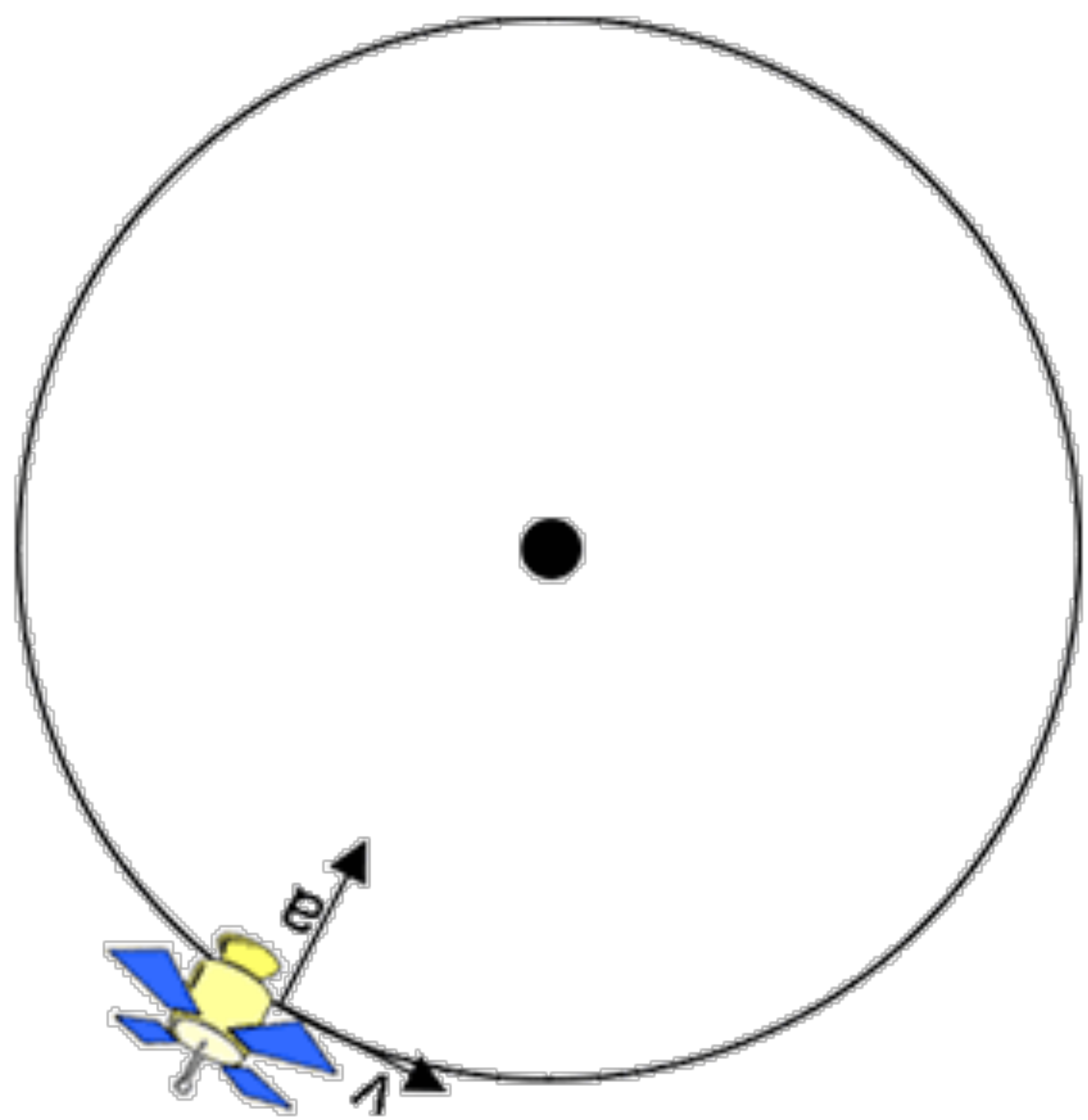
Uniform Circular Motion



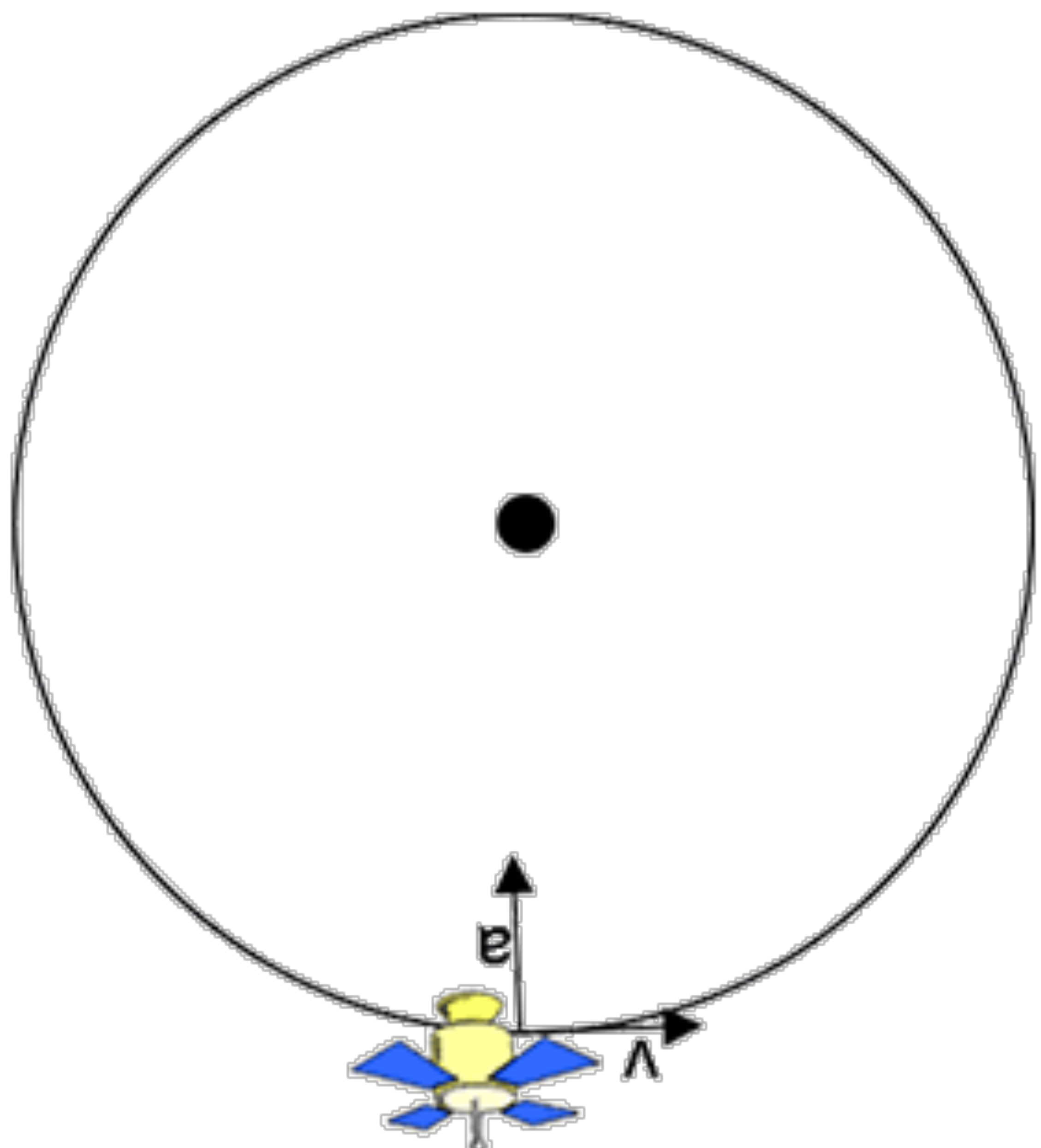
Uniform Circular Motion



Uniform Circular Motion



Uniform Circular Motion



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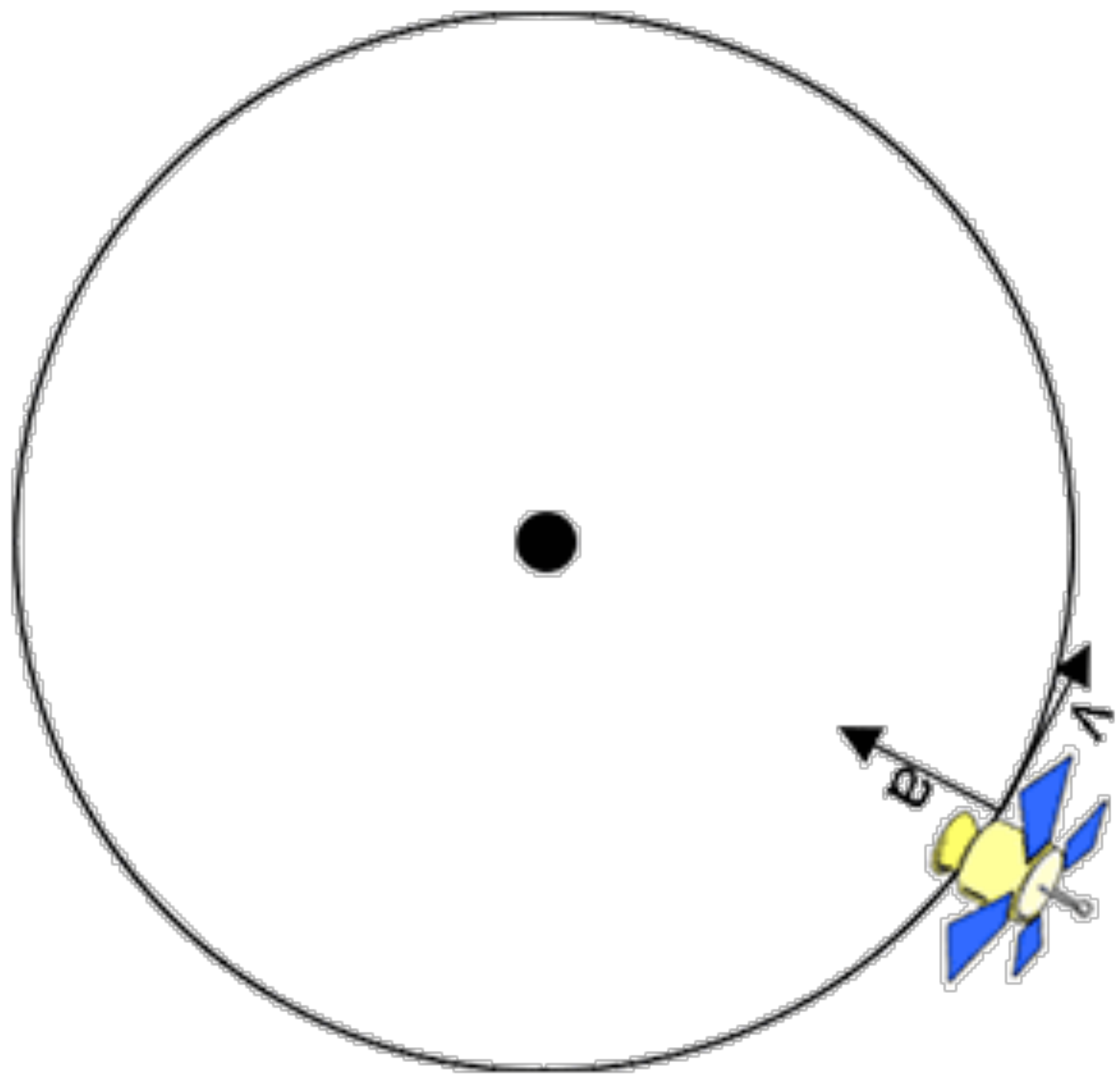

