CMACS

Computational Modeling and Analysis for Complex Systems

Performance Analysis in Parallel Programming

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January 13, 2011

Today’s agenda

- Serial Program Performance
  - An example in the cable
  - I/O statements
  - Taking times
- Parallel Program Performance Analysis
  - The cost of communication
  - Amdahl's Law
  - Work and Overhead
  - Sources of Overhead
  - Scalability

Performance of a Serial Program

Definition of Running Time:

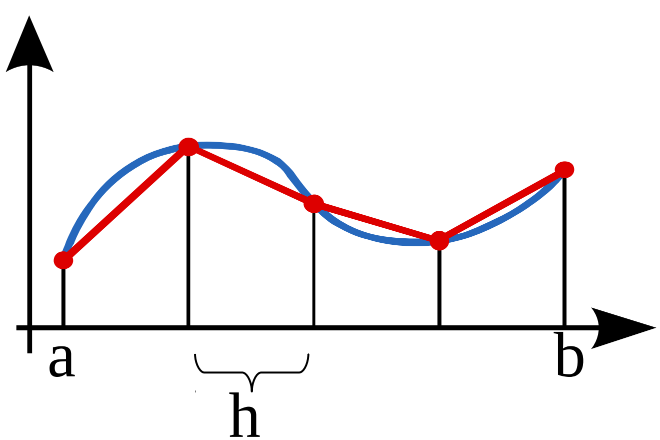
Let be T(n) units the running time of a program.

The actual time depends on:

- the hardware being used
- the programming language and compiler
- details of the input other than its size

Example of Trapezoidal Rule:

```
h = (b-a)/n;
integral = (f(a)+f(b))/2.0;          } c1
x = a;
for (i=1; i <=n-1; i++){
        x = x + h;
        integral = integral + f(x);  } c2
}
integral = integral * h;             } c1
```



$$T(n) = c_1 + c_2(n-1)$$

$T(n) \approx k_1 n + k_2$ linear polynomial in n

$$k_1 n \gg k_2 \longrightarrow T(n) \approx k_1 n$$

$$\int_a^b f(x)\,dx \approx \frac{b-a}{N}\left[\frac{f(a)+f(b)}{2} + \sum_{k=1}^{N-1} f\left(a + k\frac{b-a}{N}\right)\right]$$

What about the I/O ?

In 33 Mhz 486 PC (linux) and gcc compiler

A multiplication takes a microsecond
printf about 300 microseconds

On faster systems, the ratios may be worse,
with the arithmetic time decreasing
I/O times remaining the same

Estimation of the time

$$T(n) = T_{calc}(n) + T_{i/o}(n)$$

Parallel Program Performance Analysis

$T_\pi(n,p)$ denotes the runtime of the parallel solution with p processes.

Speedup of a parallel program:

$$S_\pi(n,p)=\frac{T_\sigma(n)}{T_\pi(n,p)}$$

Ambiguous definition

Is $T_\sigma(n)$ the fastest known serial program ?

or $T_\sigma(n)=T_\pi(n,1)$

$0<S(n,p)\leq p$

$S(n,p)=p$ **linear speedup** (very rare)

Efficiency of a parallel program:

$$E_\pi(n,p)=\frac{S(n,p)}{p}=\frac{T_\sigma(n)}{pT_\pi(n,p)}$$

Efficiency is a measure of process utilitization in a parallel program, relative to the serial program.

$0<E(n,p)\leq 1$

$E(n,p)=1$ **linear speedup** (very rare)

$E(n,p)<1/p$ **slowdown**

The Cost of Communication

For a reasonable estimatation of the performance of a parallel program, we should count also the cost of communication.

$$T(n,p)=T_{calc}(n,p)+T_{i/o}(n,p)+T_{comm}(n,p)$$

While the cost of sending a single message containig k units of data will be:

$$t_s+k\,t_c$$

t_s is called message **latency**

$\frac{1}{t_c}$ is called **bandwidth**

The Example: The Parallel Trapezoidal Rule

Serial:

```
h = (b-a)/n;
integral = (f(a)+f(b))/2.0;          C1
x = a;
for (i=1; i <=n-1; i++){
          x = x + h;
          integral = integral + f(x);  C2
}
integral = integral * h;             C1
```

Serial:

```
h = (b-a)/n;
integral = (f(a)+f(b))/2.0;          C1
x = a;
for (i=1; i <=n-1; i++){
          x = x + h;
          integral = integral + f(x);  C2
}
integral = integral * h;             C1
```

Taking Timings

Amdahl's Law

$0 \le r \le 1$ is the fraction of the program that is perfectly parallelizable

$$S(p) = \frac{T_\sigma}{(1-r)T_\sigma + rT_\sigma / p} = \frac{1}{(1-r) + r/p}$$

$$\frac{dS}{dp} = \frac{r}{[(1-r)p + r]^2} \ge 0$$

$$\lim_{p \to \infty} S(p) = \frac{1}{1-r}$$

Example: if r =0.5 the maximum speedup is 2

Work and Overhead

The amount of work done by a serial program is simply the runtime:

$$W_\sigma(n)=T_\sigma(n)$$

The amount of work done by a parallel program is the sum of the Amounts of work done by each process:

$$W_\pi(n,p)=\sum_{q=0}^{p-1} W_q(n,p)$$

Thus, an alternative definition of efficiency is:

$$E(n,p)=\frac{T_\sigma(n)}{pT_\pi(n,p)}=\frac{W_\sigma(n)}{W_\pi(n,p)}$$

Work and Overhead

Overhead is the amount of work done by the parallel program that is not done by the serial program:

$$T_o(n,p) = W_\pi(n,p) - W_o(n) = pT_\pi(n,p) - T_\sigma(n)$$

per-process overhead: difference between the parallel runtime and the ideal parallel runtime that would be obtained with linear speedup:

$$T'_o(n,p) = T_\pi(n,p) - T_\sigma(n)/p$$
$$T_o(n,p) = pT'_o(n,p)$$

Main sources of Overhead: communication, idle time and extra computation

Scalability