Saline (Density) Oscillator

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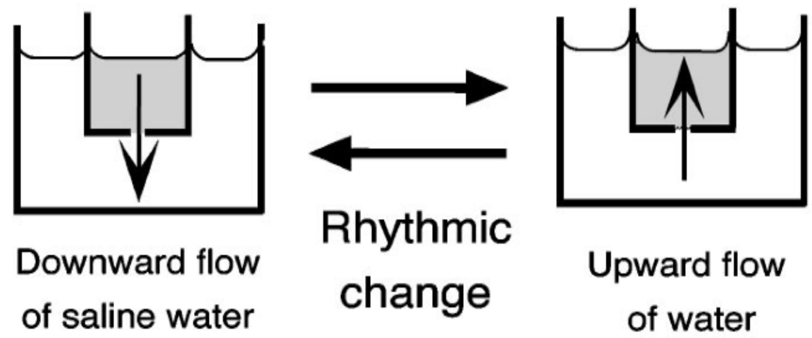
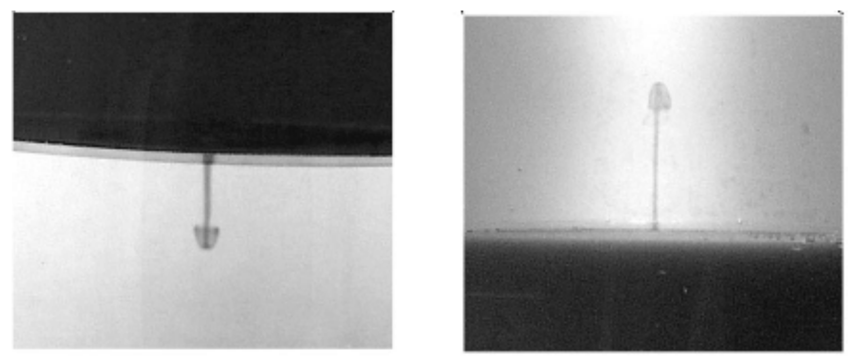
Saline Oscillator:

- A “hydrodynamic curiosity” first described by Seelye Martin in 1970.[1]
- S. Martin observed that a partially submerged syringe of salt water in fresh water exhibits oscillations.
  - downward jet of salt water followed by an upward jet of fresh water
- Oscillations were discovered by accident while setting up a demonstration of a buoyant jet for a class in meterology.[2]





Borrowed from M. Okamura and K. Yoshikawa, Phys. Rev. E. 61, 2445 (2000).

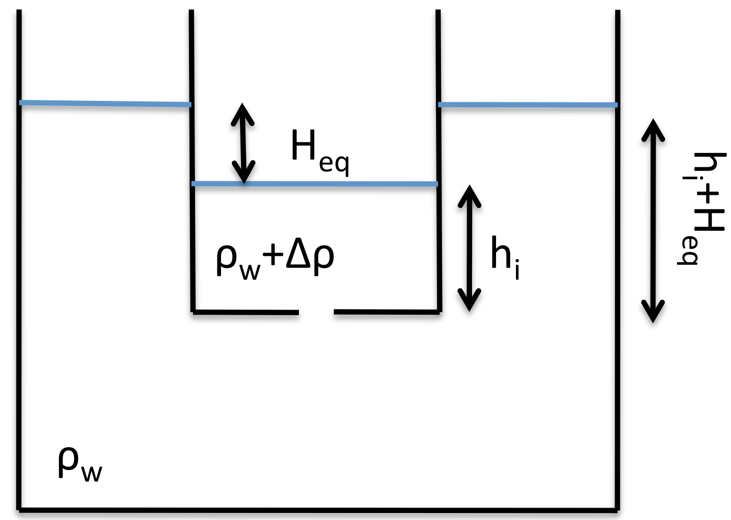
[1] Martin S., 1970, *A hydrodynamic curiosity: the salt oscillator.* Geophys. Fluid Dynamics. 1;143.

[2] Stong, C. L., 1970, *The amateur scientist*. Scientific American. 223; 221.

Physical basis for the oscillations:

- Higher density fluid (saline) lies above lower density fluid (water) with restricted access between the two fluids.
- Gravitational **instability** generates oscillations about equilibrium height (H_{eq}).