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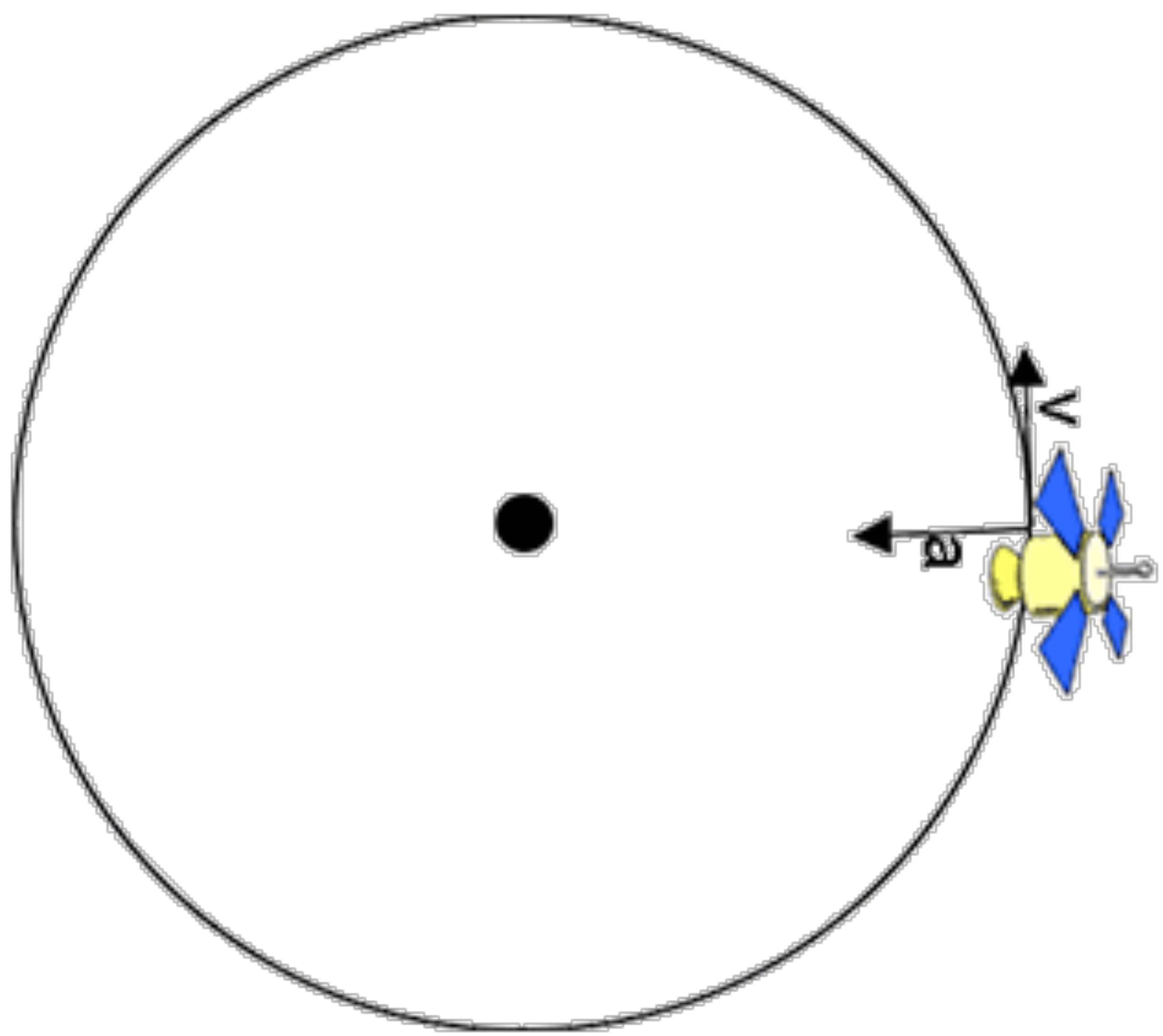
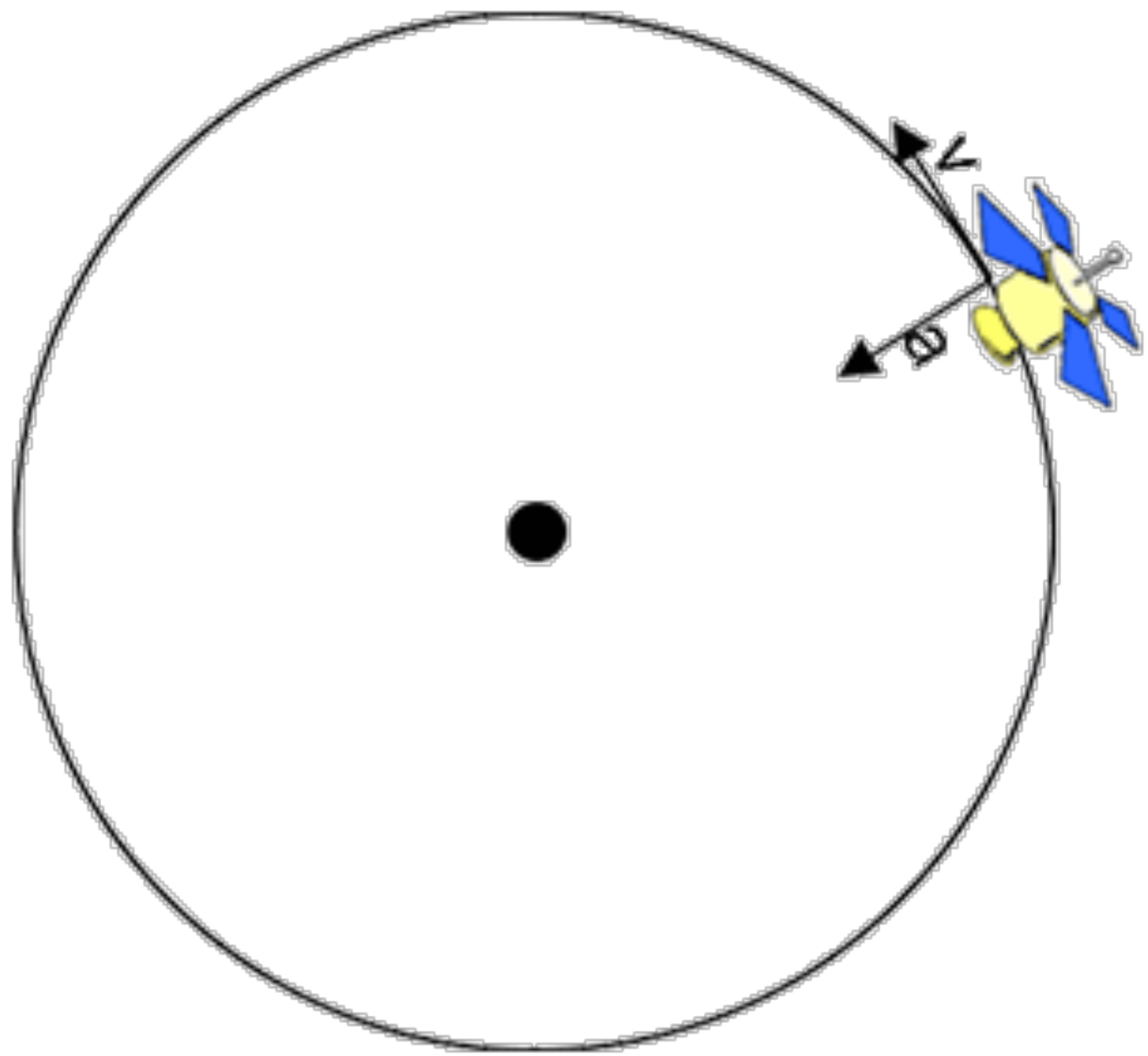
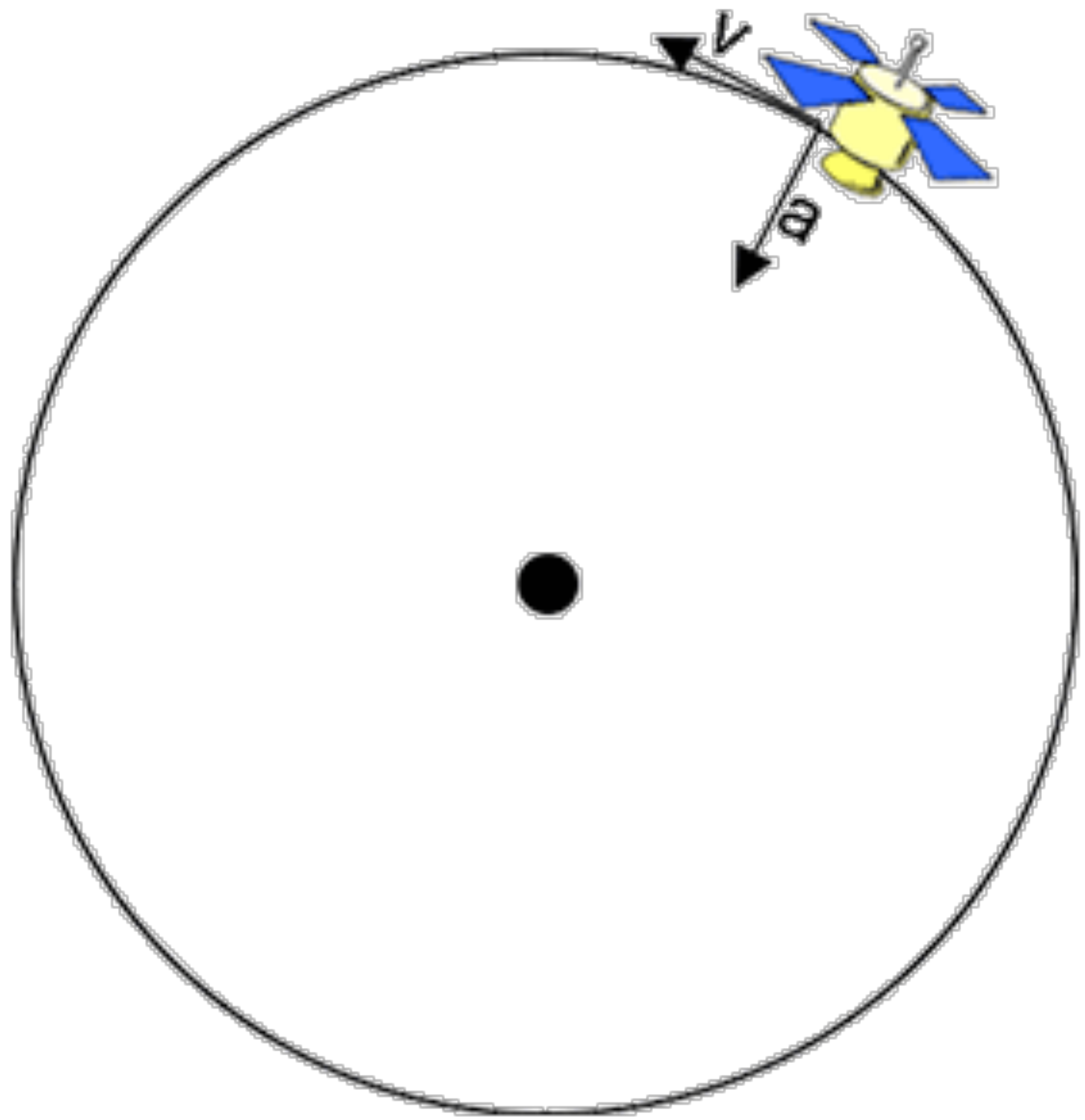


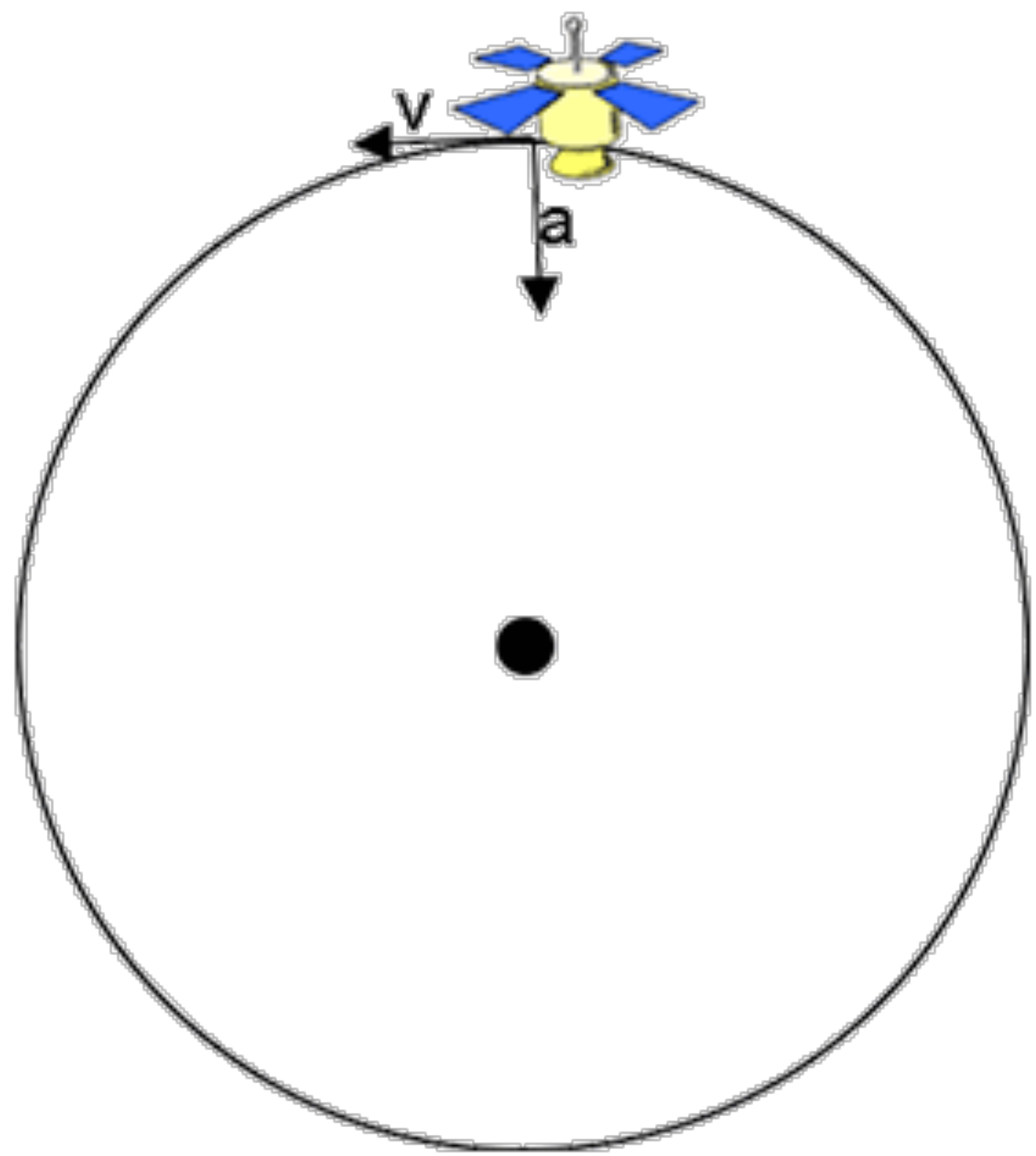
Uniform Circular Motion



Uniform Circular Motion



Uniform Circular Motion



Description of orbits

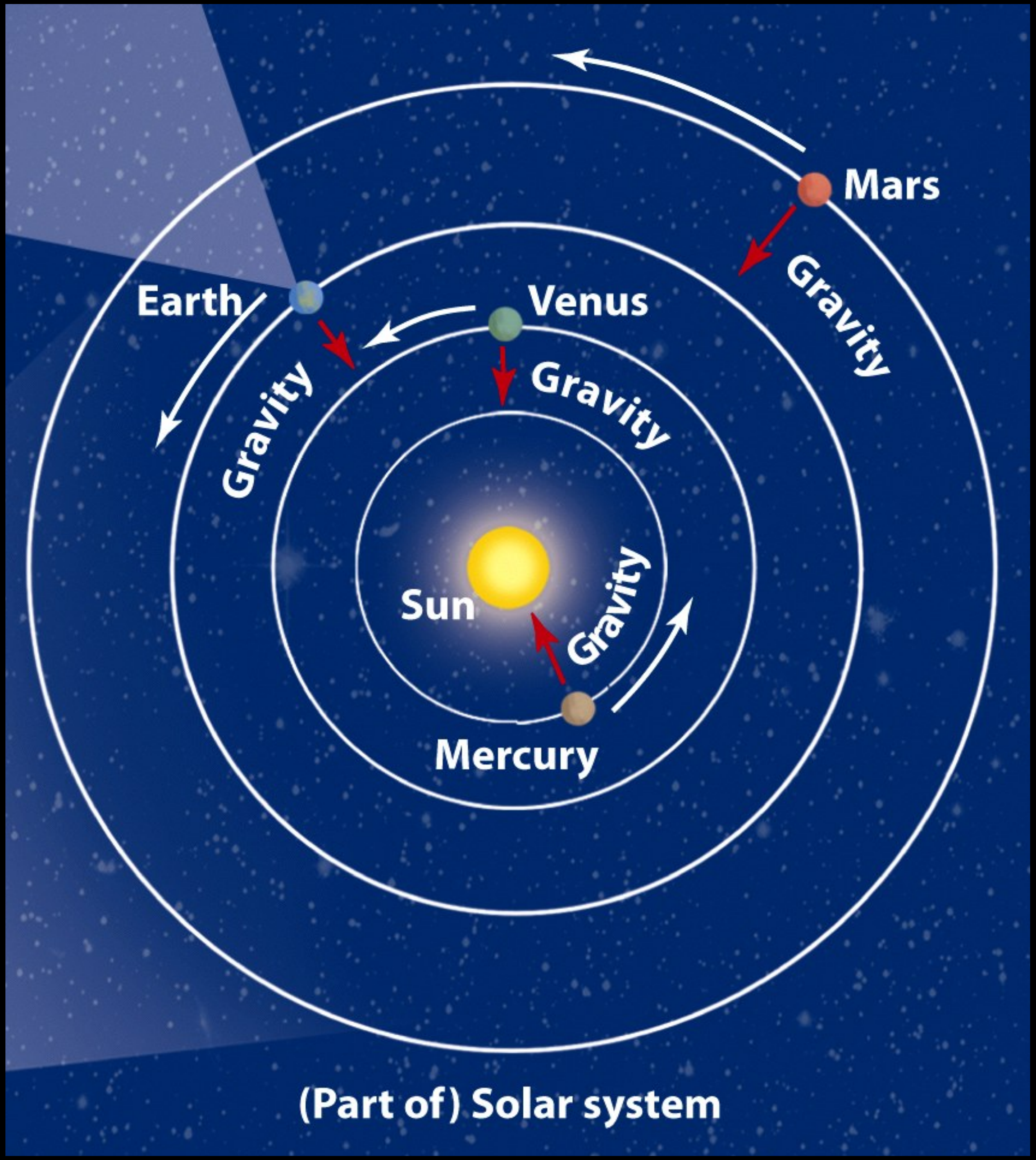
- The Sun's gravitational force pulls the planets towards it

At the same time the planet exerts a force in the opposite direction and pulls the Sun towards it

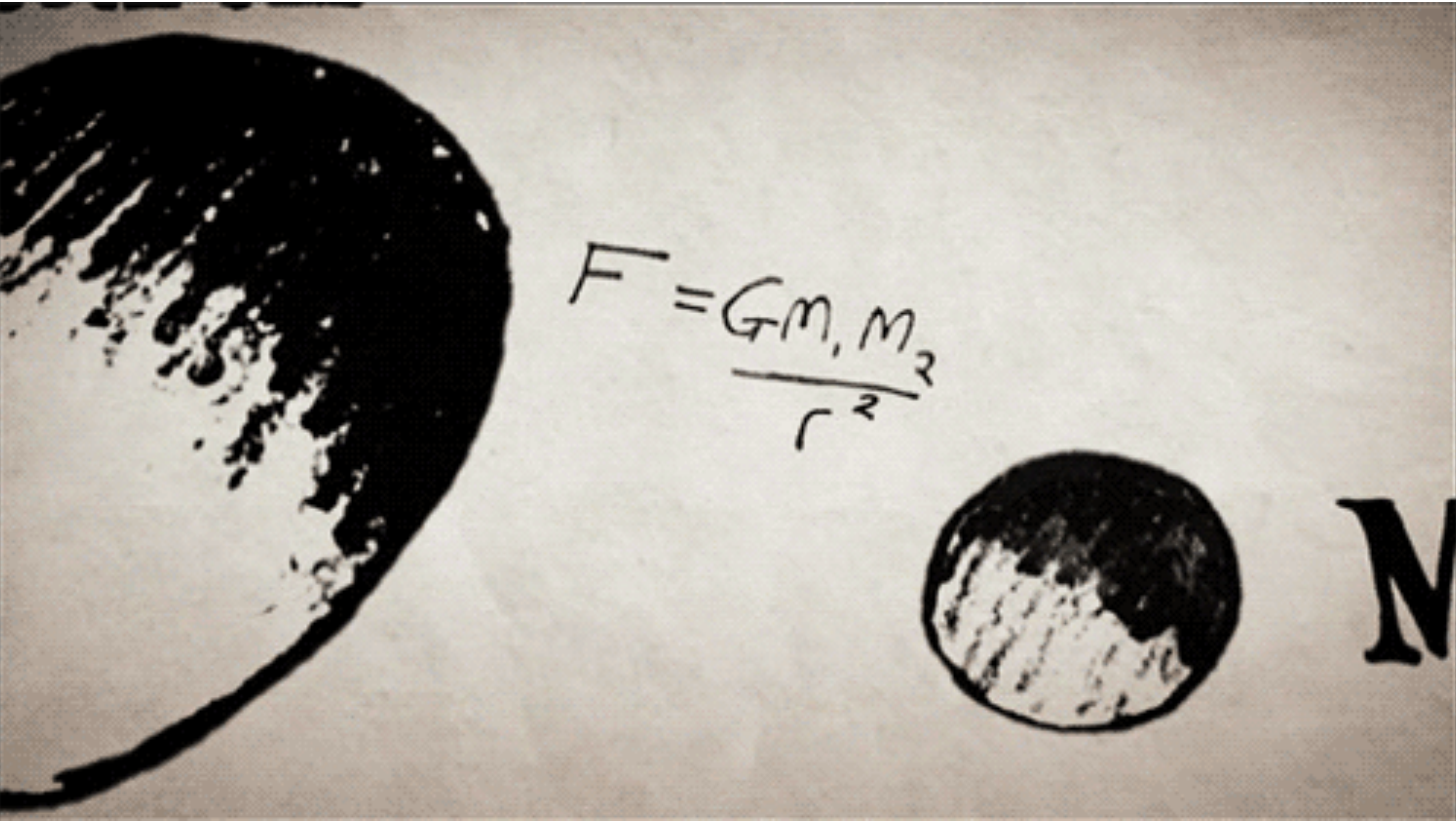
Newton's 3rd Law

- If the Sun was not there to divert the motion of the planet, it would continue in a straight line