Hydrostatic Pressure: $P = \rho g h$

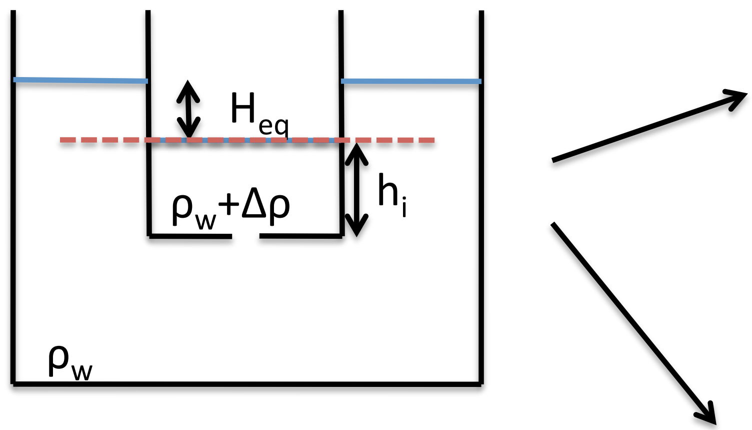


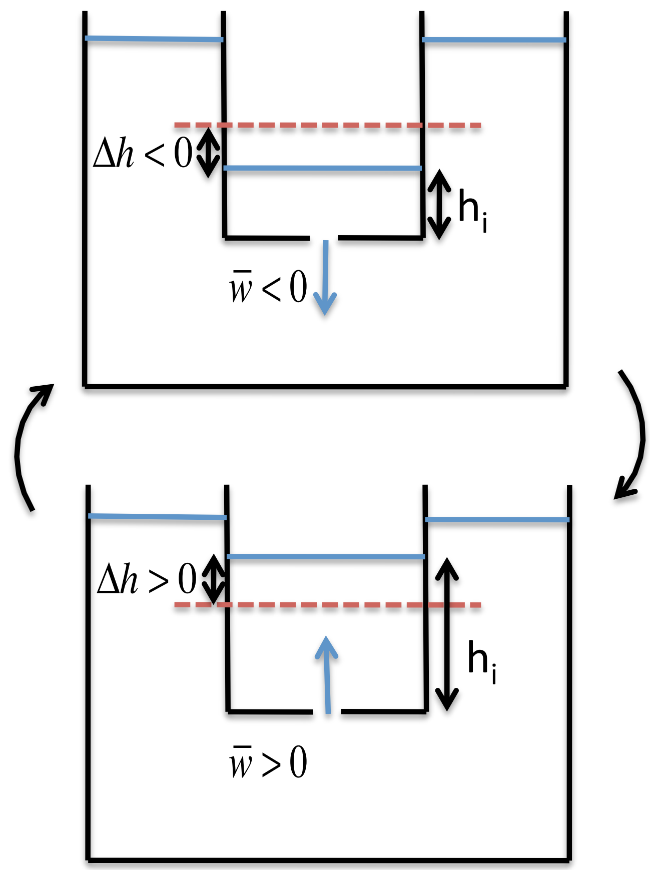
Balance hydrostatic pressures to derive equilibrium height (H_{eq}):

$$(\rho_w+\Delta\rho)\, g h_i = \rho_w g(h_i+H_{eq})$$
$$\Delta\rho/\rho_w=H_{eq}/h_i$$

Physical basis for the oscillations:

1) Equilibrium height (H_{eq}) is unstable:



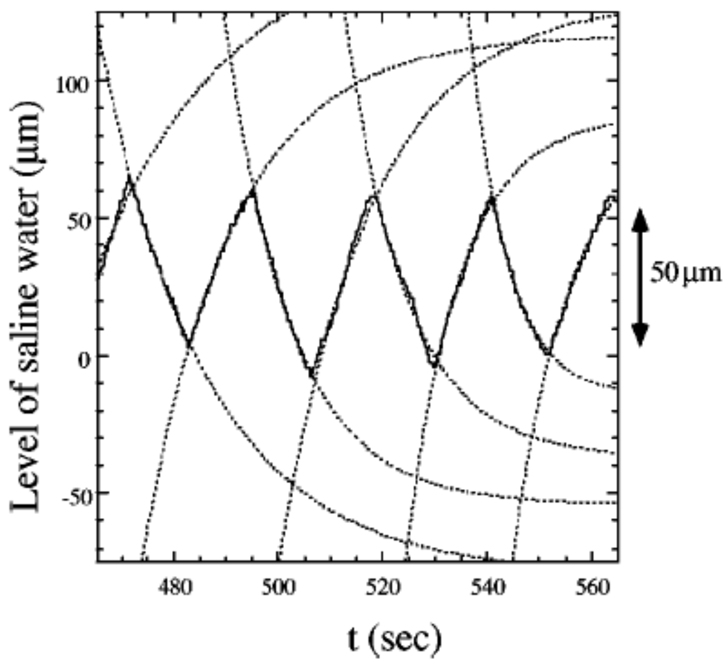


2) Flow will occur through orifice with space-average velocity, $\bar{w}(t)$

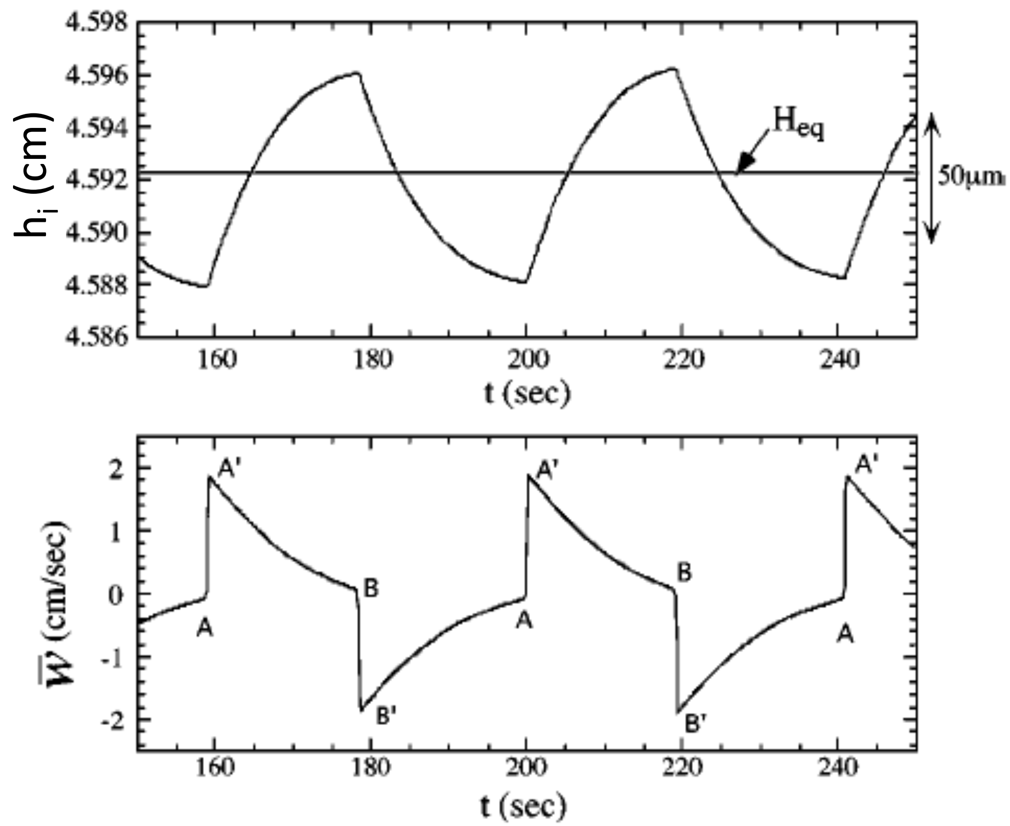
   – Height of saline water inside inner vessel will deviate periodically from equilibrium height ($\Delta h(t) = h_i(t) - H_{eq}$) leading to fluctuations in hydrostatic pressure at orifice.

Height and Velocity Profiles

Experimental:



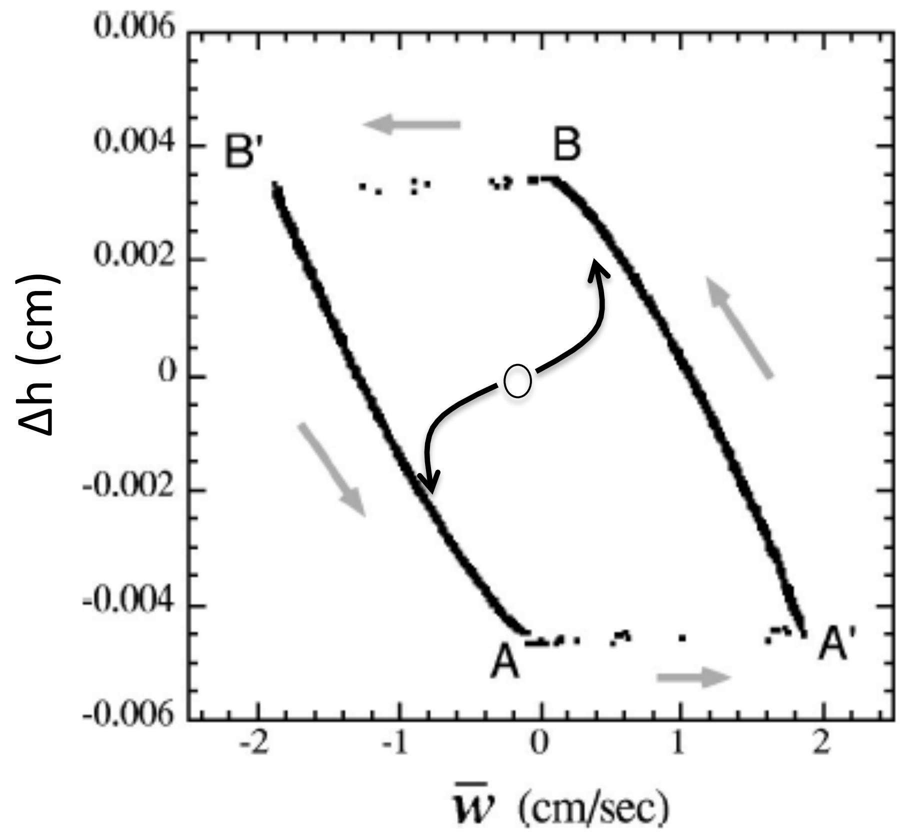
Simulation (Navier-Stokes eqs.):



Okamura, M, and K Yoshikawa. "Rhythm in a saline oscillator." *Physical Review E* (APS) 61, no. 3 (2000): 2445-2452.

Phase space trajectories

- **Trajectories** ("flow") can be visualized in **phase space**.
- Equilibrium height is an **unstable fixed point** (i.e., an unstable point of no flow).
- Trajectories ($\Delta h = h_i - H_{eq}$, $\bar{w}$) approach **stable limit cycle** (isolated closed trajectory).
- Example of a **relaxation oscillator**
  - hydrostatic pressure difference at orifice as a result of height change increases **slowly** (B' -> A, for downward flow, A'->B for upward flow)
  - This slow buildup is discharged **fast** (Transition to upward flow, A->A', or transition to downward flow, B->B')



Relaxation oscillations:

- Dynamics similar to that of other **relaxation oscillators** such as those governing **action potentials**
  - two distinct phases: fast release phase and slow recovery (**relaxing**) phase.
- Purely **nonlinear** phenomenon
  - limit cycle can't occur for linear phenomenon
- Oscillations are governed by the **structure** of the system
  - e.g. period of oscillation is intrinsic to the structure of the system and **independent** of initial conditions

Some exercises:

- Measure intrinsic period of oscillation for the setups in the following stations (try to explain the trends you observe using your physical intuition).
  - Station 1: Orifice diameter
  - Station 2: Length of orifice
  - Station 3: Vessel areas
  - Station 4: Density difference

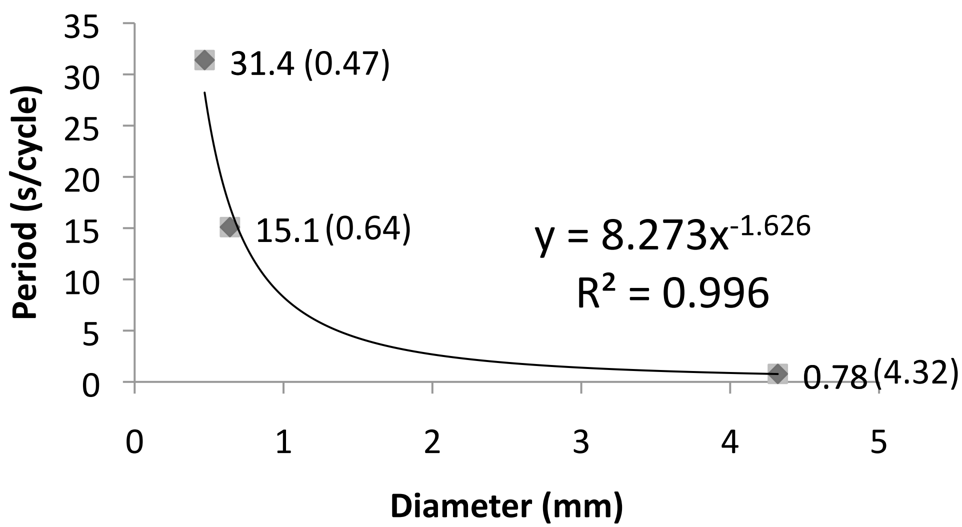
Results:

Station 1: Orifice diameter

↑ Orifice diameter ⟶ ↓ Intrinsic Period

Hagen-Poiseuille equation for fluid flowing through a pipe:

$$Q = \frac{\pi r^4}{8\mu}\frac{\Delta P}{L}$$



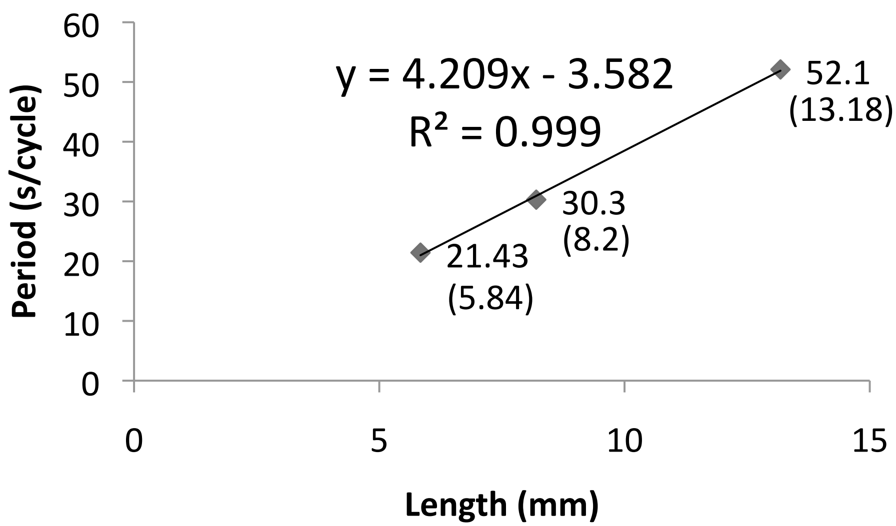
Results:

Station 2: Length of orifice

↑ Length of orifice → ↑ Intrinsic Period

Hagen-Poiseuille equation for fluid flowing through a pipe:

$$Q = \frac{\pi r^4}{8\mu}\frac{\Delta P}{L}$$

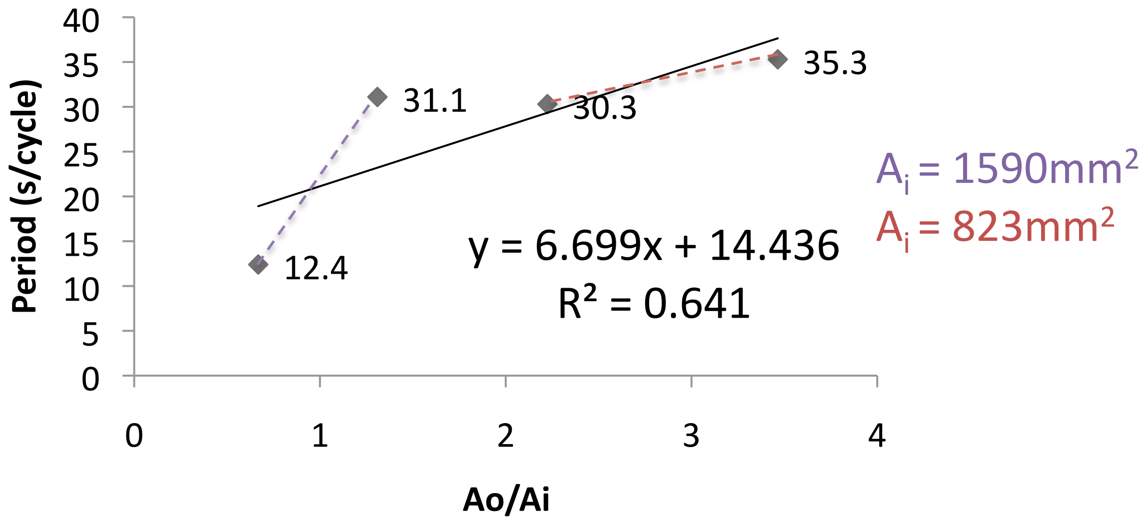


Results:

Station 3: Vessel areas

↑ A_o/A_i ⟶ ↑ Intrinsic Period

$$Q = \frac{dV}{dt} = \frac{d(Ah)}{dt} = A\frac{dh}{dt}$$



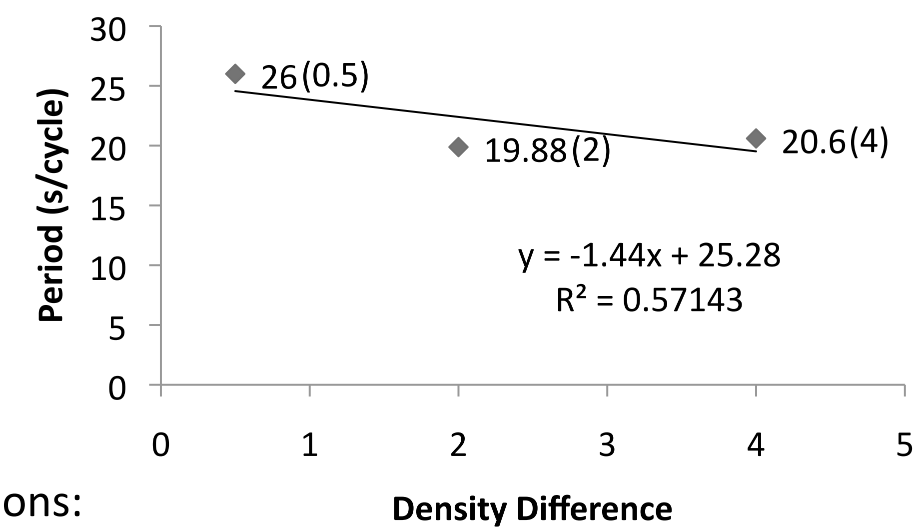
Results:

Station 4: Density difference

↑ Density Difference ⟶ ↓ Intrinsic Period

Reynolds Number:

$$\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{\rho \bar{w} d}{\mu}$$

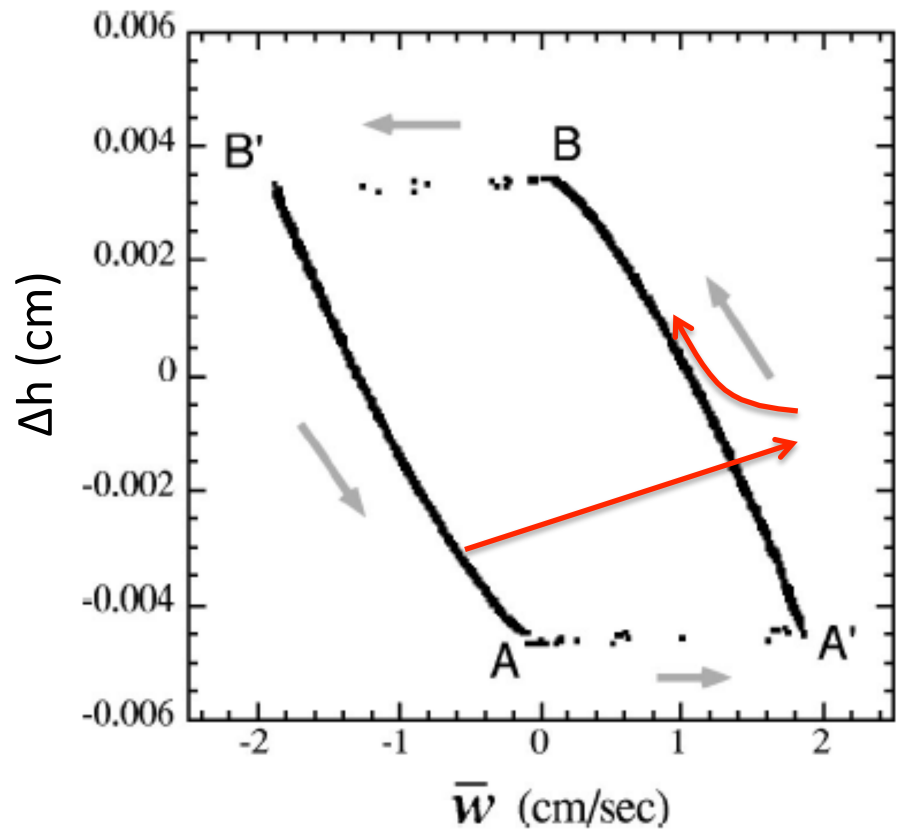


Navier-Stokes Equations:

$$\frac{d\vec{v}}{dt} + \vec{v} \cdot \nabla \vec{v} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \vec{v} + f$$