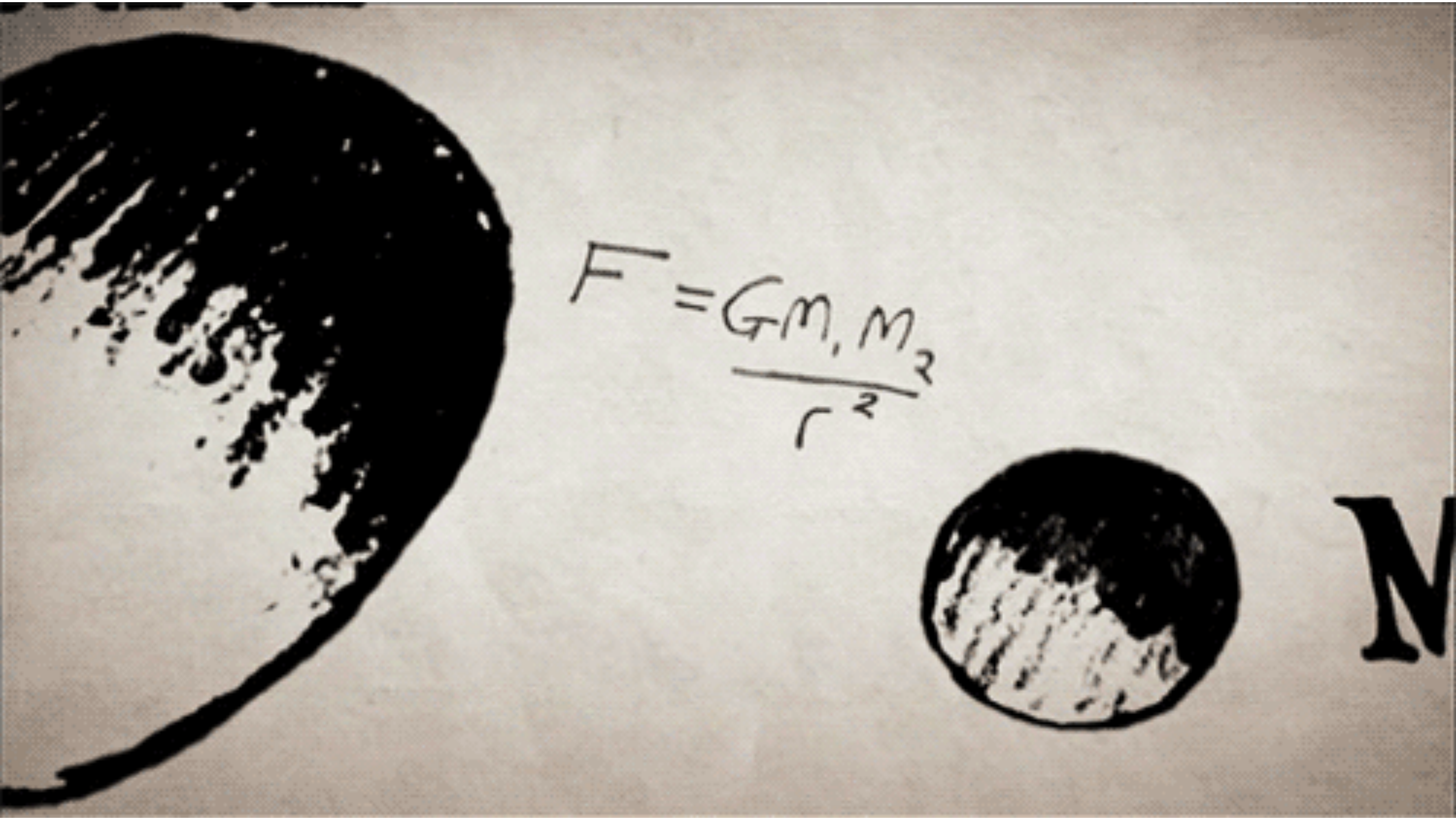
Newton's 1st Law



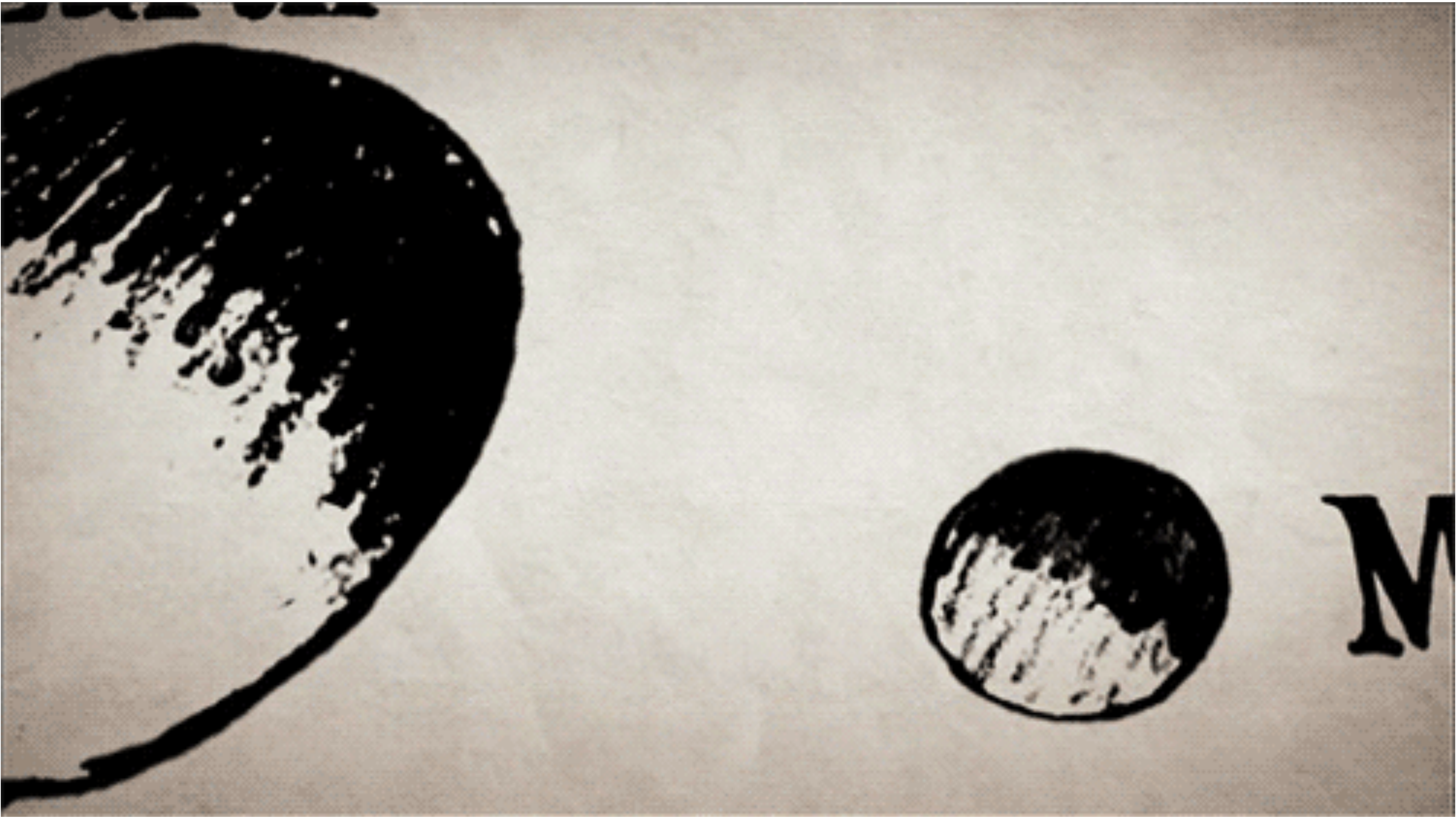
(Part of) Solar system

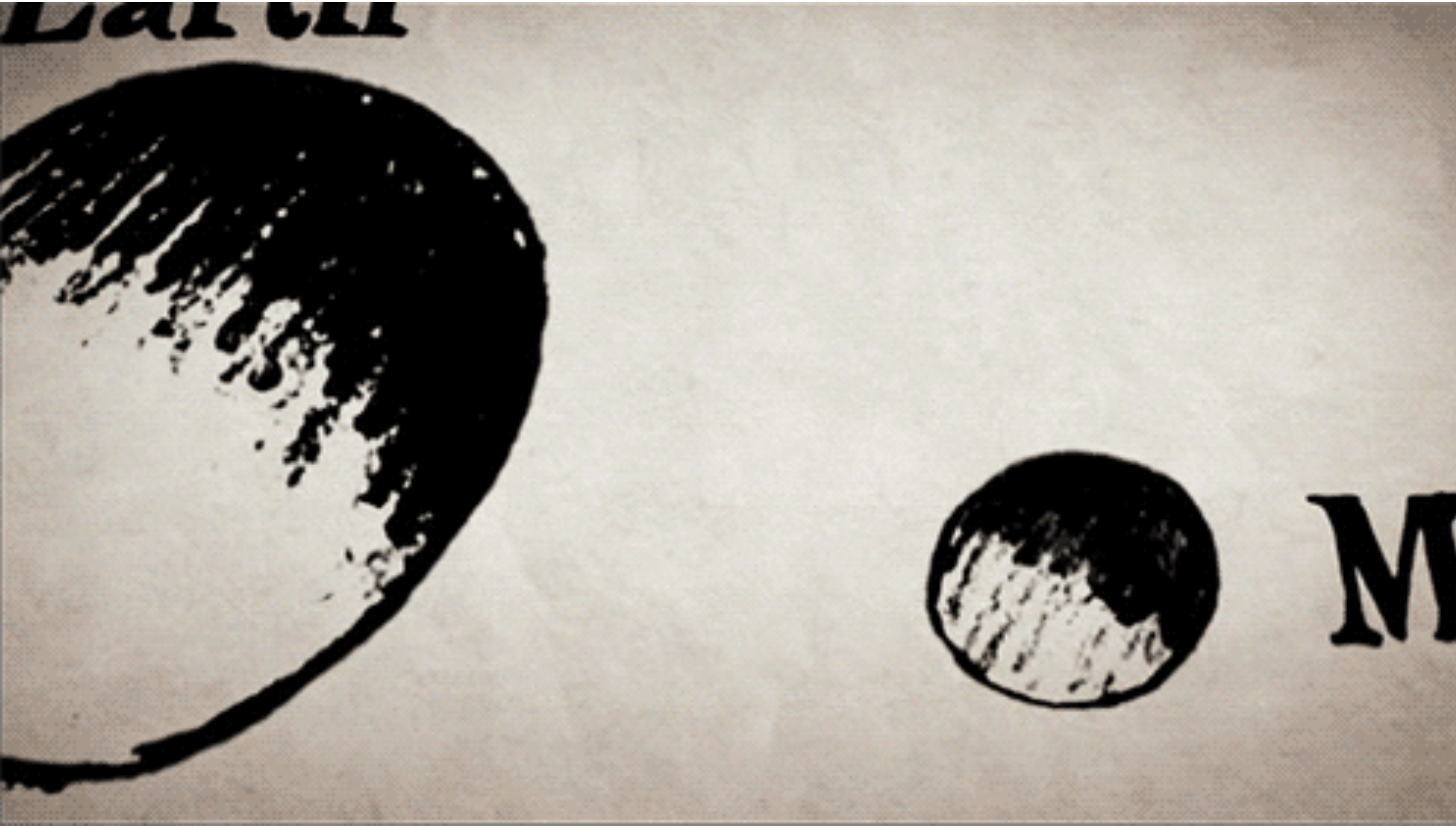


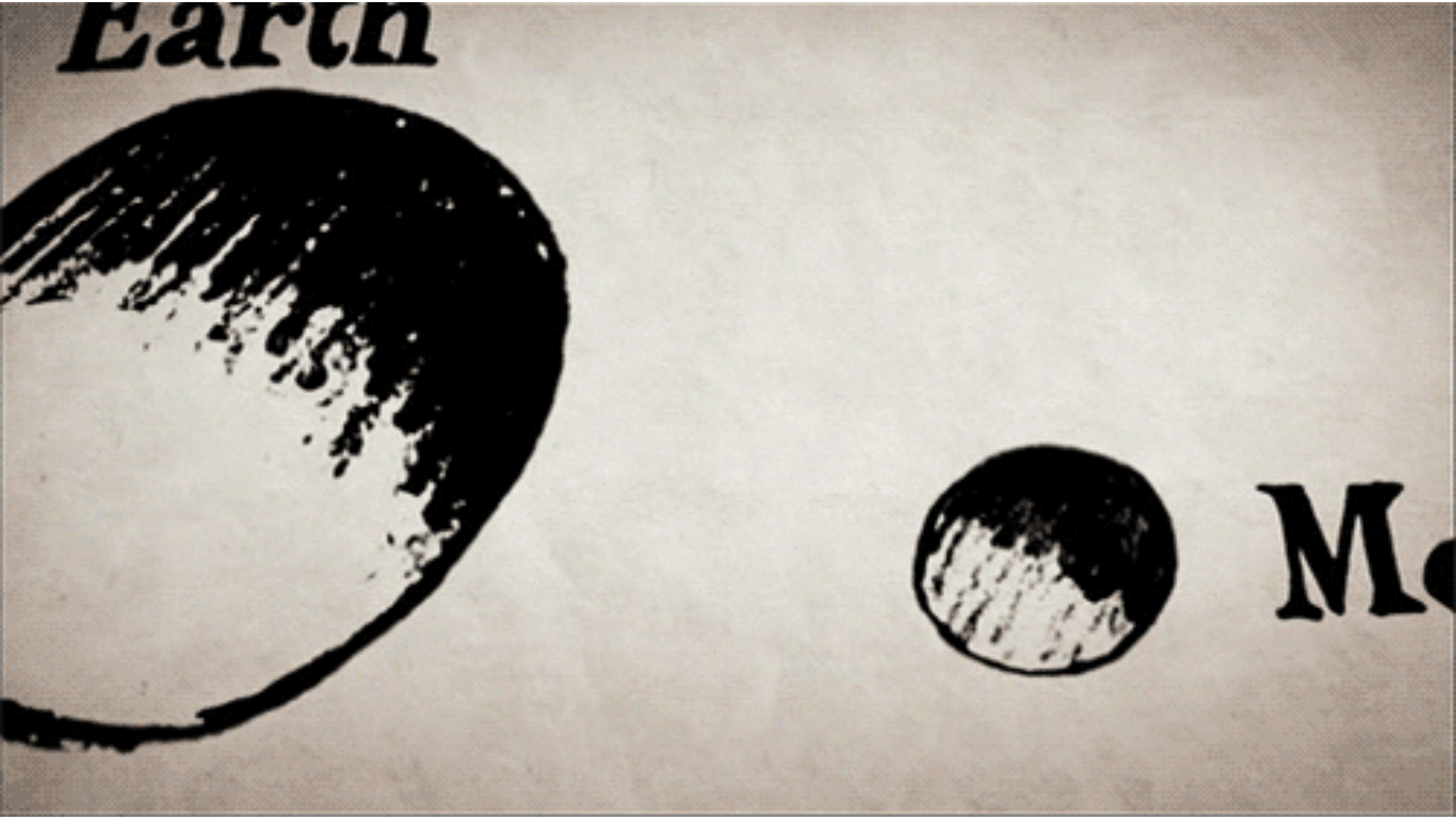
$F = \frac{Gm_1 m_2}{r^2}$

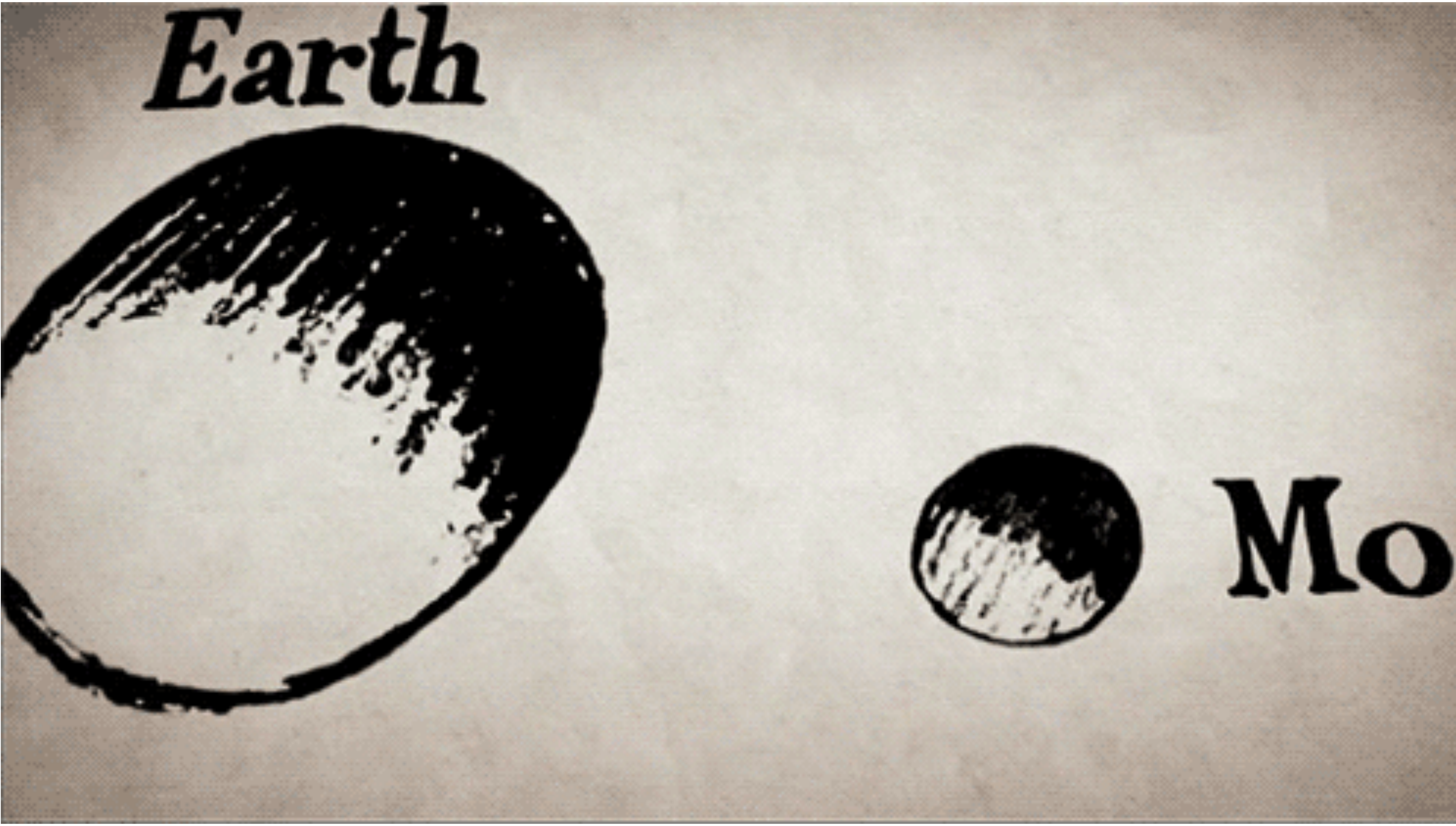


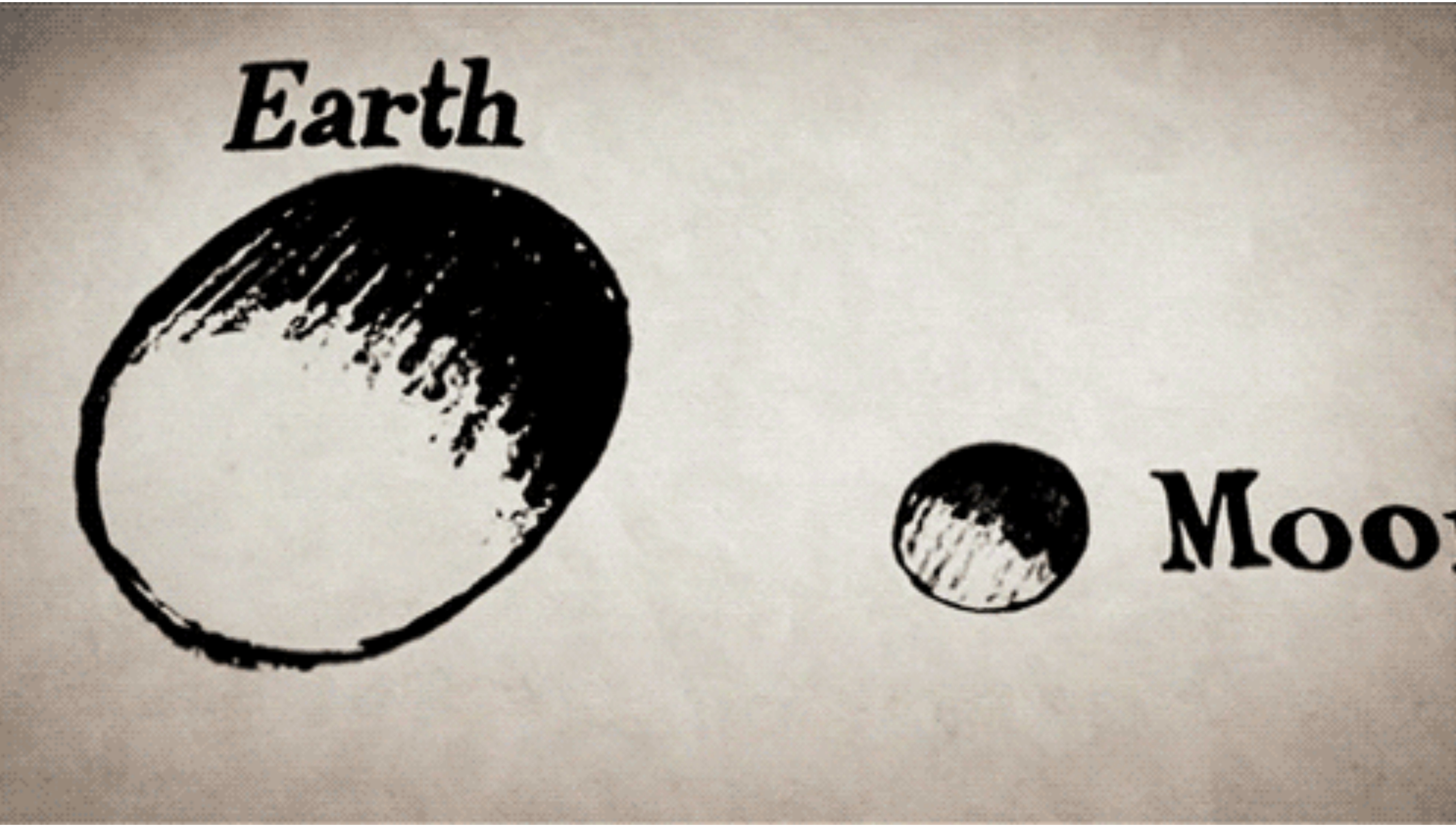
$F = \frac{G m_1 m_2}{r^2}$

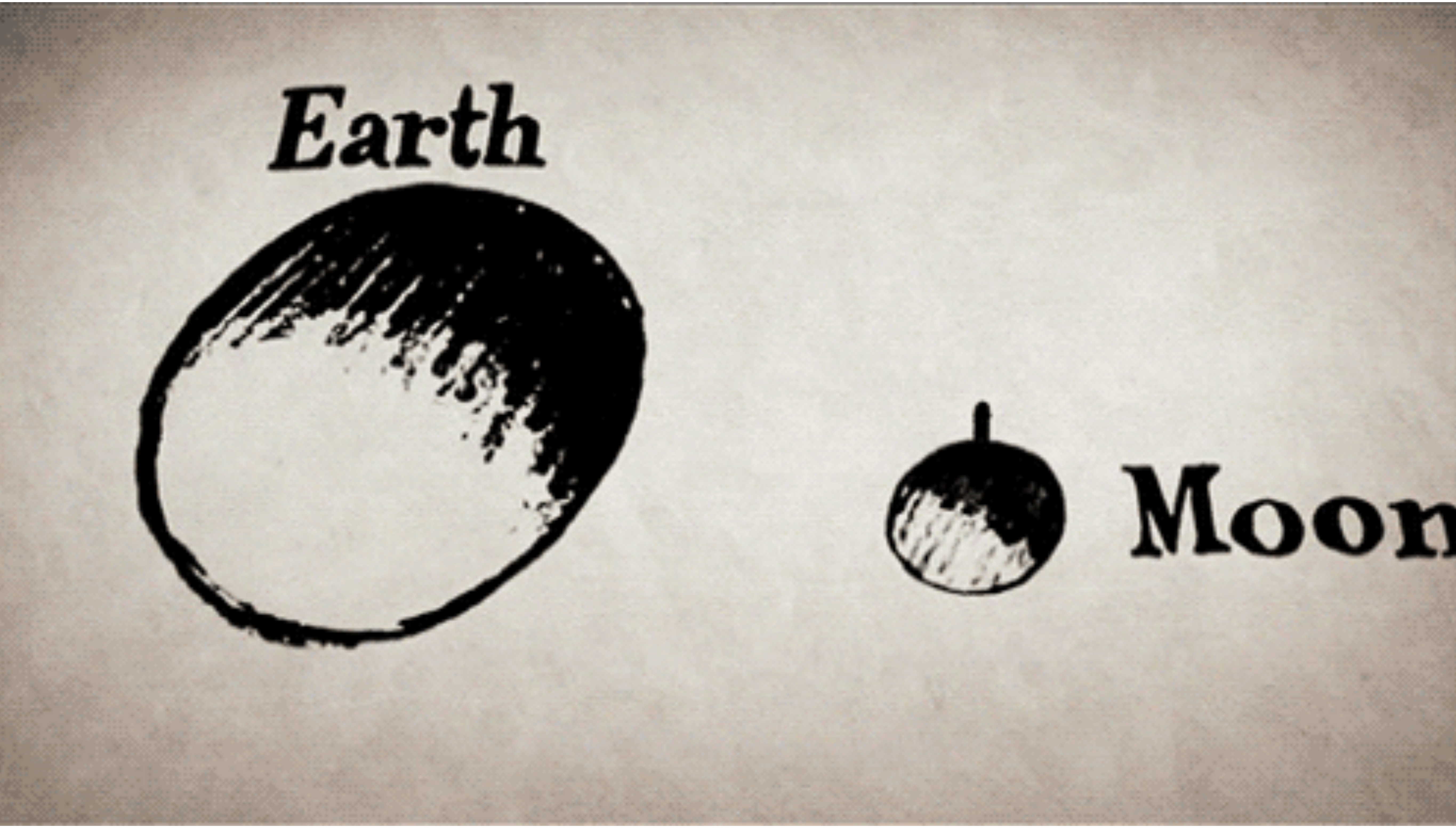


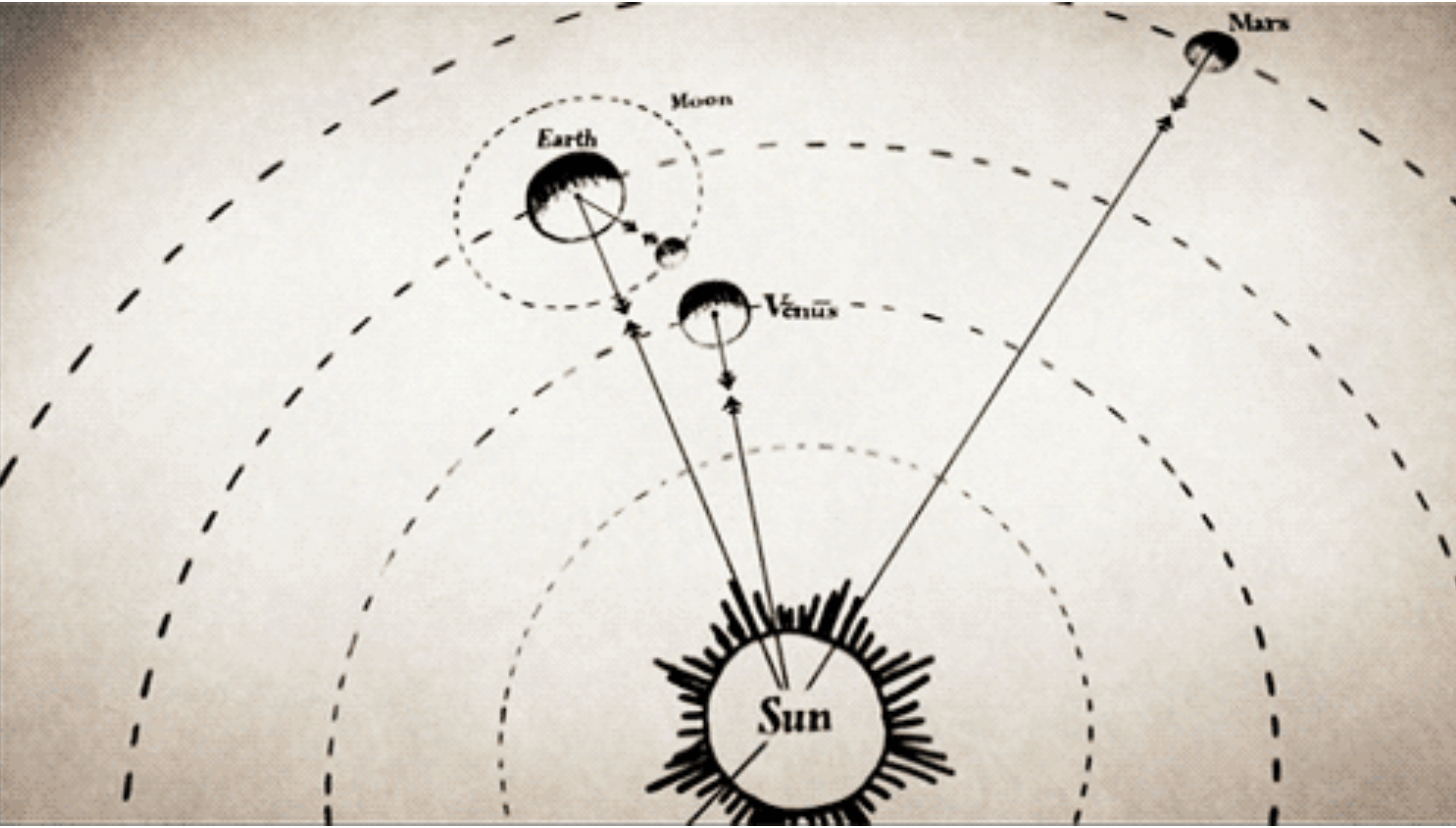


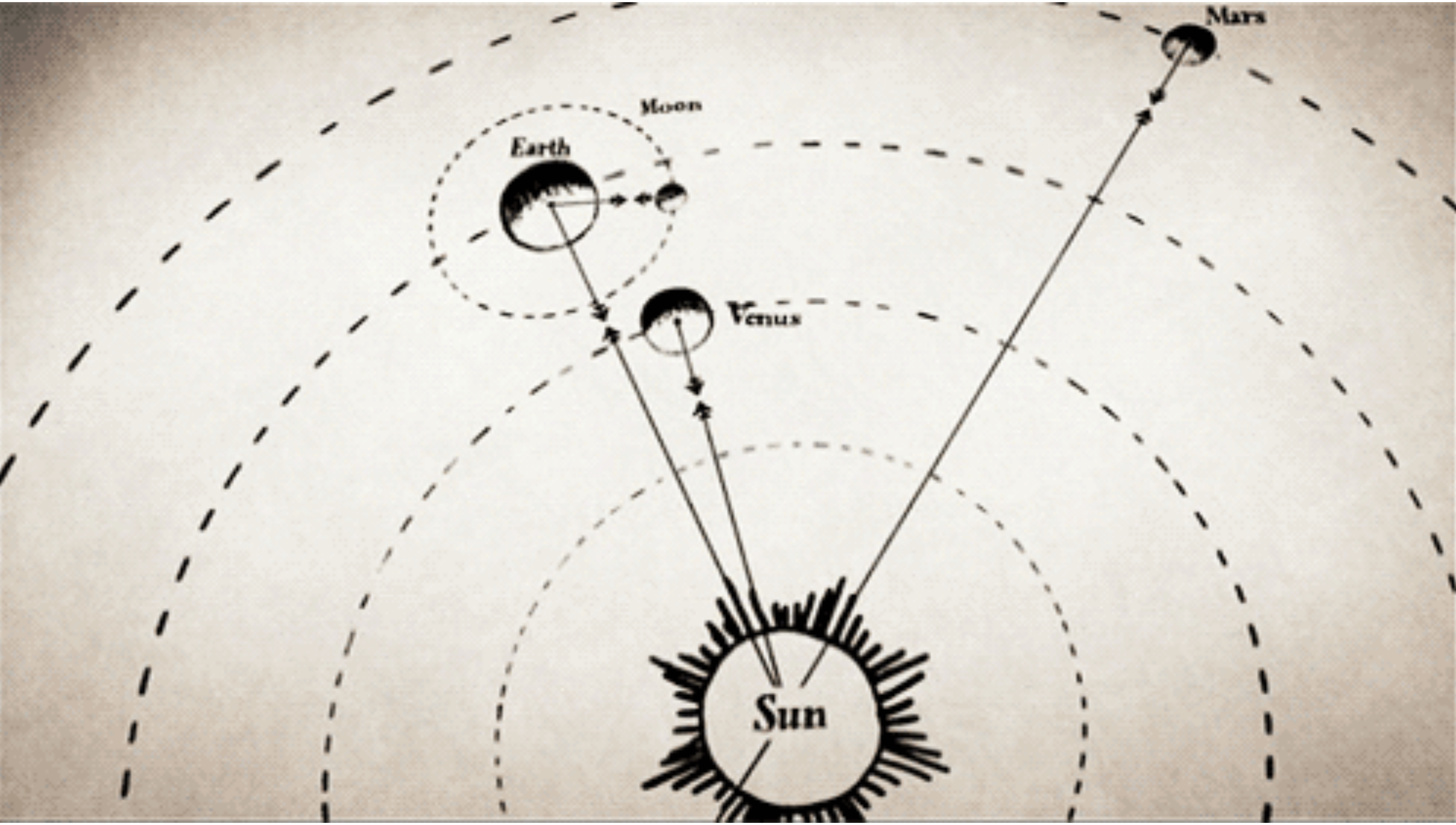

Earth

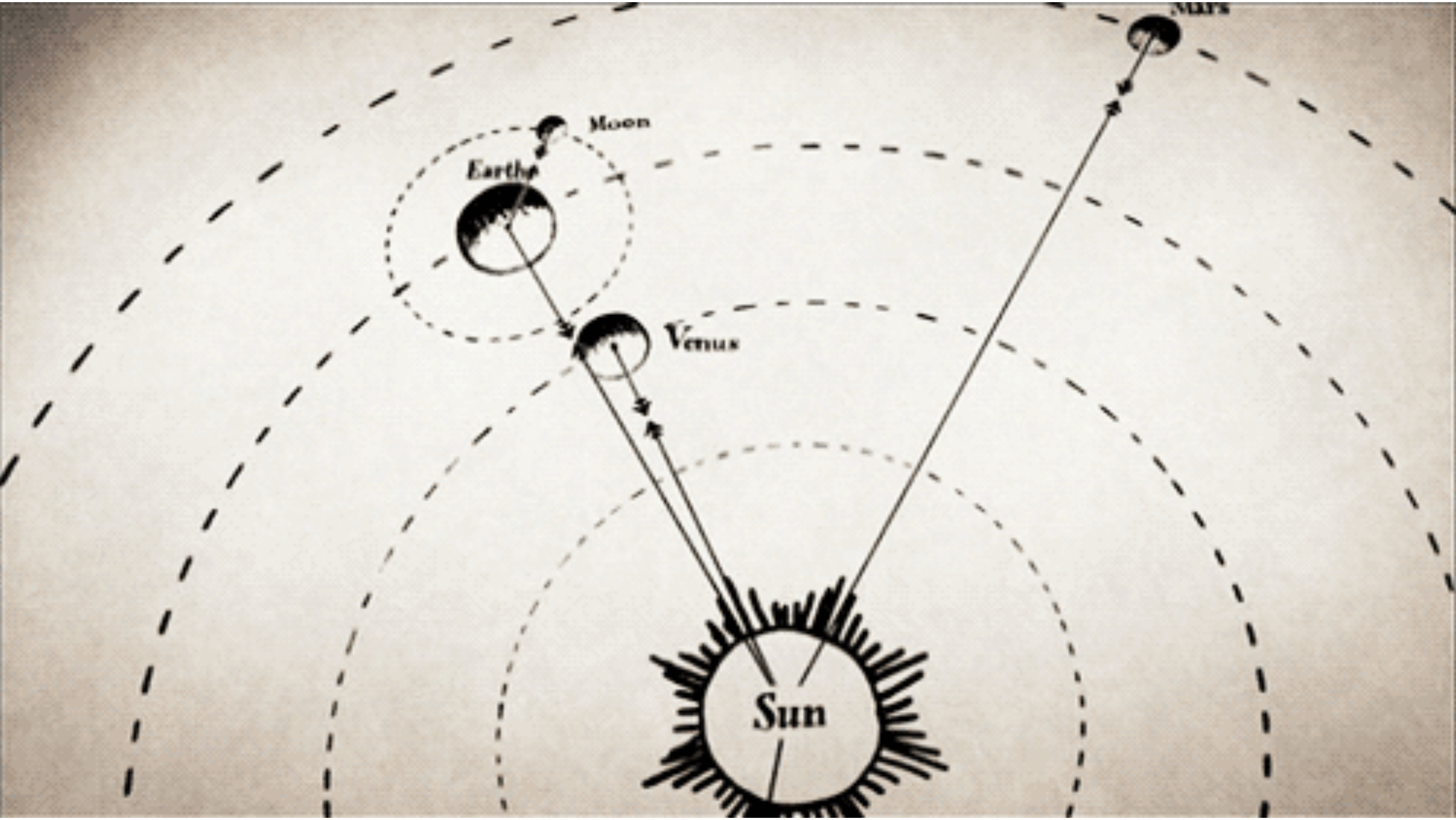

Earth
Mo


Earth
Moo


Earth
Moon


Mars
Moon
Earth
Venus
Sun


Mars
Moon
Earth
Venus
Sun


Mars
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Earth
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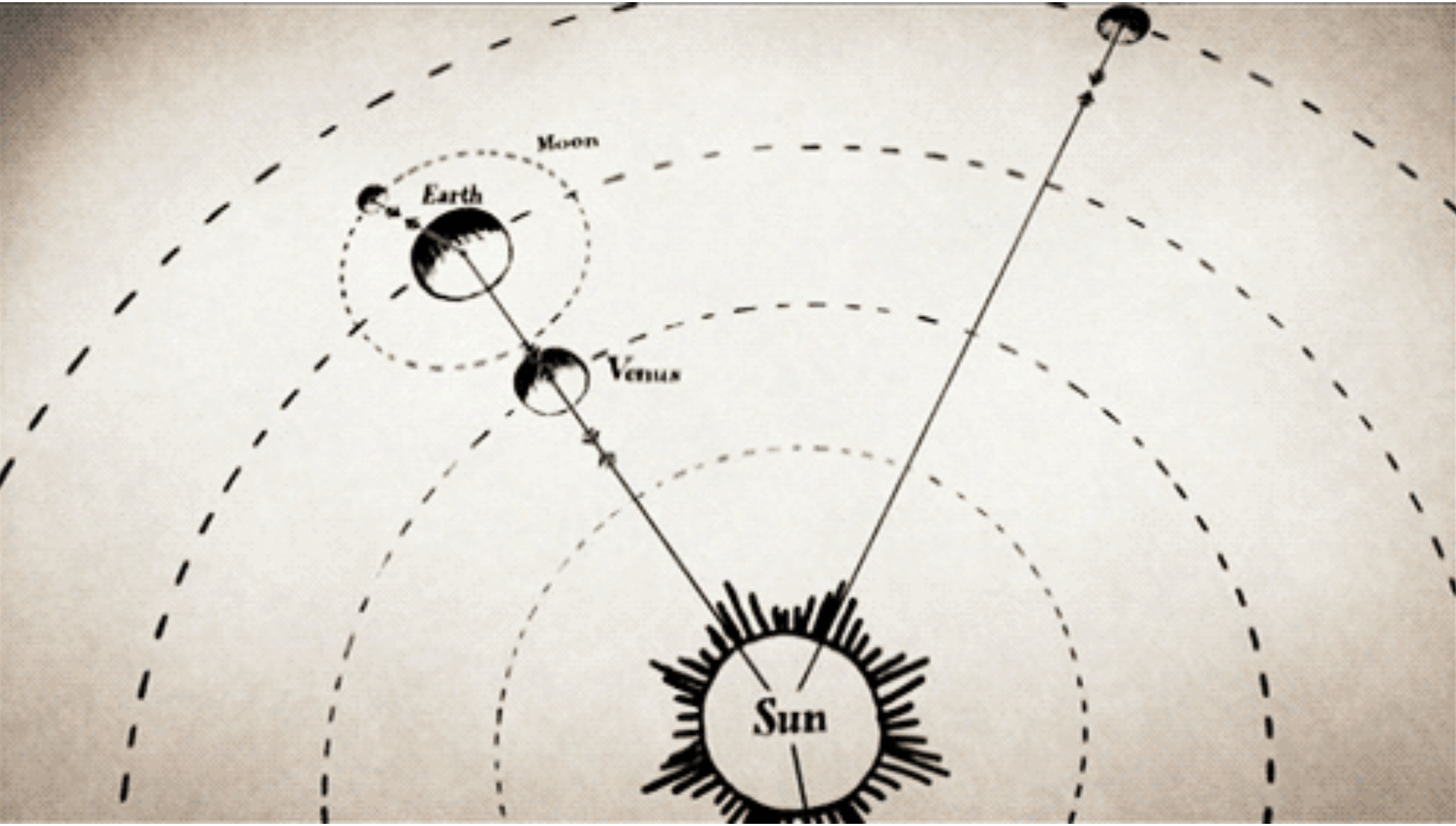
Moon
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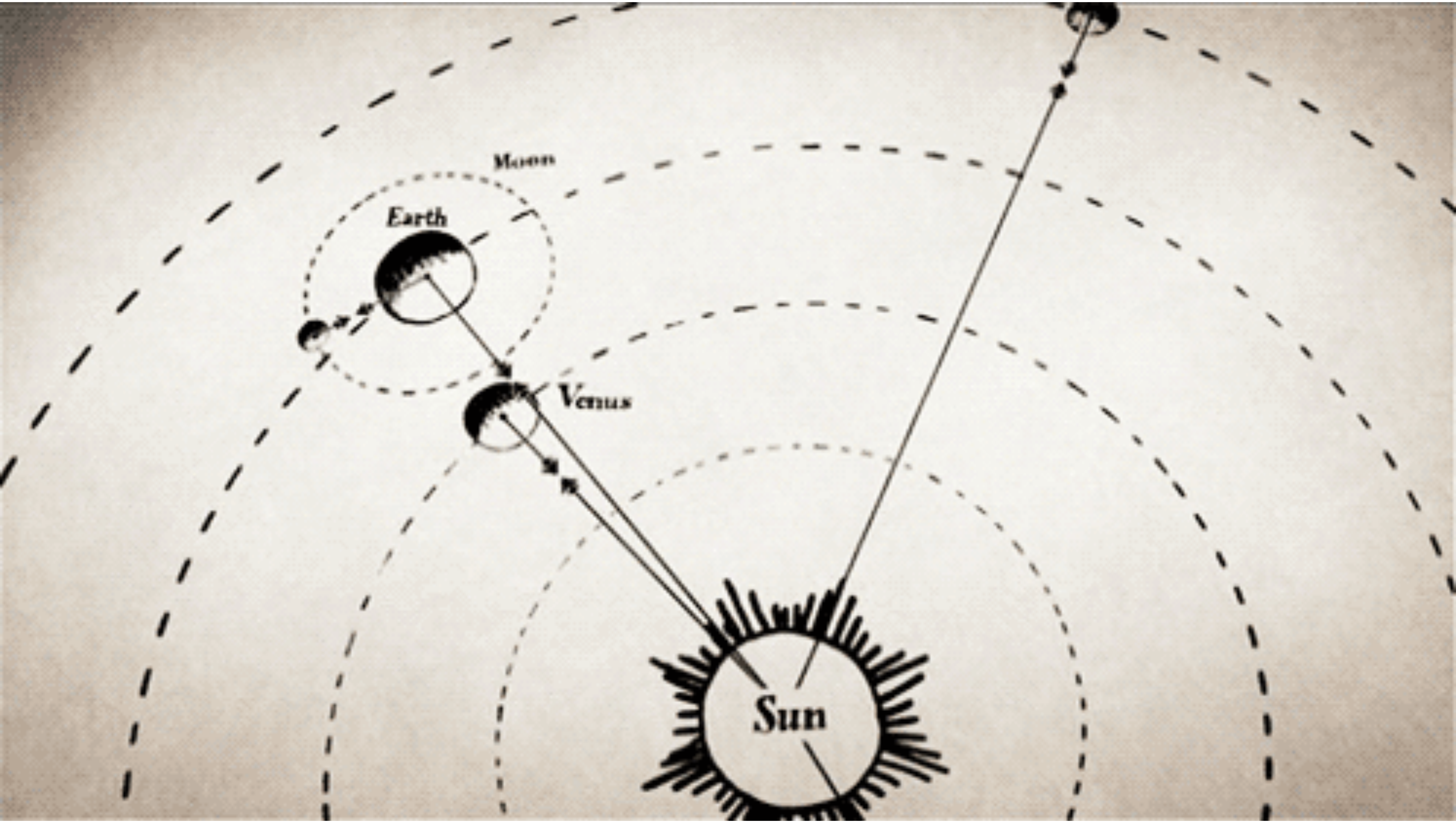