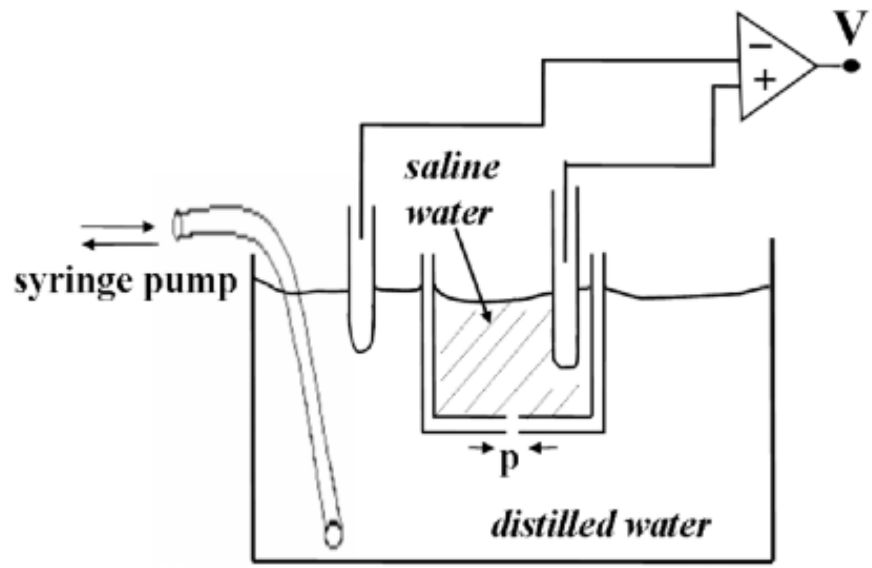
Studying Rhythms in Saline Oscillator

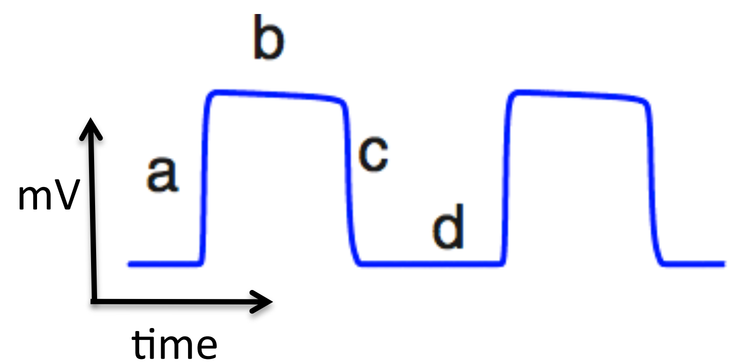
- The system can be stimulated or **paced**
  - we "push" system to different phase in **phase space** and let it return to limit cycle
  - this can be accomplished by infusing some water into outer vessel
  - similar to excitable cells