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Venus
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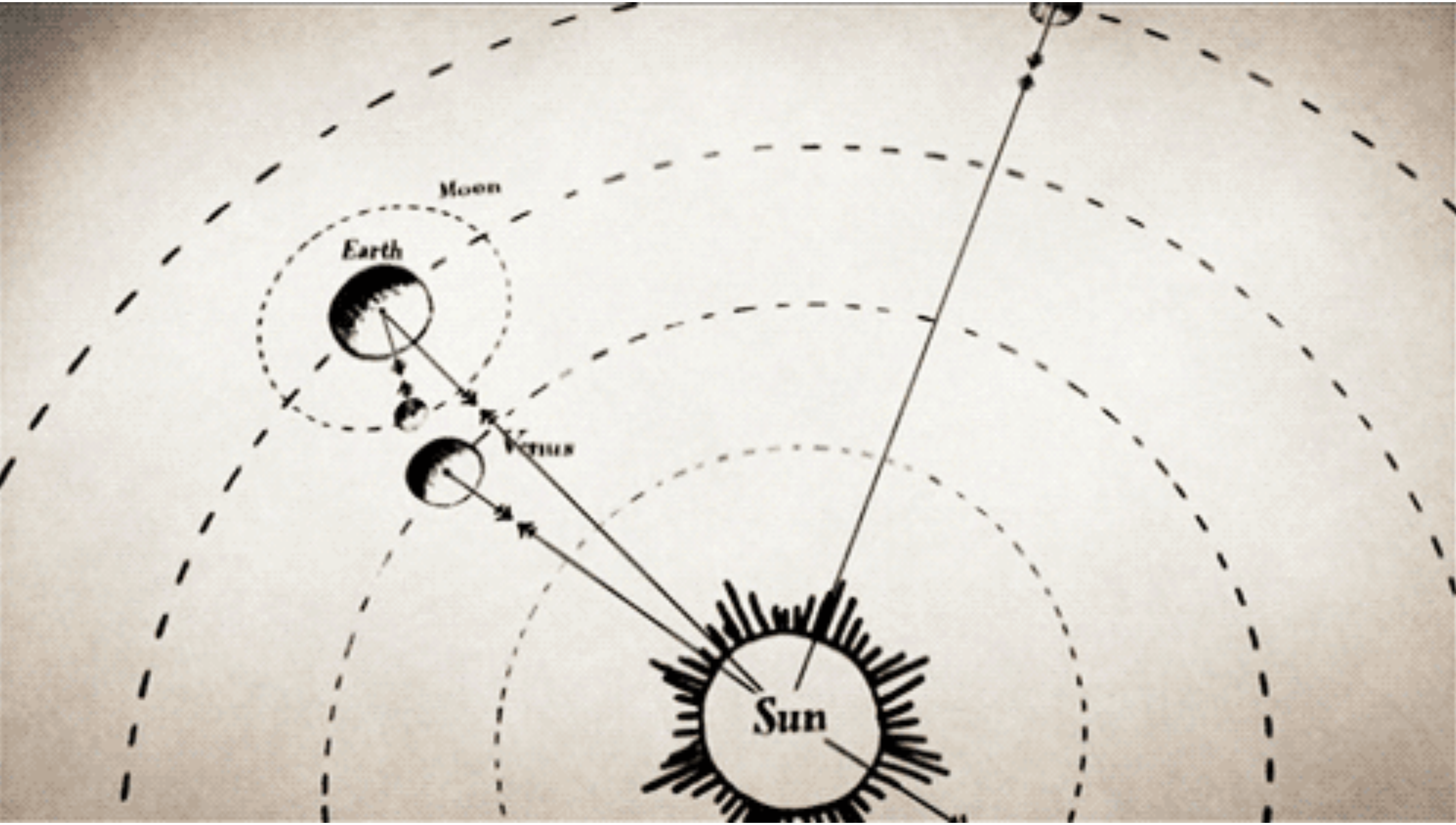
Moon
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Venus
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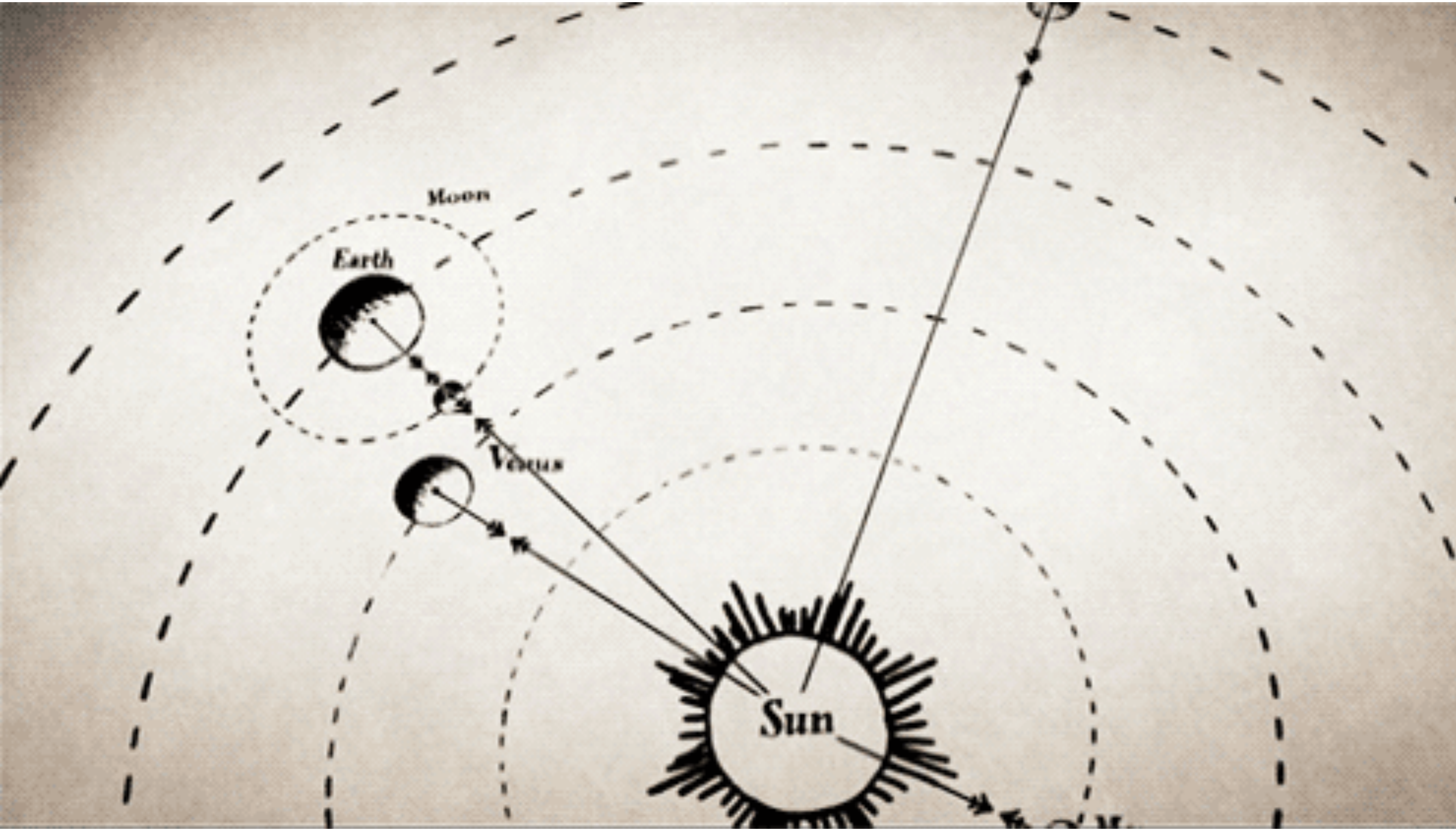
Moon
Earth
Venus
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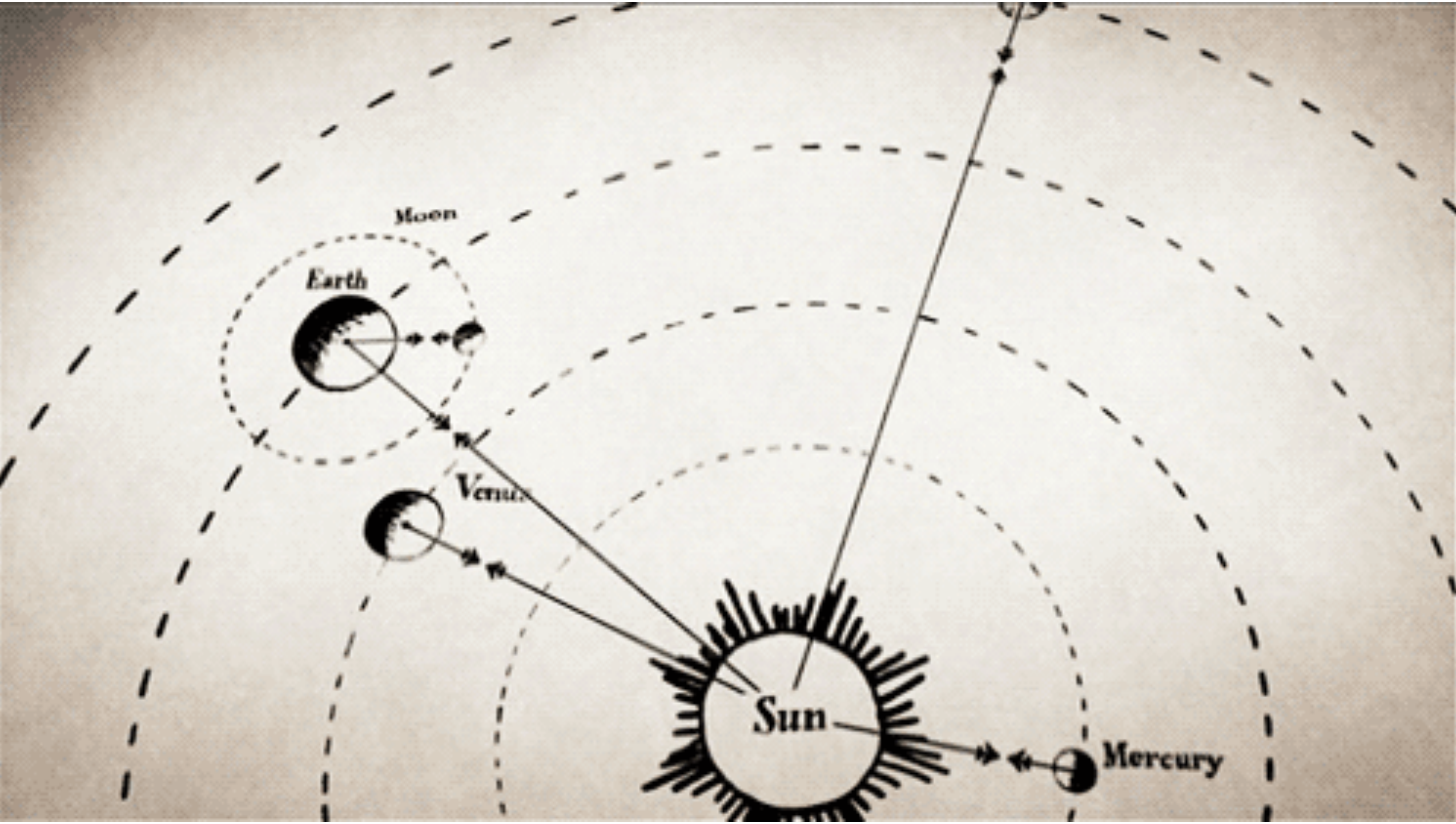
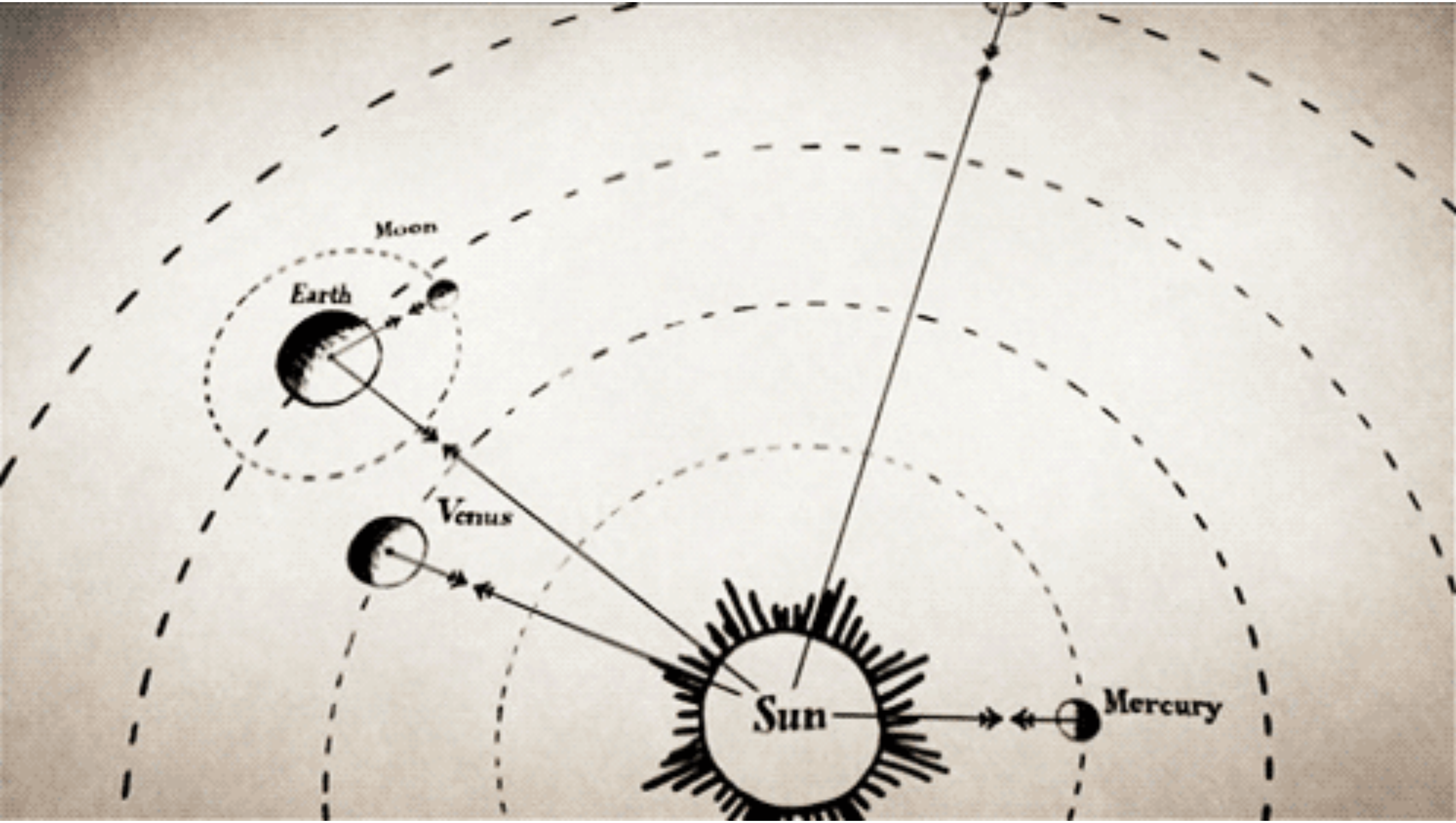

Moon
Earth
Venus
Sun
Mercury


Moon
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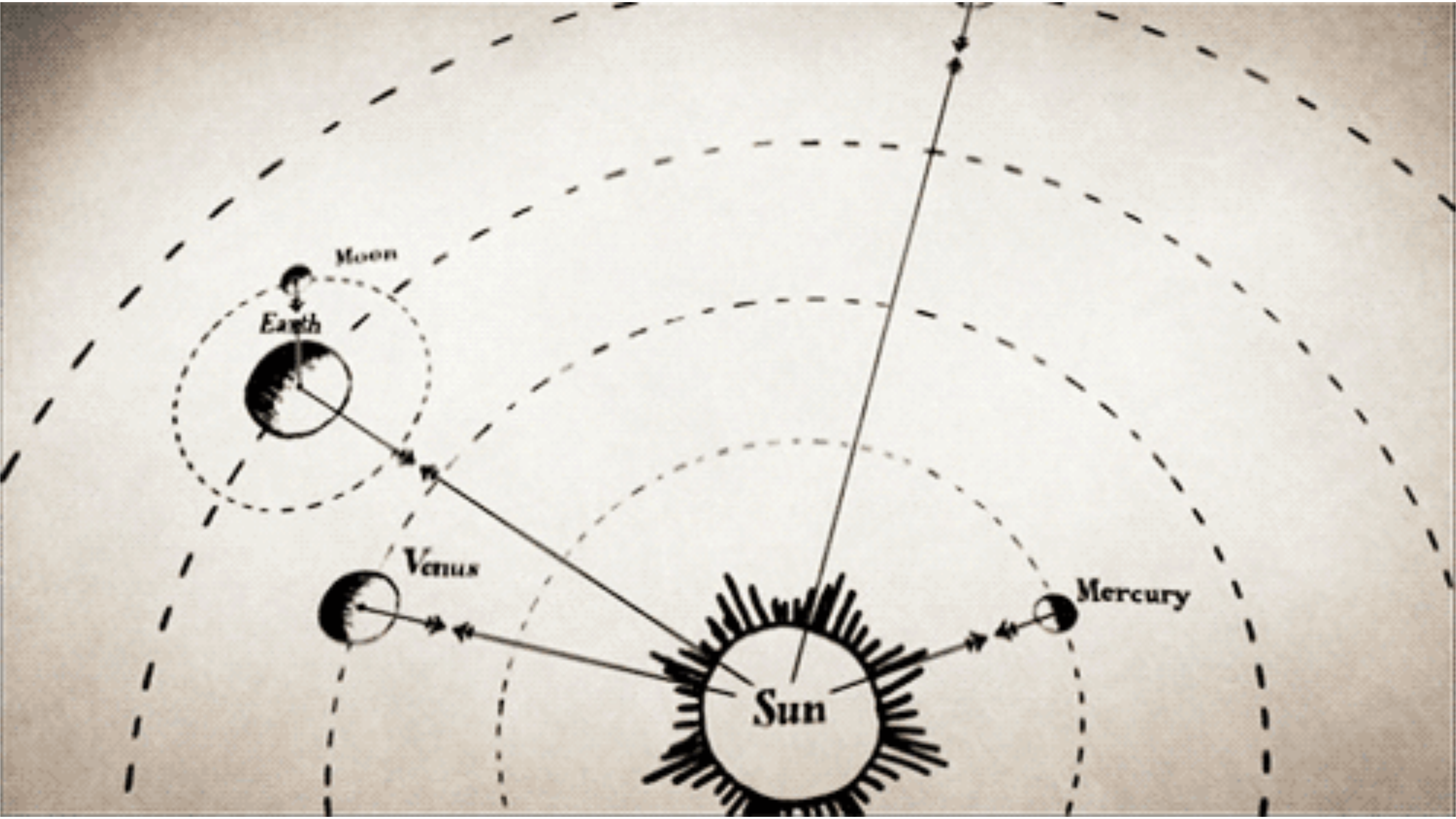
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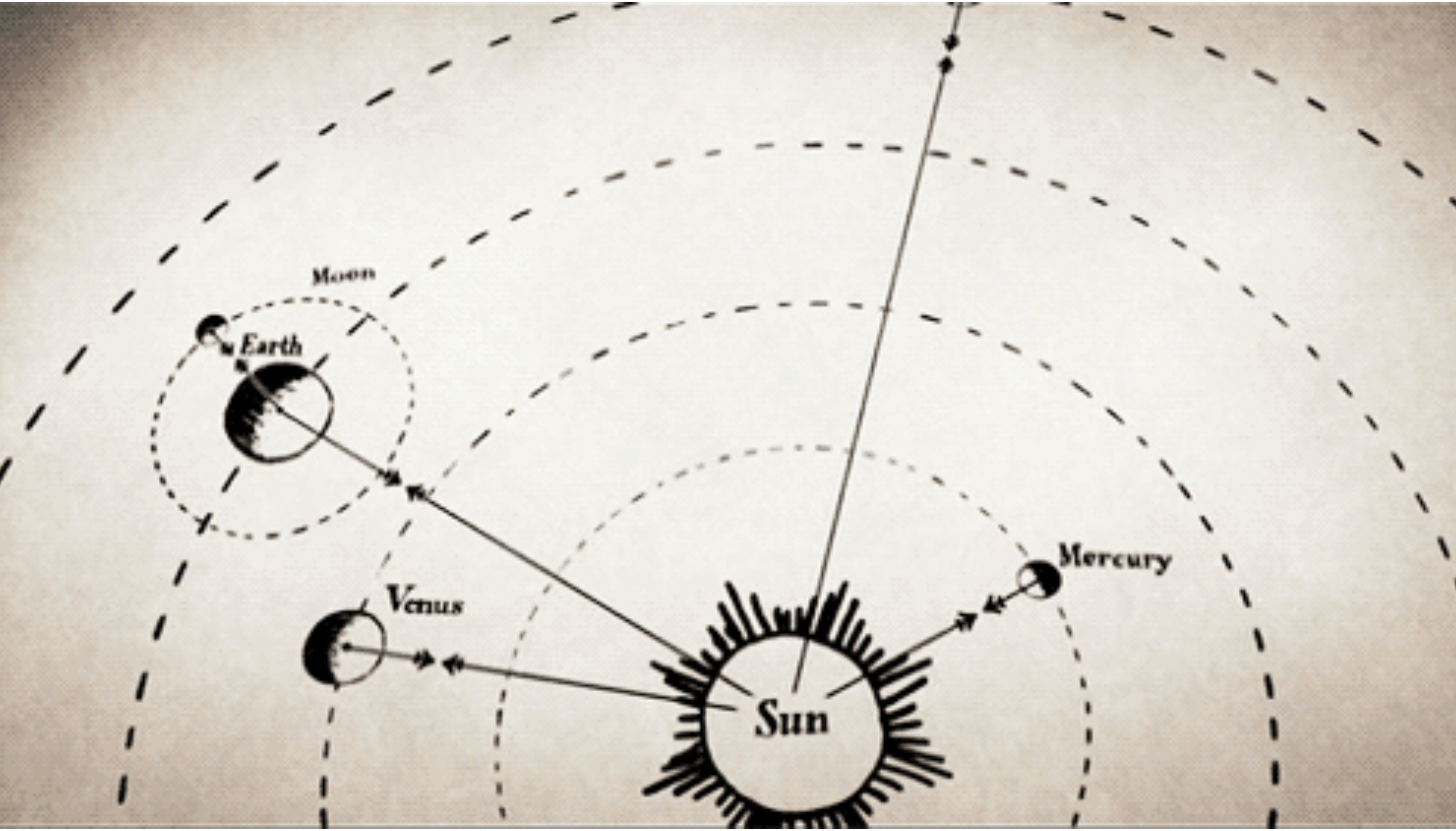
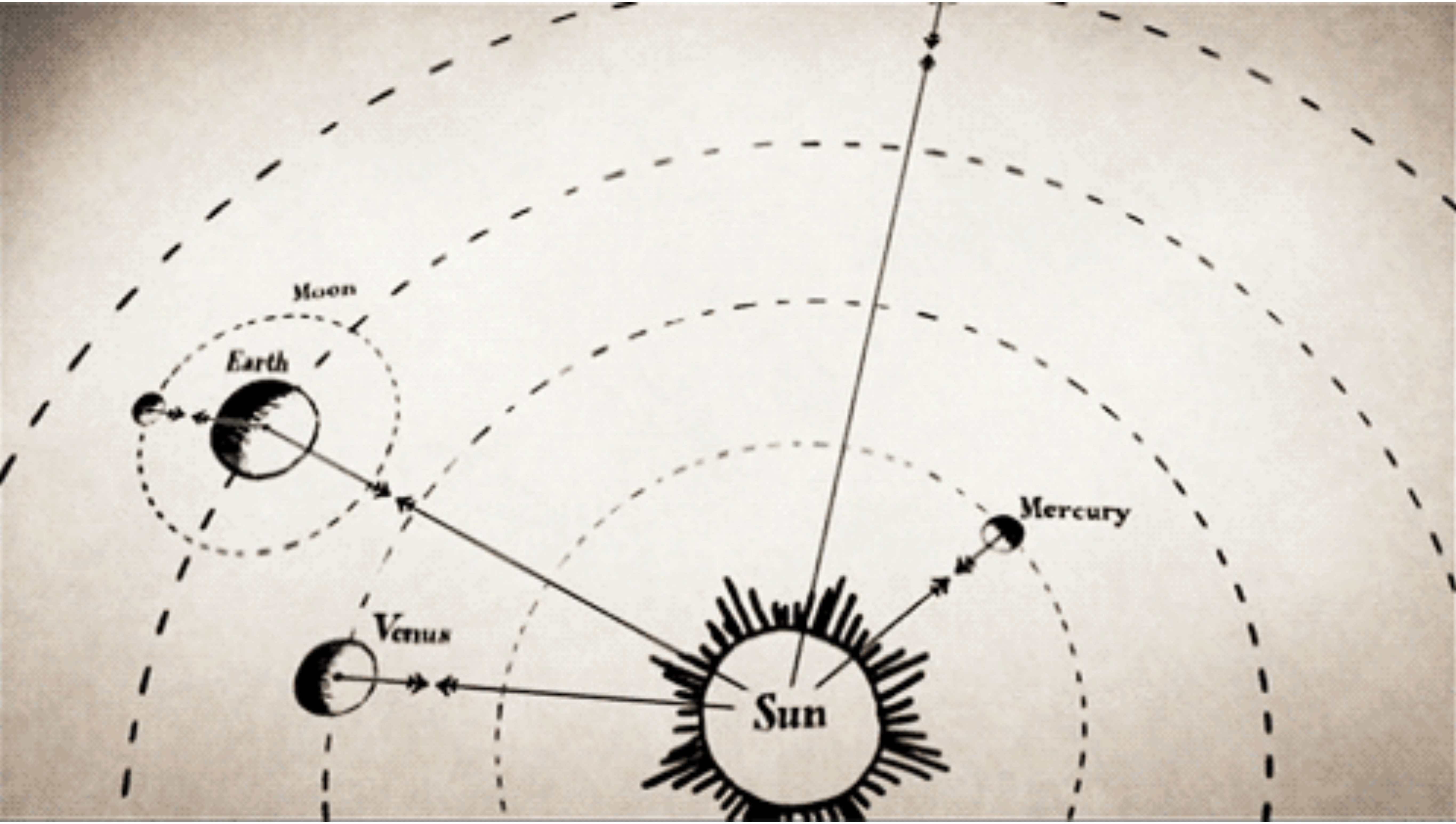
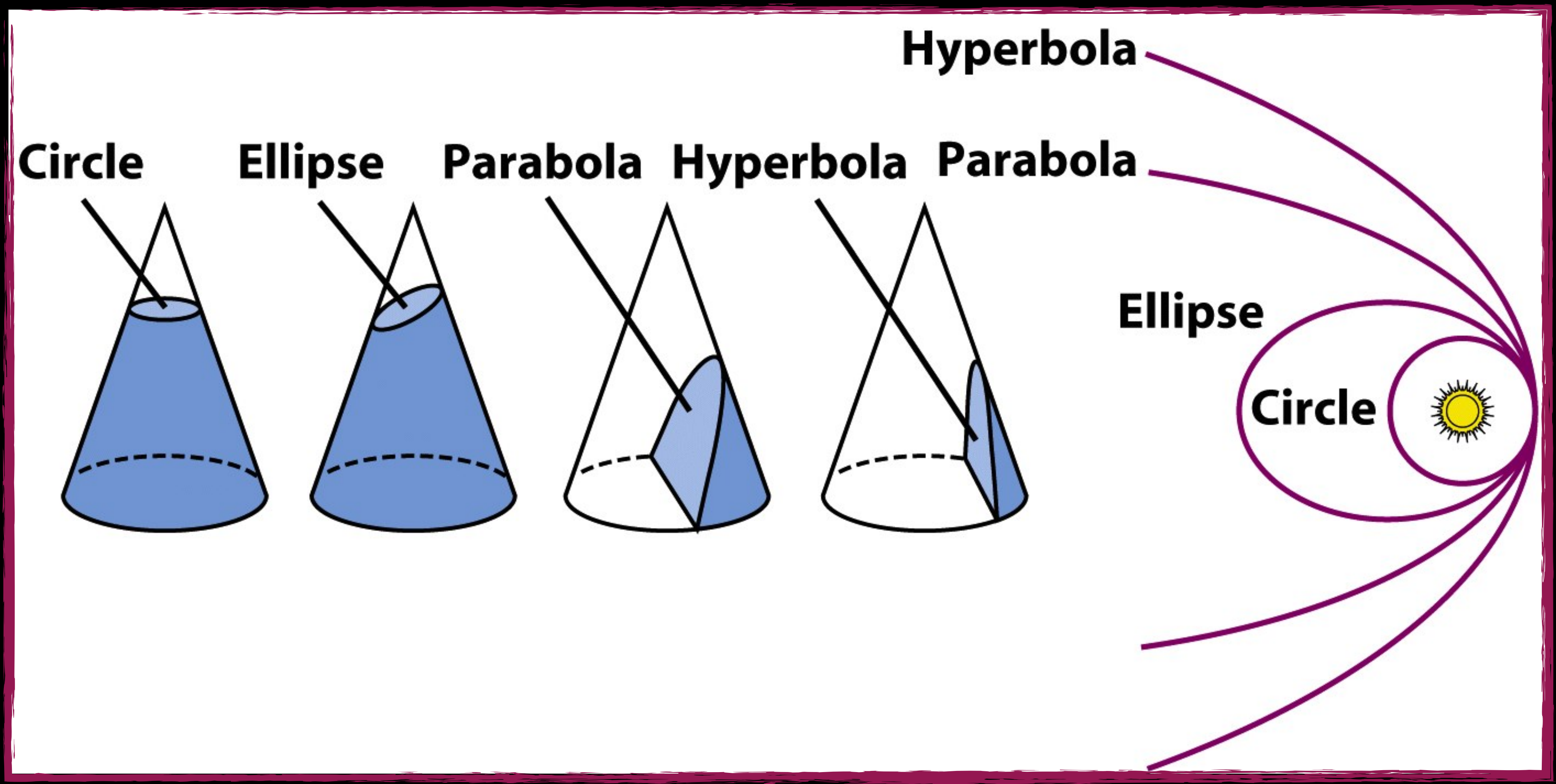