Studying Rhythms in Saline Oscillator

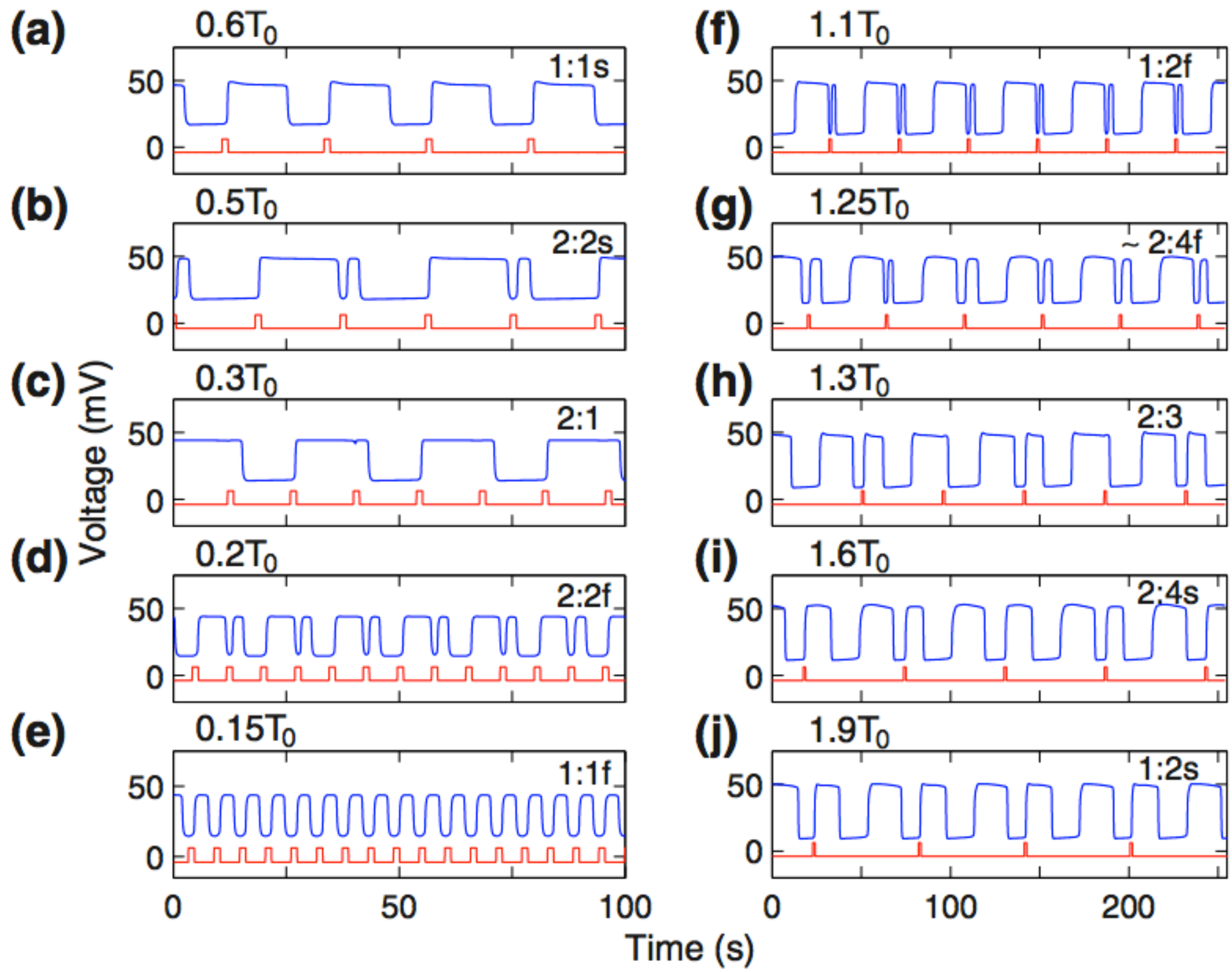
- We can study different **rhythms** of oscillation and their **bifurcations**
  - i.e., we can observe topological changes in the rhythm as we vary the pacing period
- We can record system behavior using oscilloscope because of electrode-electrolyte interface
  - bilayer generates equilibrium potential
  - equilibrium potential is different for different solutions





Various N:M Rhythms

- In a N:M rhythm, there are **N stimuli** and **M responses**, with each **N falling at its own phase** in the cycle
- Pacing at some fraction of **intrinsic period (T_0)**



González, H, H Arce, and MR Guevara. "Phase resetting, phase locking, and bistability in the periodically driven saline oscillator: Experiment and model." *Physical Review E* (APS) 78, no. 3 (2008): 36217.

Different N:M Rhythms

- To = 33s/cycle
- 0.7To = 22.4s/cycle (1:1)
- 0.5To = 16s/cycle (2:2)
- 1.9To = 60.8s/cycle (1:2)
- 0.15To = 4.8s/cycle (?)

Summary

- Saline (density) oscillators exhibit oscillatory jets from a density and hydrostatic pressure imbalance.
  - A higher density fluid suspended above a lower density fluid will exhibit a pattern of upward and downward jets through a restricted channel.
  - Equilibrium height is an unstable fixed point.
- Trajectories ($\Delta h = h_i - H_{eq}, \overline{w}$) exist on a **stable limit cycle** characterized by the structure of the system.
  - e.g., period of oscillation is defined by orifice diameter, orifice length, density difference, vessel areas, etc.
- Density oscillator is an example of a **relaxation oscillator.**
  - Limit cycle has **fast release phase** and **slow recovery phase.**
  - It's a good example of an **excitable system**.
  - Infusing a fixed amount of water can **stimulate** the system into a different state.
- Density oscillators take on various **N:M rhythms** as the period of stimulation is varied**.**
  - The transitioning point (in terms of period of stimulation) between two rhythms defines the point of **bifurcation** in the system dynamics.
  - You will see this again for other excitable systems such as neurons and cardiac cells.