Moon
Earth
Venus
Sun
Mercury


Moon
Earth
Venus
Sun
Mercury

Unbound orbits



- ➢ Newton also predicted unbound orbits

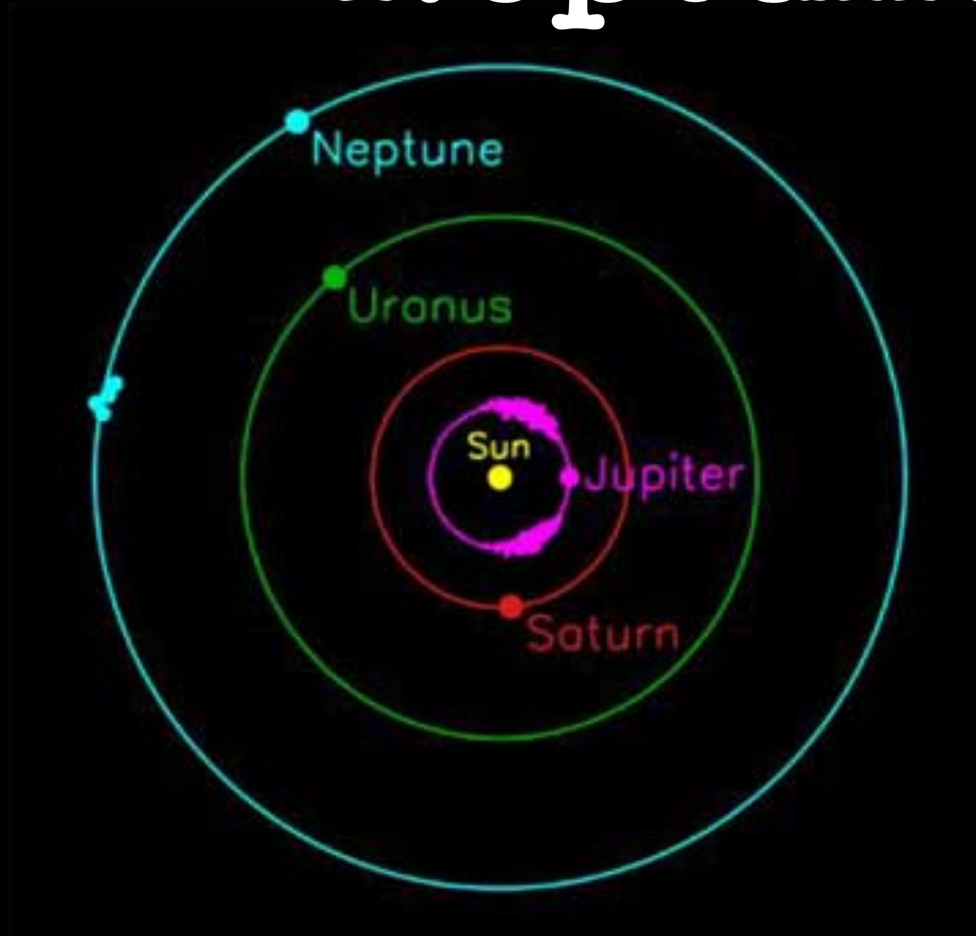
Parabolic and hyperbolic

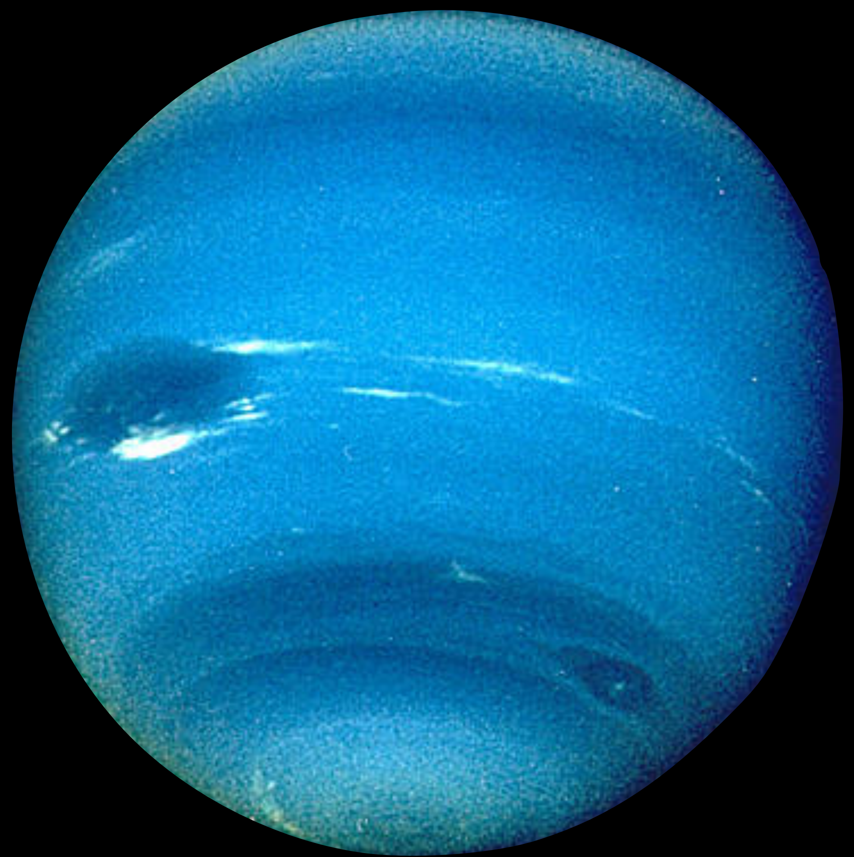
The four types of orbit are described by the conic sections

- ➢ Comets typically have such orbits

an object comes very close to the Sun then travels out of the solar system

Neptune's Discovery





- ➢ **Newton's laws were used to study the gravitational perturbations of the orbit of Uranus**
- ➢ **Adams & Leverrier predicted they were caused by a planet laying outside the orbit of Uranus**

  The planet's actual position was predicted
- ➢ **The planet, Neptune, was subsequently discovered near that position**

  A great testament to the power of Newton's laws and to science

After a cannonball is fired into frictionless space,

the amount of force needed to keep it going equals

A. zero, since no force is necessary to keep it moving

B. twice the force with which it was fired.

C. one half the force with which it was fired.

D. the same amount of force with which it was fired.

E. one quarter the force with which it was fired

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A. The feathers

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C. Neither—they both have the same mass

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A 10-kg brick and a 1-kg book are dropped in a vacuum.

The force of gravity on the 10-kg brick is

A. 10 times as much as the force on the 1-kg book

B. zero

C. the same as the force on the 1-kg book

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A player hits a ball with a bat. The action force is the impact of the bat against the ball. What is the reaction to this force?

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C. Air resistance on the ball

D. The grip of the player's hand against the bat

E. none of the above

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A person is attracted towards the center of Earth by a 440-N gravitational force. The force with which Earth is attracted toward the person is

A. 440N

B. very very small

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The law of inertia applies to

A. objects at rest

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One object has twice as much mass as another object. The first object also has twice as much

A. velocity

B. gravitational acceleration

C. inertia

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How does acceleration of an object change in relation to its mass? It is

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B. Acceleration doesn't depend on mass at all

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A girl pulls on a 10-kg wagon with a constant force of 20 N.

What is the wagon's acceleration?

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B. 2 m/s^2

C. 10m/s^2

D. 20m/s^2

E. 200 m/s^2

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A force of 3N accelerates a mass of 3kg at the rate of 1 m/s^2.

Acceleration of a mass of 6 kg acted upon by a force of 6 N is

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C. the same

D. none of the above

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Suppose a cart is being moved by a force. If suddenly a load is dumped into the cart so that the cart's mass doubles, what happens to the cart's acceleration?

A. It quarters

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C. It stays the same

D. It doubles

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You pull horizontally on a 50-kg crate with a force of 450 N and the friction force on the crate is 250 N.

The acceleration of the crate is

A. 2 m/s^2

B. 4 m/s^2

C. 9 m/s^2

D. 14 m/s^2

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According to Newton's third law, if you push gently on something, it will push

A. gently on something else

B. on you only if you aren't moving

C. gently on you

D. on something only under the right conditions

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A box is dragged without acceleration in a straight-line path across a level surface by a force of 13 N. What is the frictional force between the box and the surface?

A. 13N

B. Less than 13 N

C. More than 13 N

D. Need more information to say

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The planet Neptune was found by studying the deviations in another planet's orbit. Name the other planet:

A. Mars

B. Pluto

C. Uranus

D. Tatooine

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D. Tatoine

In each diagram below, the mass of the star is the same

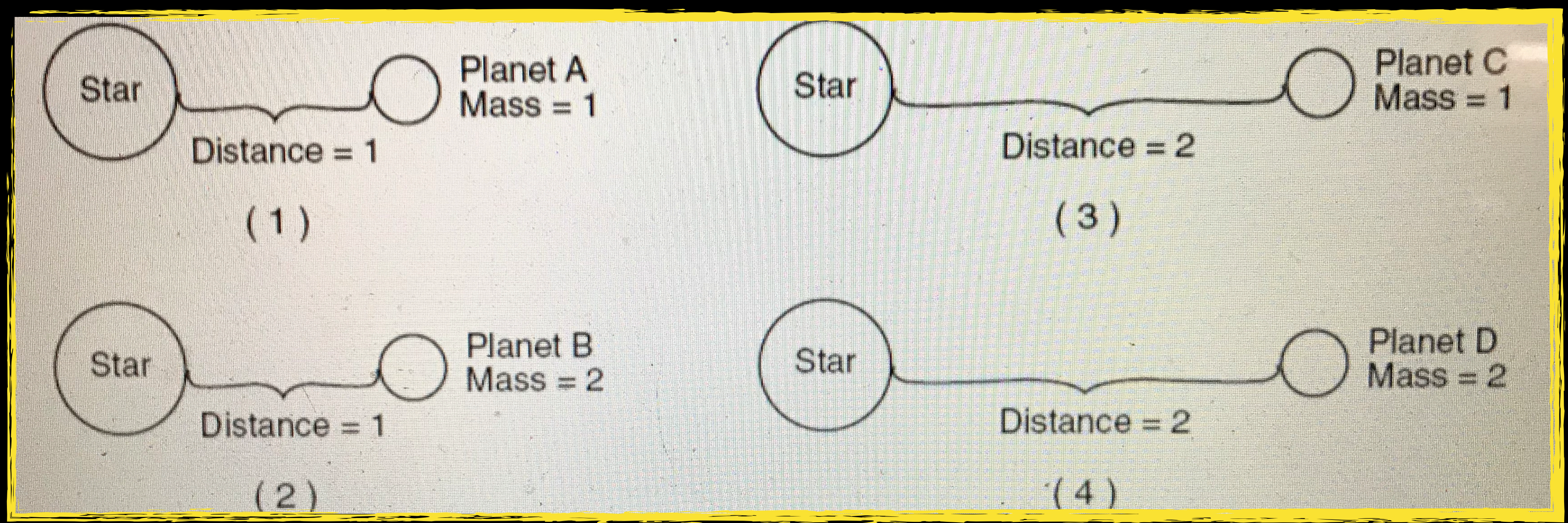
In wich diagram is the force of gravity greatest between the star and the planet shown?

A. (1)

B. (2)

C. (3)

D. (4)



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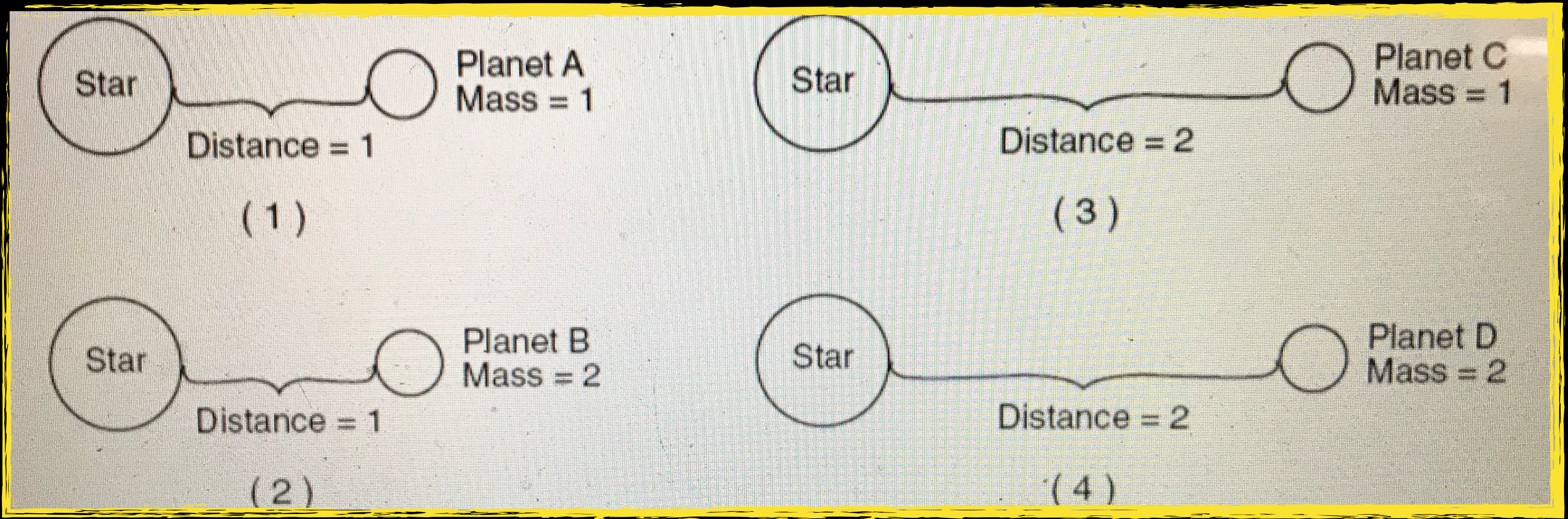

QUERY 5

The mass of Earth is 5.97×10^{24} kg, the mass of the Moon is 7.35×10^{22} kg, and the mean distance of the Moon from the center of Earth is 3.84×10^{5} km.

Use these data to calculate the magnitude of the gravitational force exerted by Earth on the Moon

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What we know ☛ $m_E = 5.97 \times 10^{24}\text{kg}$ $m_M = 7.35 \times 10^{22}\text{kg}$

$r = 3.84 \times 10^5\text{km} = 3.84 \times 10^8\text{m}$ $G = 6.673 \times 10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2$ $F_{ME} = ?$

$$F_{ME} = G\frac{m_E m_M}{r^2}$$

$$F_{ME} = (6.673x10^{-11}\ N\cdot m^2/kg^2)\left[\frac{(5.97x10^{24}\ kg)(7.35x10^{22}\ kg)}{(3.84x10^8\ m)^2}\right]$$

$$F_{ME} = (6.673x10^{-11}\ N\cdot m^2/kg^2)\left(\frac{4.39x10^{47}\ kg^2}{1.47x10^{17}\ m^2}\right)$$

$$F_{ME} = (6.673x10^{-11}\ N\cdot m^2/kg^2)(2.98x10^{30}\ kg^2/m^2)$$

$$\boxed{F_{ME} = 1.99 \times 10^{20}\text{N}}$$

Newton's Law of Gravitation implies Kepler's Third Law for circular orbit

Planet Earth moves with speed v

in approximate circular orbit of radius r about Sun

Gravitational force on Earth by Sun provides centripetal acceleration

$$\frac{GM_{\oplus} M_{\odot}}{r^2} = M_{\oplus} \frac{v^2}{r}$$

$$v^2 = \frac{GM_{\odot}}{r}$$

Because Earth moves a distance $2\pi\ r$ in time T its speed is related to period by

$$v = \frac{2\pi\ r}{T}$$

substituting

$$T^2 = \frac{4\pi^2}{GM_{\odot}}\ r^3$$

QUERY 6

The planet Mercury travels around the Sun with a mean orbital radius of 5.8×10^{10} m. The mass of the Sun is 1.99×10^{30} kg

Use Newton's version of Kepler's third law to determine how long it takes Mercury to orbit the Sun. Give your answer in Earth days.

63 QUERY 6

The planet Mercury travels around the Sun with a mean orbital radius of 5.8×10^{10} m. The mass of the Sun is 1.99×10^{30} kg

Use Newton's version of Kepler's third law to determine how long it takes Mercury to orbit the Sun. Give your answer in Earth days.

What we know ☛ $r_M = 5.810 \times 10^{10}\text{m}$ $m_S = 1.99 \times 10^{30}\text{kg}$ $T_M = ?$

$$T_M{}^2 = \left(\frac{4\pi^2}{Gm_S}\right)r^3$$

$$T_M{}^2 = \left[\frac{39.5}{(6.673x10^{-11}\ N\cdot m^2/kg^2)(1.99x10^{30}\ kg)}\right](5.810x10^{10}\ m)^3$$

$$T_M{}^2 = \left[\frac{39.5}{1.33x10^{20}\ N\cdot m^2/kg}\right](1.96x10^{32}\ m^3)$$

$$T_M{}^2 = (2.96x10^{-19}\ s^2/m^3)(1.96x10^{32}\ m^3)$$

$$T_M{}^2 = 5.82x10^{13}\ s^2$$

$$T_M = \sqrt{5.82x10^{13}\ s^2}$$

$$T_M = 7.63x10^6\ s\left(\frac{1\ hour}{3600\ s}\right)\left(\frac{1\ day}{24\ hours}\right) = \boxed{88.3\text{days}}$$

QUERY 7

The planet Venus orbits the Sun with a mean orbital radius of 1.076×10^{11} m. The mass of the Sun is 1.99×10^{30} kg. Using Newton's version of Kepler's third law, calculate the orbital period of Venus.

QUERY 7

The planet Venus orbits the Sun with a mean orbital radius of 1.076 x 10^{11} m. The mass of the Sun is 1.99 x 10^{30} kg. Using Newton's version of Kepler's third law, calculate the orbital period of Venus.

What we know ☛ $r_V = 1.076 \times 10^{11}$ m $m_S = 1.99 \times 10^{31}$ kg $T_V = ?$

$$T_V{}^2 = \left(\frac{4\pi^2}{Gm_S}\right) r_V{}^3$$

$$T_V{}^2 = \left[\frac{4\pi^2}{(6.673x10^{-11}\ N \cdot m^2/kg^2)(1.99x10^{30}\ kg)}\right](1.076x10^{11}\ m)^3$$

$$T_V{}^2 = \left[\frac{39.5}{1.33x10^{20}\ N \cdot m^2/kg}\right](1.25x10^{33}\ m^3)$$

$$T_V{}^2 = (2.97x10^{-19}\ s^2/m^3)(1.25x10^{33}\ m^3)$$

$$T_V{}^2 = 3.17x10^{14}\ s^2$$

$$T_V = \sqrt{3.17x10^{14}\ s^2}$$

$$T_V = 1.93 \times 10^7 \text{ s}$$